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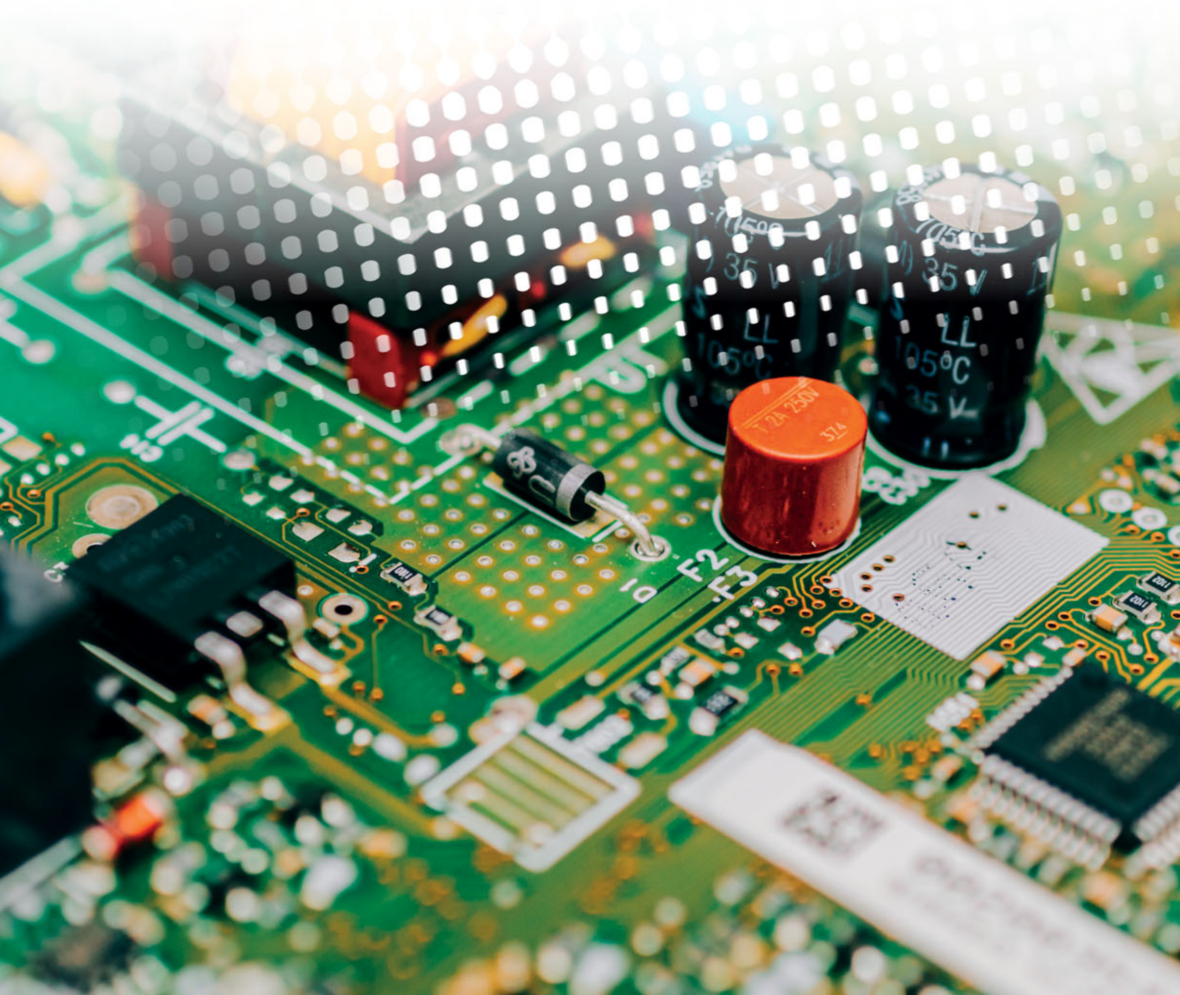
Engineering and Technology

Understandable Electric Circuits

Key concepts

2nd Edition

Meizhong Wang



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About the author

Meizhong Wang has been an instructor at the College of New Caledonia (CNC) in Canada for 29 years. She currently teaches mathematical and computing courses and has lectured in electric circuits, electronics, physics, etc. at the CNC and other college and university in Canada and China.

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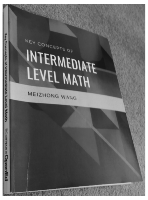
- *Algebra I* and *II – Key Concepts, Practice, and Quizzes* (The Critical Thinking Co.—U.S., 2013).
- *Math Made Easy* (CNC Press, Canada, 2011, second edition 2013).
- *Understandable Electric Circuits* (Michael Faraday House of the IET—Institution of Engineering and Technology—U.K., 2010).
- *Legends of Four Chinese Sages*—coauthor (Lily S.S.C Literary Ltd.—Canada, 2007).
- *简明电路基础*, Chinese version of *Understandable Electric Circuits* (The Higher Education Press—China, 2005).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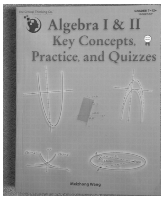
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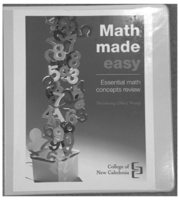
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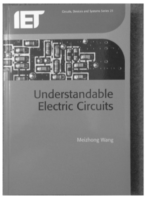
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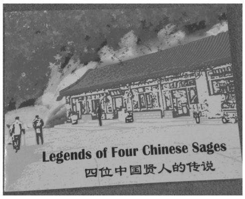
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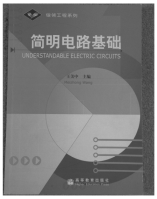
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Preface

There are many "Electric Circuits" books on the market, but this unique "Understandable Electric Circuits" provides understandable and effective introduction to the fundamentals of DC/AC circuits.

The English version of this book continues in the spirit of its successful Chinese version, which was published by Higher Education Press (one of the largest and most prominent publishers of educational books in China) in 2005, and reprinted in 2009. The second edition provides an extensive revision of the chapters in the first edition and an enlargement through the addition of three new chapters.

The new edition of the book provides updated insights on circuit analysis theory in a manner that will be more engaging to readers. It contains a design and page layout that will enhance visual interest and is a clear source of information on a complex topic.

Although the core material has not changed much, it has been extensively revised and expanded for this new edition to provide a clear source of information on this complex topic and a more concise study guide than the original edition. Each topic, key term, concept, law, etc. has a clear definition followed by examples in each section, making studying and reviewing more effective. Materials are presented visually with less text and more outlines so that the readers can quickly get to the heart of each topic.

This unique and well-structured book provides *understandable* and effective introduction to the fundamentals of DC/AC circuits, including current, voltage, power, resistor, capacitor, inductor, impedance, admittance, dependent/independent sources, basic circuit laws/rules (Ohm's Law, KVL/KCL, voltage/current divider rules), series/parallel and wye/delta circuits, methods of DC/AC analysis (branch current and mesh/node analysis), the network theorems (superposition, Thevenin's/Norton's theorems, maximum power transfer, Millman's and substitution theorems), transient analysis, RLC circuits and resonance, mutual inductance/transformers, etc.

The new edition includes challenging practice problems at the end of each chapter, and three new chapters on quantities and units, magnetism and electromagnetism, and three-phase systems.

Key features

As an aid to readers, the book provides some noteworthy features:

- A concise study guide, quickly getting to the heart of each topic, helping readers with a quick review.
- Each topic, concept, term, and phrase has a clear definition followed by examples in each section.

- Clear and easy-to-understand written format and style. Materials are presented in visual and grayscale format with less text and more outlines, boxes, etc.; clearly presenting information and making studying/reviewing more effective.
- Key terms, properties, phrases, concepts, formulas, etc. are easily located. Clear step-by-step procedures for applying theorems.
- Summary at the end of each chapter to emphasize the key points and formulas in the chapter.

Experiments after each chapter in the original edition have been replaced with practice problems, which will help students focus on the key principles, complete the connection between theory and practice, and assist readers in the learning process.

Key concepts have been explained clearly by detailed, worked examples in chapters and readers will be consistently made to apply and practice these theories in practice problems throughout the book. Practice problems allow readers to work similar problems and check their results against the odd-numbered answers provided at the end of book, and thus, provide support for readers to complete the connection between theory and practice.

Therefore, although the essential contents presented in the second edition of the book are the same as that in the first edition, the second edition contains some additions and enhancements that will ensure its applicability to readers today and for many years to come.

Suitable readers

This book is intended for college/university students, technicians, technologists, engineers, or any other professionals who require a solid foundation in the basics of electric circuits.

It targets an audience of all sectors in the fields of electrical, electronic, and computer engineering such as electrical, electronics, computer, communications, control and automation, embedded systems, signal processing, power electronics, industrial instrumentation, power systems (including renewable energy), electrical apparatus and machines, nanotechnology, biomedical imaging, information technology, artificial intelligence, and so on. It is also suitable to nonelectrical or electronics students. It provides readers with the necessary foundation for DC/AC circuits in related fields.

To make this book more reader-friendly, the concepts, new terms, laws/rules and theorems are explained in an easy-to-understand style. Clear step-by-step procedures for applying methods of DC/AC analysis and network theorems make this book easy for readers to learn electric circuits themselves.

Acknowledgments

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Chapter R

Quantities and units

Chapter outline

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Chapter R is a review of basic math fundamentals. There is a self-test at the end of the chapter that can test readers' understanding of the material. Students can take the self-test before beginning the chapter to determine how much they know about the topic. Those who do well may decide to move on to the next chapter without reading the lesson.

R.1 International system of units (SI)

R.1.1 SI units and circuit quantities

Metric system (SI – International System of Units): SI system is the world's most widely used system of measurement. It is based on the basic units of meter, kilogram, second, etc.

- SI originates from the French '*Le **S**ystème **I**nternational d'Unités*', which means the International System of Units or the metric system to most people.
- Each physical quantity has an SI unit. There are seven basic units of the SI system and they are listed in Table R.1.

Table R.1 SI base units

Quantity	Quantity symbol	Unit	Unit symbol
Length	l	Meter	m
Mass	M	Kilogram	kg
Time	t	Second	s
Electric current	I	Ampere	A
Temperature	T	Kelvin	K
Amount of substance	m	Mole	mol
Intensity of light	I	Candela	cd

Table R.2 Some circuit quantities and their SI units

Quantity	Quantity symbol	Unit	Unit symbol
Voltage	V	Volt	V
Resistance	R	Ohm	Ω
Charge	Q	Coulomb	C
Power	P	Watt	W
Energy	W	Joule	J
Electromotive force	E or V_S	Volt	V
Conductance	G	Siemens	S
Resistivity	ρ	Ohm · meter	Ω · m
… …			

SI Units	– International System of Units (SI) is the world's most widely used system of measurement. – There are seven base units of the SI system: m, kg, s, A, K, mol, and cd.

Derived quantities: All other metric units can be derived from the seven SI basic units that are called "derived quantities." Some derived SI Units for circuit quantities are given in Table R.2.

R.1.2 Metric prefixes (SI prefixes)

Metric prefixes (SI prefixes)

- Sometimes, we come across very large or small numbers when doing circuit analysis and calculation. A metric prefix (or SI prefix) is often used in the circuit calculation to reduce the number of zeroes.
- Large and small numbers are made by adding SI prefixes. A metric prefix is a modifier on the root unit that is in multiples of 10.
- In general science, the most common metric prefixes such as milli, centi, and kilo are used. In circuit analysis, more metric prefixes such as nano and pico are used. Table R.3 contains a complete list of metric prefixes.

Table R.3 Metric prefix table (the most commonly used prefixes are shown in bold.)

Prefix	**Symbol (abbreviation)**	**Exponential (power of 10)**	**Multiple value (in full)**
yotta	Y	10^{24}	1,000,000,000,000,000,000,000,000
zetta	Z	10^{21}	1,000,000,000,000,000,000,000
exa	E	10^{18}	1,000,000,000,000,000,000
peta	P	10^{15}	1,000,000,000,000,000
tera	**T**	**10^{12}**	**1,000,000,000,000**
giga	**G**	**10^{9}**	**1,000,000,000**
mega	**M**	**10^{6}**	**1,000,000**
myria	my	10^{4}	10,000
kilo	**k**	**10^{3}**	**1,000**
hecto	h	10^{2}	100
deka	da	10	10
deci	d	10^{-1}	0.1
centi	**c**	**10^{-2}**	**0.01**
milli	**m**	**10^{-3}**	**0.001**
micro	**μ**	**10^{-6}**	**0.000 001**
nano	**n**	**10^{-9}**	**0.000 000 001**
pico	**p**	**10^{-12}**	**0.000 000 000 001**
femto	f	10^{-15}	0.000 000 000 000 001
atto	a	10^{-18}	0.000 000 000 000 000 001
zepto	z	10^{-21}	0.000 000 000 000 000 000 001
yocto	y	10^{-24}	0.000 000 000 000 000 000 000 001

Note: μ is a Greek letter called "mu" (see "Appendix A" for a list of Greek letters).

R.1.3 Metric conversion

Metric conversion table

Power of 10	10^3	10^2	10^1	1	·	10^{-1}	10^{-2}	10^{-3}
Prefix	kilo	hecto	deka	Example: meter, ampere, volt, etc.	·	deci	centi	milli
Symbol	k	h	da		·	d	c	m

Larger ______________________________ Smaller

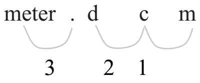
Steps for metric conversion through decimal movement

- Identify the number of places to move on the metric conversion table.
- Move the decimal point.
  - Convert a *smaller* unit to a *larger* unit: move the decimal point to the *left*.
  - Convert a *larger* unit to a *smaller* unit: move the decimal point to the *right*.

Example R.1: 326 mm = (?) m

- Identify mm (millimeters) and m (meters) on the conversion table.

  Count places from mm to m: 3 places



- Move three decimal places. (1 m = 1,000 mm)

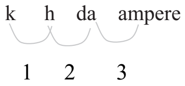
  Convert a smaller unit (mm) to a larger (m) unit: move the decimal point to the left.

  $\underbrace{326}.\text{mm} = \boxed{0.326\text{ m}}$ Move the decimal point three places to the left (326 = 326.).

Example R.2: 4.675 kA = (?) A

- Identify kA (kilo amperes) and A (amperes) on the conversion table.

  Count places from kA to A: three places



- Move three decimal places. (1 kA = 1,000 A)

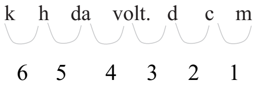
  Convert a larger unit (kA) to a smaller (A) unit: move the decimal point to the right.

  $4.\underbrace{765}\text{ kA} = \boxed{4{,}765\text{ A}}$ Move the decimal point three places to the right.

Example R.3: 30.5 mV = (?) kV

- Identify mV (millivolts) and km (kilometers) on the conversion table.

  Count places from mV to kV: six places



- Move six decimal places. (1 kV = 1,000,000 mV)

  Convert a smaller unit (mV) to a larger (kV) unit: move the decimal point to the left.

  $30.5\text{ mV} = \boxed{0.\underbrace{000030}5\text{ kV}}$ Move the decimal point six places to the left (add 0s).

R.1.4 The unit factor method

Convert units using the unit factor method (or the factor-label method)

- Write the original term as a fraction (over 1). Example: 10 g can be written as $\frac{10\text{ g}}{1}$
- Write the conversion formula as a fraction $\frac{1}{(\ \ )}$ or $\frac{(\ \ )}{1}$.

Example: 1 m = 100 cm can be written as $\frac{1\text{ m}}{(100\text{ cm})}$ or $\frac{(100\text{ cm})}{1\text{ m}}$

(Put the desired or unknown unit on the top.)

- Multiply the original term by $\frac{1}{(\ \ )}$ or $\frac{(\ \ )}{1}$. (Cancel out the same units.)

Metric conversion using the unit factor method:

Example R.4: 1,200 V = (?) kV

- Write the original term (the left side) as a fraction: $1{,}200\text{ V} = \frac{1{,}200\text{ V}}{1}$
- Write the conversion formula as a fraction. $1\text{ kV} = 1{,}000\text{ V}$: $\frac{1\text{ kV}}{(1{,}000\text{ V})}$

  "kV" is the desired unit.
- Multiply: $$1{,}200\text{ V} = \frac{1{,}200\,\cancel{\text{V}}}{1}\cdot\frac{1\text{ kV}}{(1{,}000\,\cancel{\text{V}})}$$ The units "V" cancel out. $$= \frac{1{,}200\text{ kV}}{1{,}000}$$ $$= \boxed{1.2\text{ kV}}$$

Example R.5: 30 cm = (?) mm

- Write the original term (the left side) as a fraction: $30\text{ cm} = \frac{30\text{ cm}}{1}$
- Write the conversion formula as a fraction. $1\text{ cm} = 10\text{ mm}$: $\frac{(10\text{ mm})}{1\text{ cm}}$

  "mm" is the desired unit.
- Multiply: $$30\text{ cm} = \frac{30\,\cancel{\text{cm}}}{1}\cdot\frac{10\text{ mm}}{(1\,\cancel{\text{cm}})}$$ The units "cm" cancel out. $$= \frac{(30)(10)\text{ mm}}{1}$$ $$= \boxed{300\text{ mm}}$$

Adding and subtracting SI measurements:

Example R.6:

$$\begin{array}{r} 3\ \text{A} \\ \underline{-\ 2{,}000\ \text{mA}} \end{array} \Longrightarrow \begin{array}{r} 3{,}000\ \text{mA} \\ \underline{-2{,}000\ \text{mA}} \\ \boxed{1{,}000\ \text{mA}} \end{array} \qquad 1\ \text{A} = 1{,}000\ \text{mA}$$

Combine after converting to the same unit.

Example R.7:

$$\begin{array}{r} 25\ \text{kW} \\ \underline{+4\ \text{W}} \end{array} \Longrightarrow \begin{array}{r} 25{,}000\ \text{W} \\ \underline{+\qquad 4\ \text{W}} \\ \boxed{25{,}004\ \text{W}} \end{array} \qquad 1\ \text{kW} = 1{,}000\ \text{W}$$

R.2 Scientific notation

R.2.1 Write in scientific notation

Scientific notation is a special way of concisely expressing very *large* and *small* numbers.

Example R.8: $300{,}000{,}000 = 3 \times 10^{8}\ \text{m/s}$ The speed of light.

$0.00000000000000000016 = 1.6 \times 10^{-19}\ \text{C}$ An electron.

Scientific notation

It is a product of a number $\underbrace{\underline{\text{between 1 and 10}}}_{\searrow}$ and $\underbrace{\underline{\text{power of 10}}}_{\swarrow}$.

$$N \times 10^{\pm n}$$

Scientific notation		Example
$N \times 10^{\pm n}$	$1 \leq N < 10$ n-integer	$\underset{\substack{\uparrow \\ \text{Standard form}}}{67504.3} = \underset{\substack{\uparrow \\ \text{Scientific notation}}}{6.75043 \times 10^{4}}$

Scientific vs. non-scientific notation

Scientific notation	**Not scientific notation**		
7.6×10^3	76×10^2	$76 > 10$	76 is not between 1 and 10.
8.2×10^{13}	0.82×10^{14}	$0.82 < 1$	0.82 is not between 1 and 10.
5.37×10^7	53.7×10^6	$53.7 > 10$	53.7 is not between 1 and 10.

Writing a number in scientific notation

Example R.9: **Step**

- Move the decimal point *after* the *first nonzero digit*.

0.0079 37213000.

$n = 3$ $n = 7$

- Determine n (the power of 10) by counting the number of places you moved the decimal.
- If the decimal point is moved to the *right*: $\times 10^{-n}$

$0.0079 = \boxed{7.9 \times 10^{-3}}$

3 places to the right

- If the decimal point is moved to the *left*: $\times 10^{n}$

$37213000. = \boxed{3.7213 \times 10^{7}}$

7 places to the left

Example R.10: Write in scientific notation.

1. $\mathbf{2340000} = 2340000. = \boxed{2.34 \times 10^6}$ 6 places to the left, $\times 10^n$

2. $\mathbf{0.000000439} = \boxed{4.39 \times 10^{-7}}$ 7 places to the right, $\times 10^{-n}$

Example R.11: Write in standard (or ordinary) form.

1. $\mathbf{6.4275 \times 10^4} = \boxed{64{,}275}$ 2. $2.9 \times 10^{-3} = \boxed{0.0029}$

Example R.12: Simplify and write in scientific notation.

1. $(\mathbf{4.9 \times 10^{-3}})(\mathbf{3.82 \times 10^{8}}) = (4.9 \times 3.82)(10^{-3+8})$ Multiply coefficients of $10^{\pm n}$, $a^m a^n = a^{m+n}$

$= (18.718 \times 10^{5})$ $18.718 > 10$, this is not in scientific notation.

$= \boxed{(1.8718 \times 10^{6})}$ $1.8718 < 10$, this is in scientific notation.

2. $\dfrac{(\mathbf{5 \times 10^{5}})(\mathbf{2.3 \times 10^{-2}})}{\mathbf{4.5 \times 10^{7}}} = \dfrac{5 \times 2.3}{4.5} \times \dfrac{(10^{5} \times 10^{-2})}{10^{7}}$ Regroup coefficients of $10^{\pm n}$

$\approx \boxed{2.556 \times 10^{-4}}$ $a^m a^n = a^{m+n}$, $\dfrac{a^m}{a^n} = a^{m-n}$

R.3 Engineering notation

R.3.1 Write in engineering notation

Engineering notation is a version of scientific notation that the power of ten is always a multiple of 3.

Engineering notation

It is a product of a number and power of 10 that are multiples of 3.

(10 to the ±3, ±6, ±9, etc.).

Engineering notation	**Example**
$N \times 10^{\pm 3}$, $N \times 10^{\pm 6}$, $N \times 10^{\pm 9}$ …	$\underset{\text{Standard form}}{\underset{\uparrow}{6750.43}} = \underset{\text{Engineering notation}}{\underset{\uparrow}{6.75043}} \times 10^{3}$

Engineering vs. non-engineering notation

Engineering notation	**Not Engineering notation**
7.6×10^{3}	76×10^{2}
8.2×10^{6}	0.82×10^{5}
5.37×10^{-9}	53.7×10^{-8}

Writing a number in engineering notation

Example R.13: **Step**

- If the decimal point is moved to the *right*: $\times 10^{-n}$ — $00079 = \boxed{7.9 \times 10^{-3}}$

  3 places to the right

- If the decimal point is moved to the *left*: $\times 10^{n}$ — $\mathbf{37}213000. = \boxed{37.213 \times 10^{6}}$

  6 places to the left

Example R.14: Write in engineering notation.

1. $\mathbf{3450000} = 3450000. = \boxed{3.45 \times 10^{6}}$ — 6 places to the left, $\times 10^{n}$

2. $\mathbf{0.0000000324} = \boxed{32.4 \times 10^{-9}}$ — 9 places to the right, $\times 10^{-n}$

Example R.15: Write in standard (or ordinary) form.

1. $\mathbf{5.1437 \times 10^{6}} = \boxed{5143700}$

2. $\mathbf{3.4 \times 10^{-3}} = \boxed{0.0034}$

Example R.16: Simplify and write in engineering notation.

1. $$\left(\mathbf{2.4}\times\mathbf{10^{-3}}\right)\left(\mathbf{7.53}\times\mathbf{10^{8}}\right) = (2.4 \times 7.53)(10^{-3+8})$$

   Multiply coefficients of $10^{\pm n}$, $a^m a^n = a^{m+n}$

   $$= \left(18.072 \times 10^{5}\right)$$

   10^5, this is not in engineering notation.

   $$= \boxed{(1.8072 \times 10^{6})}$$

   This is in engineering notation.

2. $$\frac{(\mathbf{5}\times\mathbf{10^{5}})(\mathbf{2.3}\times\mathbf{10^{-2}})}{\mathbf{4.5}\times\mathbf{10^{7}}} = \frac{5 \times 2.3}{4.5} \times \frac{(10^{5} \times 10^{-2})}{10^{7}}$$

   $$\approx \boxed{2.556 \times 10^{-4}}$$

   $$\approx \boxed{25.56 \times 10^{-3}}$$

   Regroup coefficients of $10^{\pm n}$

   $a^m a^n = a^{m+n}$, $\dfrac{a^m}{a^n} = a^{m-n}$

Summary

Metric system (SI – International System of Units): the most widely used system of measurement in the world. It is based on the basic units of meter, kilogram, second, etc.

Imperial system of units: a system of measurement units originally defined in England, including the foot, pound, quart, ounce, gallon, mile, yard . . .

Metric prefixes (SI prefixes): large and small numbers are made by adding SI prefixes, which is based on multiples of 10.

Steps for metric conversion through decimal movement:

- Identify the number of places to move on the metric conversion table.
- Move the decimal point.
  - Convert a *smaller* unit *to* a *larger* unit: move the decimal point to the *left*.
  - Convert a *larger* unit *to* a *smaller* unit: move the decimal point to the *right*.

Metric conversion table

Value	1,000	100	10	1	.	0.1	0 .01	0.001
Prefix	kilo	hecto	deka	**meter (m)** **gram (g)** **liter (L)**	.	deci	centi	milli
Symbol	k	h	da		.	d	c	m

Larger ______________________________ Smaller

Convert units using the unit factor method (or the factor-label method):

- Write the original term as a fraction (over 1). Example: 10 g can be written as $\frac{10\text{ g}}{1}$
- Write the conversion formula as a fraction $\frac{1}{(\ \ )}$ or $\frac{(\ \ )}{1}$.

  Example: 1 m = 100 cm can be written as $\frac{1\text{ m}}{(100\text{ cm})}$ or $\frac{(100\text{ cm})}{1\text{ m}}$

  (Put the desired or unknown unit on the top.)
- Multiply the original term by $\frac{1}{(\ \ )}$ or $\frac{(\ \ )}{1}$. (Cancel out the same units.)

Scientific notation: a product of a number between 1 and 10 and power of 10.

$N \times 10^{\pm n}$

Scientific notation		**Example**
$N \times 10^{\pm n}$	$1 \leq N < 10$ n-integer	$\underset{\substack{\uparrow \\ \text{Standard form}}}{6750.43} = \underset{\substack{\uparrow \\ \text{Scientific notation}}}{6.75043 \times 10^3}$

Writing a number in scientific notation:

- Move the decimal point *after* the *first nonzero digit*.
- Determine n (the power of 10) by counting the number of places you moved the decimal.
- If the decimal point is moved to the *right*: $\times 10^{-n}$
- If the decimal point is moved to the *left*: $\times 10^{n}$

Engineering notation is a version of scientific notation that the power of ten is always a multiple of 3.

Engineering notation: a product of a number and power of 10 that are multiples of 3.

(10 to the ±3, ±6, ±9, etc.).

Engineering notation	**Example**
$N \times 10^{\pm 3}$, $N \times 10^{\pm 6}$, $N \times 10^{\pm 9}$ …	$6750.43 = 6.75043 \times 10^3$ ↑ Standard form ↑ Engineering notation

Key metric prefix

Prefix	**Symbol (abbreviation)**	**Power of 10**	**Multiple value**	**Example**
tera	T	10^{12}	1,000,000,000,000	1 Tm = 1,000,000,000,000 m
giga	G	10^{9}	1,000,000,000	1 Gm = 1,000,000,000 m
mega	M	10^{6}	1,000,000	1 Mm = 1,000,000 m
kilo	k	10^{3}	1,000	1 km = 1,000 m
hecto	h	10^{2}	100	1 hm = 100 m
deka	da	10^{1}	10	1 dam = 10 m
		1	10^{0}	Example: meter, ampere, volt, etc.
deci	d	10^{-1}	0.1	1 m = 10 dm
centi	c	10^{-2}	0.01	1 m = 100 cm
milli	m	10^{-3}	0.001	1 m = 1,000 mm
micro	μ	10^{-6}	0.000001	1 m = 1,000,000 μm
nano	n	10^{-9}	0.000000001	1 m = 1,000,000,000 nm
pico	p	10^{-12}	0.000000000001	1 m = 1,000,000,000,000 pm

Self-test

R.1

1. Complete the following unit conversion:
- (a) 439 mm = (?) m
- (b) 2.236 hA = (?) A
- (c) 48.3 mV = (?) kV
- (d) 2.5 kW = (?) hW
- (e) 0.89 mV = (?) μV
- (f) 167 W = (?) kW
- (g) 0.00003 A = 30 (?) A

2. Complete the following unit conversion:
- (a) 7,230 V = (?) kV
- (b) 52 cm = (?) mm
- (c) 3.4 dA= (?) A
- (d) 52 dam = (?) cm
- (e) 1,500 kΩ = (?) MΩ
- (f) 0.025 A = (?) mA

3. Combine.
- (a) 7 A – 3,000 mA = (?) mA
- (b) 63 kV + 6 V = (?) V
- (c) 0.72 A + 4.58 A – 10 mA = (?) mA
- (d) 25.3 kΩ + 357 daΩ = (?) kΩ

R.2

4. Express the following numbers in scientific notation:
- (a) 45,600,000
- (b) 0.00000523
- (c) 0.0006
- (d) 932,000
- (e) 23,000
- (f) 0.012

5. Write in standard (or ordinary) form.
- (a) 3.578×10^3
- (b) 4.3×10^{-5}

6. Simplify and write in scientific notation.
- (a) $(5.42 \times 10^{-2})(4.38 \times 10^7)$
- (b) $\dfrac{(5 \times 10^5)(2.4 \times 10^{-3})}{3.2 \times 10^8}$

R.3

7. Express the following numbers in engineering notation:
- (a) 36,700,000
- (b) 0.00000456

8. Write in standard (or ordinary) form.

(a) 7.456×10^{3}
(b) 4.3×10^{-3}

9. Simplify and write in engineering notation.

(a) $(3.65 \times 10^{-2})(4.78 \times 10^{11})$
(b) $\dfrac{(4 \times 10^{5})(3.2 \times 10^{-3})}{5.6 \times 10^{8}}$

10. Express the following numbers in engineering notation:

(a) 6,900 Ω (? kΩ)
(b) 0.00004 A (? μA)
(c) 63,200 V (? kV)
(d) 0.02 A (? mA)

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Chapter 1

Basic concepts of electric circuits

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1.1 Introduction to electric circuits

1.1.1 Why study electric circuits?

- Electrical energy is the great driving force and the supporting pillar for modern industry and civilization.
- Our everyday life would be unthinkable without electricity or the use of electronic products.

- Any complex electrical and electronic device or control system is founded from the basic theory of electric circuits.
- Only when you have grasped and understood the basic concepts and principles of electric circuits can you further study electrical, electronic, and computer engineering and other related areas.
- When you start reading this book, perhaps you have already chosen the electrical or the electronic fields as your professional goal—a wise choice!
- Electrical, electronic, and computer engineering has made and continues to make incredible contributions to most aspects of human society—a truth that cannot be neglected. Moreover, it may have a bigger impact on human civilization in the future.
- Since increase in interest and the rise of computer technology, artificial intelligence, etc., electric circuits are playing an important fundamental role in the digital age.
- Experts forecast that demand for professionals in this field will grow continuously. This is good news for people who have chosen these areas of study.
- Reading this book or other electric circuit book is a first step into the electrical, electronic, and computer world that will introduce you to the foundation of the professions in these areas.

1.1.2 Careers in electrical, electronic, and computer engineering

- Nowadays, electrical, electronic, and computer technology is developing so rapidly that many career options exist for those who have chosen this field.
- As long as you have gained a solid foundation in electric circuits and electronics, the training that most employers provide in their branches will lead you into a brand new professional career very quickly.
- There are many types of jobs for electrical and electronic engineering technology. Only a partial list is as follows:
  - Electrical engineer
  - Electronics engineer
  - Electrical design engineer
  - Control and automation engineer
  - Process and system engineer
  - Instrument engineer
  - Robotics engineer
  - Product engineer
  - Field engineer
  - Reliability engineer
  - Integrated circuits (IC) design engineer
  - Computer engineer
  - Power electronics engineer
  - Electrical and electronics engineering professor/lecturer
  - Designer and technologist
  - Electrical and electronics technician
  - Hydro technician

  - Electrician
  - Equipment maintenance technician
  - Electronic test technician
  - Calibration/lab technician
  - Technical writer for electronic products
  - Electronic repair
  - … …
- Electrical and electronic technicians, technologists, engineers, and experts will be in demand in the future, so you definitely do not want to miss this golden opportunity.

1.1.3 Milestones of electric circuit theory

Many early scientists have made great contributions in developing the theorems of electrical circuits. The laws and physical quantities that they discovered are named after them, and all are important milestones in the field of electric engineering. We list here only the ones that are described in this book.

Milestones of electric circuit theory

- **Coulomb** is the unit of electric charge; it was named in honor of Charles Augustin de Coulomb (1736–1806), a French physicist. Coulomb developed *Coulomb's law,* which is the definition of the electrostatic force of attraction and repulsion, and the principle of charge interactions (attraction or repulsion of positive and negative electric charges).
- **Faraday** is the unit of capacitance; it was named in honor of Michael Faraday (1791–1867), an English physicist and chemist. He discovered that relative motion of the magnetic field and conductor can produce electric current, which we know today as the *Faraday's law* of electromagnetic induction. Faraday also discovered that the electric current originates from the chemical reaction that occurs between two metallic conductors.
- **Ampere** is the unit of electric current; it was named in honor of André-Marie Ampère (1775–1836), a French physicist. He was one of the main discoverers of electromagnetism and is best known for defining a method to measure the flow of current.
- **Ohm** is the unit of resistance; it was named in honor of Georg Simon Ohm (1789–1854), a German physicist. He established the relationship between voltage, current, and resistance, and formulated the most famous electric circuit law—*Ohm's law*.
- **Volt** is the unit of voltage; it was named in honor of Alessandro Volta (1745–1827), an Italian physicist. He constructed the first electric battery that could produce a reliable, steady current.
- **Watt** is the unit of power; it was named in honor of James Watt (1736–1819), a Scottish engineer and an inventor. He made great improvements in the steam engine and made important contributions in the area of magnetic fields.
- **Lenz**'s law was named in honor of Heinrich Friedrich Emil Lenz (1804–1865), a Baltic German physicist. He discovered that the polarity of the induced

current that is produced in the conductor of the magnetic field always resists the change of its induced voltage; this is known as *Lenz's law*.

- **Maxwell** is the unit of magnetic flux; it was named in honor of James Clerk Maxwell (1831–1879), a Scottish physicist and mathematician. The German physicist Wilhelm Eduard Weber (1804–1891) shares the honor with Maxwell (1 Wb $= 10^8$ Mx). Maxwell had established the *Maxwell's equations* that represent perfect ways to state the fundamentals of electricity and magnetism.
- **Hertz** is the unit of frequency; it was named in honor of Heinrich Rudolf Hertz (1857–1894), a German physicist and mathematician. He was the first person to broadcast and receive radio waves. Through the low-frequency microwave experiment, Hertz confirmed Maxwell's electromagnetic theory.
- **Henry** is the unit of inductance; it was named in honor of Joseph Henry (1797–1878), a Scottish-American scientist. He discovered self-induction and mutual inductance.
- **Joule** is the unit of energy; it was named in honor of James Prescott Joule (1818–1889), an English physicist. He made great contributions in discovering the law of the conservation of energy. This law states that energy may transform from one form into another, but is never lost. *Joule's law* was named after him and states that heat will be produced in an electrical conductor.

The majority of the laws and units of measurement stated above will be used in the later chapters of this book. Being familiar with them will be beneficial for further study of electric circuits.

1.2 Electric circuits and schematic diagrams

1.2.1 Basic electric circuits

Electric circuit: It is a closed loop of pathway with electric charges flowing through it.

- It is the sum of all electric components in the closed loop of pathway with flowing electric charges.
- An example of an electric circuit includes resistors, capacitors, inductors, power sources, wires, switches, etc. (These electric components will be explained later.)

A basic electric circuit contains three components: the power supply, the electrical load, and the wires (conductors) (Figure 1.1).

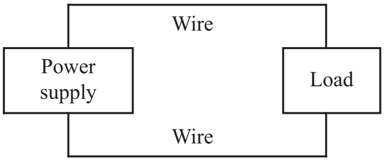


Figure 1.1 Requirements of a basic circuit

Electric circuit	A closed loop of pathway with electric charges or current flowing through it.

Wires connect the power supply and the load, and carry electric charges through the circuit.

A power supply (power source) is a device that supplies electrical energy to the load of the circuit; it can convert other forms of energy to electrical energy. The electric battery and generator are examples of power supply.

- The battery converts chemical energy into electrical energy.
- The hydroelectric generator converts hydro energy (the energy of moving water) into electrical energy.
- The thermo power generator converts heat energy into electrical energy.
- The nuclear power generator converts nuclear energy into electrical energy.
- The wind generator converts wind energy into electrical energy.
- The solar generator converts solar energy into electrical energy.

An electrical load is a device that is usually connected to the output terminal of an electric circuit.

- The load consumes or absorbs electrical energy from the source.
- The load may be any device that can receive electrical energy and convert it into other forms of energy.

Examples of electric loads:

- Electric lamp converts electrical energy into light energy.
- Electric stove converts electrical energy into heat energy.
- Electric motor converts electrical energy into mechanical energy.
- Electric fan converts electrical energy into wind energy.
- Speaker converts electrical energy into sound energy.
- Solar cell converts sunlight into electrical energy.
- Microphone converts sound energy into electrical energy.
- . . .

Requirements of a basic circuit	– Power supply (power source) is a device that supplies electrical energy to a load; it can convert other energy forms into electrical energy. – Load is a device that is connected to the output terminal of an electric circuit, and consumes electrical energy. – Wires connect the components in a circuit together, and carry electric charges through the circuit.

Figure 1.2 is an example of a simple electric circuit—a flashlight (or electric torch) circuit. In this circuit, the battery is the power supply unit and the small light bulb is the load, and they are connected together by wires. Figure 1.3 is the schematic of the flashlight.

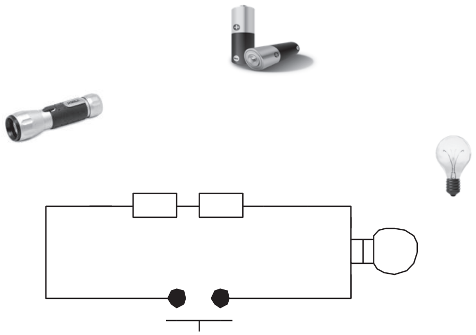


Figure 1.2 The flashlight circuit

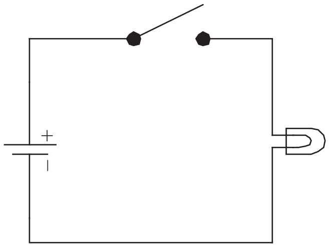


Figure 1.3 Schematic of the flashlight circuit

1.2.2 Circuit schematics (diagrams) and symbols

Circuit diagrams

- Studying electric circuits usually requires drawing or recognizing circuit diagrams. Circuit diagrams can make electric circuits easier to understand, analyze, and calculate.
- When studying more theories of electric circuits, circuits can be more and more complex and drawing the pictorial representation of the circuits will not be very realistic.
- The more common electric circuits are usually represented by schematics.

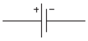
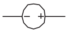
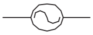
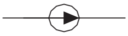
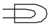
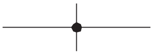
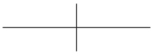
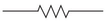
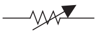
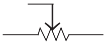
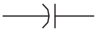
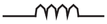
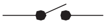
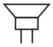
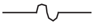
Schematics

- A schematic is a simplified circuit diagram that shows the interconnection of circuit components.
- It uses standard graphic circuit symbols according to the layout of the actual circuit connection.
- This is a way to draw circuit diagrams far more quickly and easily.

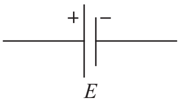
Circuit symbols

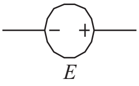
- The circuit symbols are the idealization and approximation of the actual circuit components (Table 1.1).

Table 1.1 The commonly used circuit schematic symbols

Component	**Circuit symbol**
DC power supply	 
AC power supply	
Current source	
Lamp	
Connected wires	
Unconnected wires	
Fixed resistor	
Variable resistor	 or 
Capacitor	
Inductor	
Switch	
Speaker	
Ground	
Fuse	
Ohm meter	
Ammeter	
Voltmeter	
Transformer	

- For example, both the battery and the direct current (DC) generator can convert other energy forms into electrical energy and produce DC voltage. Therefore, they are represented by the same circuit symbol—the DC power supply E.





1.3 Electric current

1.3.1 Current

Water current analogy electric current

- There are several key circuit quantities in electric circuit theory: electric current, voltage, power, etc. These circuit quantities are very important to study in electric circuits.

- Electric charges and electric current are analogous to the flow of water in a water hose or pipe.
- Water current is a flow of water through a water circuit (faucet, pipe or hose, etc.).
- Electric current is a flow of electric charges through an electric circuit (wires, power supply, load, etc.).
- Water is measured in liters or gallons, so you can measure the amount of water that flows out of the tap at certain time intervals, i.e., liters or gallons per minute or hour.
- Electric current is measured by the amount of electric charges that flows past a given point at a certain time interval in an electric circuit.

Calculating electric current

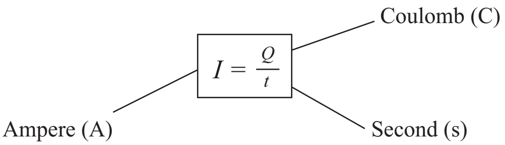
- If Q represents the amount of charges that is moving past a point at time t, then the current I is:

$$I = \frac{Q}{t}$$

If you have learned calculus, current also can be expressed by the derivative: $i = \frac{dq}{dt}$

- $\boxed{\text{Current} = \frac{\text{Charge}}{\text{Time}}}$ or $\boxed{\boldsymbol{I} = \frac{Q}{t}}$

- Units:



Note: – Italic letters represent the quantity symbols.
– Non-italic letters represent unit symbols.

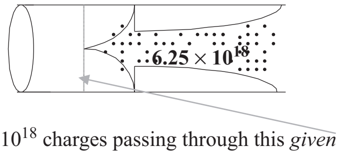
Electric current (I)	– Current is a flow of electric charges through an electric circuit. – Current I is measured by the amount of charge Q that flows past a given point at a certain time t: $I = \frac{Q}{t}$

1.3.2 1-ampere current

What is a 1-ampere current?

A current of 1 A (ampere) means that there is 1 C (coulomb) of electric charge passing through a given cross-sectional area of wire in 1 s (second).

$$\boxed{1\ \text{A} = \frac{1\ \text{C}}{1\ \text{s}}}$$



There are 6.25 × 10^{18} charges passing through this *given cross-sectional* area in 1 s

Figure 1.4 1 A of current

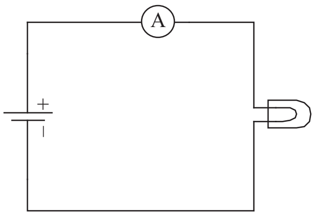


Figure 1.5 Measuring current with an ammeter

1 A of current actually means there are about 6.25×10^{18} charges passing through a given cross-sectional area of wire in 1 s (Figure 1.4).

Since 1 C is approximately equal to 6.25×10^{18} charges ($1\text{C} \approx 6.25 \times 10^{18}$ charges).

Example 1.1: If a charge of 100 C passes through a given cross-sectional area of wire in 50 s, what is the current?

Solution: Since $Q = 100$ C and $t = 50$ s

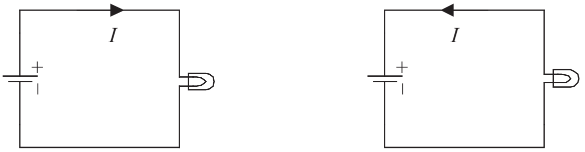
$$I = \frac{Q}{t} = \frac{100\text{ C}}{50\text{ s}} = \boxed{2\text{ A}}$$

An ammeter is an instrument that can be used to measure current, and its symbol is Ⓐ. It must be connected in series with the circuit to measure current, as shown in Figure 1.5.

1.3.3 The direction of electric current

Which way does electric charge really flow?

- When early scientists started to work with electricity, the structure of atoms was not very clear, and they assumed at that time the current was a flow of positive charges (protons) from the positive terminal of a power supply (such as a battery) to its negative terminal.
- Later on, scientists discovered that electric current is in fact a flow of negative charges (electrons) from the negative terminal of a power supply to its positive terminal.



(a) Conventional current flow (b) Electron flow

Figure 1.6 The direction of electric current

- But by the time the real direction of current flow was discovered, a flow of positive charges (protons) from the positive terminal of a power supply to its negative terminal had already been well established and used commonly in electrical circuitry.

Two methods to express the direction of electric current

- Conventional current flow version: The current is defined as a flow of positive charges (protons) from the positive terminal of a power supply to its negative terminal (Figure 1.6(a)).
- Electrons flow version: The current is defined as a flow of negative charges (electrons) from the negative terminal of a power supply to its positive terminal (Figure 1.6(b)).

It will make no difference as to which method is used

- It will not affect the analysis, design, calculation, measurement, and applications of the electric circuits as long as one method is used consistently.
- In this book, the conventional current flow version is used.

Conventional current flow vs. electron flow	– A flow of *positive* charges (*protons*) from the positive terminal of a power supply to its negative terminal. – A flow of *negative* charges (*electrons*) from the negative terminal of a power supply to its positive terminal.

1.4 Electric voltage

1.4.1 Voltage/electromotive force

Water gun and voltage: The concept of another important circuit quantity—voltage works on the principle of a water gun.

The trigger of a water gun is attached to a pump that squirts water out of a tiny hole at the muzzle.

- If there is no pressure from the gun (the trigger is not pressed), there will be no water out of the muzzle.
- Low-pressure squirting produces thin streams of water over a short distance.
- High pressure produces a very powerful stream over a longer distance.

Water pressure vs. voltage: Just as water pressure is required for a water gun or water circuit, electric pressure or voltage is required for an electric circuit.

- Voltage is responsible for the pushing and pulling of electrons or current through an electric circuit.
- The higher the voltage, the greater the current will be.

Flashlight or torch circuit and voltage (Figure 1.2):

- If only a small lamp is connected with wires without a battery, the flashlight will not work. Since electric charges in the wire (conductor) randomly drift in different directions, a current cannot form in a specific direction.
- Once the battery (voltage source) is connected to the load (lamp) by wires, it will produce electric current in the circuit. The positive electrode of the battery attracts the negative charges (electrons), and the negative electrode of the battery repels the electrons. This causes the electrons to flow in one direction and produce electric current.

Electromotive force (EMF): the battery is one example of a voltage source that produces electromotive force (EMF) between its two terminals.

- EMF moves electrons around the circuit or causes current to flow through the circuit since EMF is actually "the electron-moving force."
- EMF is the electric pressure or force that is supplied by a voltage source, which causes current to flow in a circuit.

EMF produced by a voltage source is analogous to water pressure produced by a pump in a water circuit.

Units of voltage and EMF: voltage is symbolized by V (italic letter), and its unit is volts (non-italic letter V). EMF is symbolized by $\boldsymbol{E}$, and its unit is also volts (V).

1.4.2 Potential difference/voltage

Water-level difference

Assuming there are two water tanks A and B, water will flow from tank A to B only when tank A has a higher water level than tank B, as shown in Figure 1.7.

Water-level difference vs. electrical potential difference

- Common sense tells us that "water flows to the lower end," so water will only flow when there is a water-level difference. It is the water-level difference that

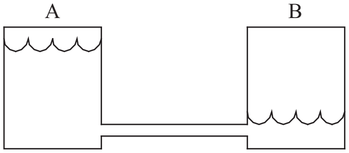


Figure 1.7 Water-level difference

produces the potential energy for tank A, and work is done when water flows from tank A to B.

- Water will flow between two places in a water circuit only when there is a water-level difference. This concept can also be used in the electric circuit.
- Current will flow between two points in an electric circuit only when there is an electrical potential difference.

Potential difference or voltage

- If a light bulb is continuously kept on, i.e., to maintain continuous movement of electrons in the circuit, the two terminals of the lamp need to have an electrical potential difference.
- The potential difference or voltage is produced by the EMF of the voltage source.
- Potential difference or voltage is the amount of energy or work that would be required to move electrons between two points.
- Current will flow between two points in a circuit only when there is a potential difference. The voltage or the potential difference always exists between two points.

Calculating voltage

- $\boxed{\text{Voltage} = \frac{\text{Work}}{\text{Charge}}}$ or $\boxed{V = \frac{W}{Q}}$
- Units: Volt (V) $\boxed{V = \frac{W}{Q}}$ Joule (J) Charge (C)

If you have learned calculus, voltage can also be expressed by the derivative $v = \frac{dw}{dq}$.

Example 1.2: If 1 J of energy is used to move a 1 C charge from point a to b, it will have a 1 V potential difference or voltage across two points, as shown in Figure 1.8:

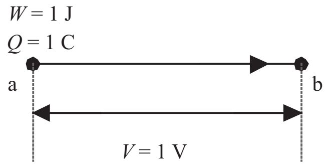


Figure 1.8 Potential difference or voltage

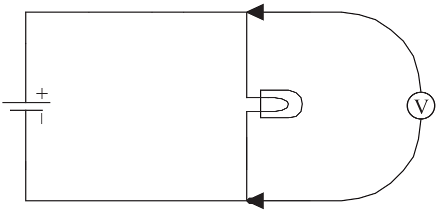


Figure 1.9 Measuring voltage with a voltmeter

Voltage V (or potential difference)	V is the amount of energy or work required to move electrons between two points: $V = \frac{w}{Q}$

Although voltage and potential difference are not exactly the same, the two are used interchangeably.

Electromotive Force (EMF)	EMF is an electric pressure or force that is supplied by a voltage source, which causes electric current to flow in a circuit.

There are different names representing voltage or potential difference in electric circuits, such as the source voltage, applied voltage, load voltage, voltage drop, and voltage rise. What are the differences between them?

- EMF can be called source voltage or applied voltage ($\boldsymbol{E}$ or $\boldsymbol{V}_{\mathbf{S}}$) since it is supplied by a voltage source and applied to the load in a circuit.
- Load voltage: voltage across the two terminals of the load.
- Voltage drop: voltage across a component when current flows from a higher potential point to a lower potential point in the circuit.
- Voltage rise: voltage across a component when current flows from a lower potential point to a higher potential point in the circuit.

Voltmeter is an instrument that can be used to measure voltage. Its symbol is Ⓥ. The voltmeter should be connected in parallel with the circuit component to measure voltage, as shown in Figure 1.9.

1.5 Resistance and Ohm's law

1.5.1 Resistance/resistor

Resistance

- Water resistance: what will happen when we throw some rocks into a small creek? The speed of the water current will slow down in the creek. This is because the rocks (water resistance) "resist" the flow of water.

  A similar concept may also be used in an electric circuit.

- Current resistance (R): the resistor (current resistance) “resists” the flow of electrical current.
  - The higher the value of resistance, the smaller the current will be.
  - The resistance of a conductor is a measure of how difficult it is to resist the current flow.

Resistor

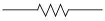
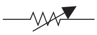
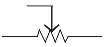
- The lamp, electric stove, motor, and other such loads may be represented by the resistor R because once this kind of load is connected to an electric circuit, it will consume electrical energy, cause resistance, and reduce current in the circuit.
- Sometimes resistor R will need to be adjusted to a different level for different applications. For example,
  - The intensity of light of an adjustable lamp can be adjusted by using resistors.
  - A resistor can also be used to maintain a safe current level in a circuit.
- A resistor is a two-terminal component of a circuit that is designed to resist or limit the flow of current. There are a variety of resistors with different resistance values for different applications.

The resistor and resistance of a circuit have different meanings

Resistor (R)	A two-terminal component of a circuit that limits the flow of current.
Resistance (*R*)	The measure of a material’s opposition to the flow of current, and its unit is ohms (Ω).

Fixed or variable resistors

- A fixed resistor has a “fixed” resistance value and cannot be changed.
- A variable resistor has a resistance value that can be easily changed or adjusted manually or automatically.

Symbols of the resistor	Fixed resistor:  Variable resistor:  or 

1.5.2 Factors affecting resistance

There is no “perfect” electrical conductor; every conductor that makes up the wires has some level of resistance regardless of the material it is made from.

There are four main factors affecting the resistance in a conductor: the cross-sectional area of the wire (A), length of the conductor (ℓ), temperature (T), and resistivity of the material (ρ) (Figure 1.10).

- Cross-sectional area of the wire A: More water will flow through a wider pipe than that through a narrow pipe. Similarly, the larger the diameter of the wire,

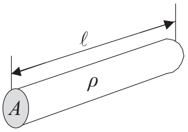


Figure 1.10 Factors affecting the resistance

Table 1.2 Table of resistivity (ρ)

Material	**Resistivity ρ (Ω · m)**
Copper	1.68×10^{-8}
Gold	2.44×10^{-8}
Aluminum	2.82×10^{-8}
Silver	1.59×10^{-8}
Iron	1.0×10^{-7}
Brass	0.8×10^{-7}
Nichrome	1.1×10^{-6}
Tin	1.09×10^{-7}
Lead	2.2×10^{-7}

the greater the cross-sectional area, the less the resistance in the wire and the more the flow of current.

- Length ℓ: The longer the wire, the more the resistance and the more the time taken for the current to flow.
- Resistivity ρ: It is a measure of the opposition to flowing current through a material of wire, or how difficult it is for current to flow through a material. The different materials have different resistivity, i.e., more or less resistance in the materials.
- Temperature *T:* Resistivity of a material is dependent upon the temperature surrounding the material. Resistivity increases with an increase in temperature for most materials. Table 1.2 lists resistivity of some materials at 20°C.

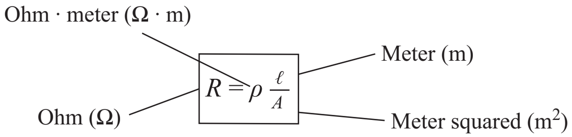
Factors affecting resistance	$R = \rho \frac{\ell}{A}$, where A is the cross-sectional area ℓ the length T the temperature, and ρ the resistivity (conducting ability of a material for a wire).

Note: ρ is a Greek letter pronounced "rho" (see "Appendix A" for a list of Greek letters).

Calculating resistance

- $\boxed{\text{Resistance} = \text{resistivity}\frac{\text{length}}{\text{area}}}$ or $\boxed{R = \rho\frac{\ell}{A}}$

- Units:



Example 1.3: There is a copper wire 50 m in length with a cross-sectional area of 0.13 cm^2. What is the resistance of the wire?

Solution:

$$\ell = 50\,\text{m} = 5{,}000\,\text{cm}$$

$$A = 0.13\,\text{cm}^2$$

$$\rho = 1.68 \times 10^{-8}\,\Omega \cdot \text{m} = 1.68 \times 10^{-6}\,\Omega \cdot \text{cm (copper)}$$

$$R = \rho\frac{\ell}{A} = \frac{(1.68 \times 10^{-6}\,\Omega \cdot \text{cm})(5{,}000\,\text{cm})}{0.13\,\text{cm}^2} \approx \boxed{0.0646\,\Omega}$$

- The resistance of this copper wire is 0.0646 Ω. Although there is resistance in the copper wire, it is very small. A 50-m-long wire only has 0.0646 Ω resistance; thus, we can say that copper is a good conducting material.
- Copper and aluminum are commonly used conducting materials with reasonable price and better conductivity.

Ohmmeter is an instrument that can be used to measure the resistance. Its symbol is (Ω). The resistor must be removed from the circuit to measure resistance as shown in Figure 1.11.

Ammeter Ⓐ	– Ⓐ is an instrument that is used to measure current; it should be connected in series in the circuit.
Voltmeter Ⓥ	– Ⓥ is an instrument that is used to measure voltage; it should be connected in parallel with the component.
Ohmmeter (Ω)	– (Ω) is an instrument that is used to measure resistance, and the resistor must be removed from the circuit to measure the resistance.

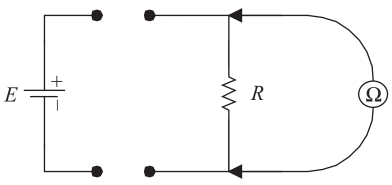


Figure 1.11 Measuring resistance with an ohmmeter

1.5.3 Conductance

Conductance *G* is the conductivity of the material.

- It is the ability of a material to pass current rather than resist it.
- It is how easy rather than how difficult it is for current to flow through a circuit.
- It is a term that is opposite to the term “resistance.” The less the resistance R of the material, the greater the conductance G, the better the conductivity of the material, and vice versa.

Factors affecting conductance

- The factors that affect resistance are the same for conductance, but in the opposite way. Mathematically, conductance is the reciprocal of resistance, i.e.,

$$\boxed{G = \frac{1}{R}} \quad \text{or} \quad \boxed{G = \frac{A}{\rho \ell}} \qquad \left(\because R = \rho \frac{\ell}{A}\right)$$

- Increasing the cross-sectional area (A) of the wire or reducing the wire length (ℓ) can get better conductivity. (This can be seen from the equation of conductance.)

Unit of conductance

- The SI unit of conductance is the siemens (S).
- Some books use a unit mho (℧) for conductance, which was derived from the spelling ohm backward and with an upside-down Greek letter omega.
  Mho actually is the reciprocal of ohm, just as conductance G is the reciprocal of resistance R.

Conductance *G*	G is the conductivity of the material, and it is the reciprocal of resistance: $G = \frac{1}{R}$

Calculating conductance

- $\boxed{\text{Conductance} = \frac{1}{\text{Resistance}}}$ or $\boxed{G = \frac{1}{R}}$

- Units: Siemens (S) Or Mho (℧) $\boxed{G = \frac{1}{R}}$ Ohm (Ω)

Example 1.4: What is the conductance if the resistance R is 22 Ω?

Solution: $$G = \frac{1}{R} = \frac{1}{22\ \Omega} \approx \boxed{0.0455\ \text{S or } 0.0455\ \mho}$$

It is often preferable and more convenient to use conductance in parallel circuits. This will be discussed in the later chapters.

1.5.4 Ohm's law

Ohm's law is a very important and useful equation in electric circuit theory. It precisely expresses the relationship between current, voltage, and resistance with a simple mathematical equation.

Ohm's law states that current through a conductor in a circuit is directly proportional to the voltage across it and inversely proportional to the resistance in it, i.e.,

$$\boxed{I = \frac{V}{R}} \qquad \text{or} \qquad \boxed{I = \frac{E}{R}}$$

Any form of energy conversion from one type to another can be expressed as the following equation:

$$\boxed{\text{Effect} = \frac{\text{Cause}}{\text{Opposition}}}$$

In an electric circuit, it is the voltage that causes current to flow, so current flow is the result or effect of voltage, and resistance is the opposition to the current flow.

Replacing voltage, current, and resistance into the above expression will obtain Ohm's law:

$$\text{Current} = \frac{\text{Voltage}}{\text{Resistance}} \qquad \text{or} \qquad I = \frac{V}{R}$$

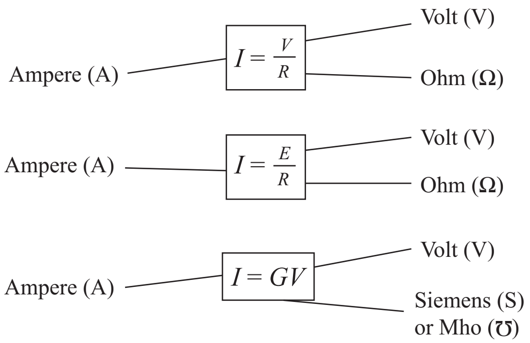
Conductance form of Ohm's law

Ohm's law can be written in terms of conductance as follows:

$$\boxed{I = GV} \qquad \left(\text{since } G = \frac{1}{R} \quad \text{and} \quad I = \frac{V}{R}\right)$$

Ohm's law	– Ohm's law expresses the relationship between I, V, and R. – I through a conductor is directly proportional to V, and inversely proportional to R: $I = \frac{V}{R}$ or $I = \frac{E}{R}$ or $I = GV$

Units:


$I = \frac{V}{R}$
Ampere (A)
Volt (V)
Ohm (Ω)
$I = \frac{E}{R}$
Ampere (A)
Volt (V)
Ohm (Ω)
$I = GV$
Ampere (A)
Volt (V)
Siemens (S)
or Mho (℧)

Memory aid for Ohm's law: Using mathematics to manipulate Ohm's law, and solving for V and R, respectively, we can write Ohm's law in several different forms:

$$V = IR \qquad I = \frac{V}{R} \qquad R = \frac{V}{I}$$

- These three equations can be illustrated in Figure 1.12 as a memory aid for Ohm's law.
- By covering one of the three variables from Ohm's law in the diagram, we can get the right form of Ohm's law to calculate the unknown.

The experimental circuit of Ohm's law

- The experimental circuit with a resistor of 125 Ω in Figure 1.13 may prove Ohm's law.
- If a voltmeter is connected in the circuit and the source voltage is measured $E = 2.5$ V. Also, connecting an ammeter and measuring the current in the circuit will result in $I = 0.02$ A.
- With Ohm's law we can confirm that current in the circuit is indeed 0.02 A:

$$I = \frac{E}{R} = \frac{2.5\ V}{125\ \Omega} = 0.02\ \text{A}$$

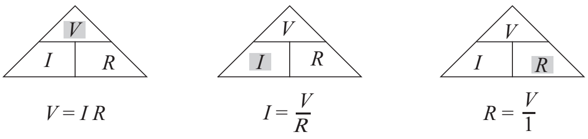

V
I
R
$V = I\,R$
V
I
R
$I = \frac{V}{R}$
V
I
R
$R = \frac{V}{1}$

Figure 1.12 Memory aid for Ohm's law

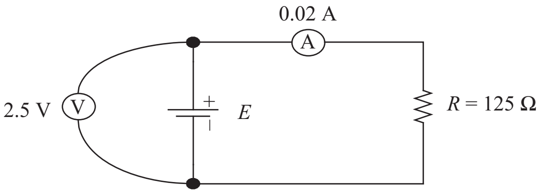

0.02 A
A
2.5 V
V
+
E
R = 125 Ω

Figure 1.13 The experimental circuit of Ohm's law

Example 1.5: A source of 12 V is connected to a resistive lamp and a current of 3 A flows. What is the resistance of the circuit?

Solution: $R = \frac{E}{I} = \frac{12\text{ V}}{3\text{ A}} = \boxed{4\ \Omega}$

1.5.5 *I–V characteristics of Ohm's law*

***I–V* characteristics of Ohm's law:** Using a Cartesian coordinate system, voltage V (x-axis) is plotted against current I (y-axis); this graph of current versus voltage will be a straight line, as shown in Figure 1.14.

- The straight line in Figure 1.14 describes the current–voltage relationship of a 10-Ω resistor.
  - When voltage V is 10 V and current is 1 A, $R = \frac{V}{I} = \frac{10\text{ V}}{1\text{ A}} = 10\ \Omega$.
  - When voltage V is 5 V and current is 0.5 A, $R = \frac{V}{I} = \frac{5\text{ V}}{0.5\text{ A}} = 10\ \Omega$.
- The different lines with different slopes on the *I–V* characteristic represent the different values of resistors. For example, a 20-Ω resistor can be illustrated as in Figure 1.15.
- Since *I–V* characteristic shows the relationship between current I and voltage V for a resistor, it is called the *I–V* characteristic of Ohm's law.

The *I–V* characteristic of the straight line illustrates the behavior of a linear resistor, i.e., the resistance does not change with the voltage or current. If the

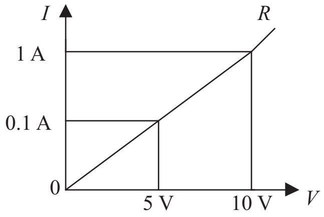


Figure 1.14 I–V characteristics (R = 10 Ω)

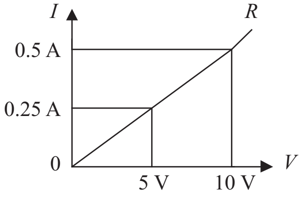


Figure 1.15 I–V characteristics (R = 20 Ω)

voltage decreases from 10 to 5 V, the resistance still equals 20 Ω as shown in Figure 1.15.

When the relationship of voltage and current is not a straight line, the resultant resistor will be a non linear resistor.

1.6 Reference direction of voltage and current

1.6.1 Reference direction of current

Direction of current

- The actual current direction: When performing circuit analysis and calculations in many situations, the actual current direction through a specific component or branch may change sometimes, and it may be difficult to determine the actual current direction for a component or branch.
- Reference direction of current: It is convenient to assume an arbitrarily chosen current direction (with an arrow), which is the concept of reference direction of current.
  - If $I > 0$: If the resultant mathematical calculation for current through that component or branch is positive ($I > 0$), the actual current direction is consistent with the assumed or reference direction.
  - If $I < 0$: If the resultant mathematical calculation for the current of that component is negative ($I < 0$), the actual current direction is opposite to the assumed or reference direction.
- The solid line arrows indicate the reference current directions and the dashed line arrows indicate the actual current directions (as shown in Figure 1.16).

Two methods to represent the reference direction of current (Figure 1.17)

- Expressed with an arrow, the direction of the arrow indicates the reference direction of current.
- Expressed with a double subscription, for instance, I_{ab} indicates the reference direction of current is from point a to b.

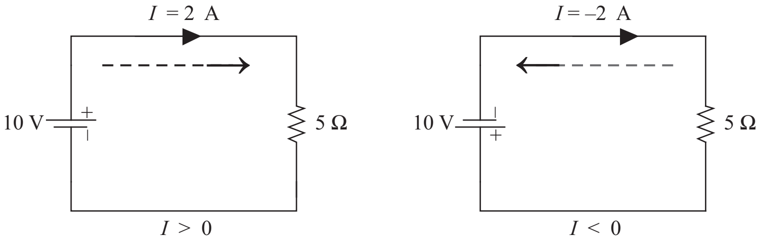
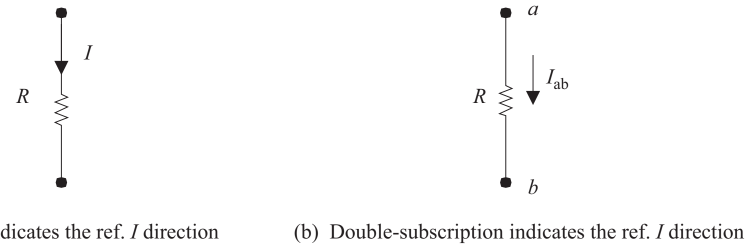


Figure 1.16 Reference direction of current



(a) Arrow indicates the ref. *I* direction (b) Double-subscription indicates the ref. *I* direction

Figure 1.17 Reference direction of current I

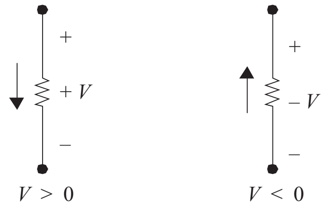


Figure 1.18 Reference polarity of voltage

1.6.2 Reference polarity of voltage

Reference polarity of voltage

Similar to the current reference direction, the voltage reference polarity is also an assumption of arbitrarily chosen polarity.

- If $V > 0$: If the resultant calculation for voltage across a component is positive ($V > 0$), the actual voltage polarity is consistent with the assumed reference polarity.
- If $V < 0$: If the resultant calculation is negative ($V < 0$), the actual voltage polarity is opposite to the assumed reference polarity.

As shown in Figure 1.18, the positive (+) and negative (−) polarities represent the reference voltage polarities, and arrows represent the actual voltage polarities.

Three methods to indicate the reference polarity of voltage

(Figure 1.19)

- Expressed with an arrow, the direction of the arrow points from positive to negative.
- Expressed with polarities, positive sign (+) indicates a higher potential position, and negative sign (−) indicates a lower potential position.
- Expressed with a double subscription, for instance, V_{ab} indicates that the potential position a is higher than the potential position b.

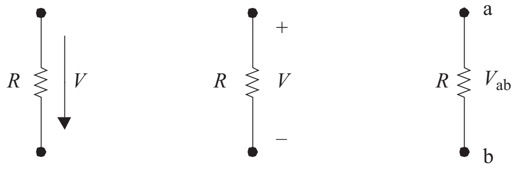
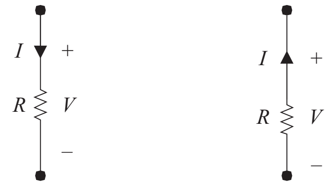


Figure 1.19 Methods indicating the reference polarity of voltage



(a) (b)

Figure 1.20 (a) Mutually related reference polarity of I and V (b) non-mutually related reference polarity of I and V

1.6.3 Mutually related reference polarity of current/voltage

Mutually related reference direction or polarity of current/voltage

- If the reference direction of current is assigned by the flow from the positive side to the negative side of voltage across a component, then the reference current direction and reference voltage polarity is consistent.
- In other words, along with the current reference direction (the reference arrow pointing from + to −) is the voltage from positive to negative polarity. This is called the mutually related reference direction or polarity of current/voltage.

If we only know one reference direction or polarity, it is possible to determine the other, and this is shown in Figure 1.20.

Reference direction of current	Assuming an arbitrarily chosen direction as the reference direction of current I: – If $I > 0$: the actual current direction is consistent with the reference current direction. – If $I < 0$: the actual current direction is opposite to the reference current direction.

Reference polarity of voltage	Assuming an arbitrarily chosen voltage polarity as the reference polarity of voltage: – If $V > 0$: the actual voltage polarity is consistent with the reference voltage polarity. – If $V < 0$: the actual voltage polarity is opposite to the reference voltage polarity.

Mutually related ref. polarity of *V* and *I*	If the reference *I* direction is assigned by an arrow pointing from + to − of *V* across a component, then the reference *I* direction and reference *V* polarity is consistent.

Summary

Milestones of the electric circuits

Name of the scientist	Nationality	Name of the unit/law	Named for
Charles Augustin de Coulomb	French	Coulomb	Unit of charge (C)
Alessandro Volta	Italy	Volt	Unit of voltage (V)
André-Marie Ampère	French	Ampere	Unit of current (A)
Georg Simon Ohm	German	Ohm	Unit of resistance (Ω)
James Watt	Scotland	Watt	Unit of power (W)
Friedrich Emil Lenz	German	Lenz	Lenz's law
James Clerk Maxwell	Scotland	Maxwell	Unit of flux (maxwell), Maxwell magnetic field equation
Wilhelm Eduard Weber	German	Webber	Unit of flux (web) 1 Wb $= 10^8$ Mx
Heinrich Rudolf Hertz	German	Hertz	Unit of frequency (Hz)
Kirchhoff	German	Kirchhoff	Kirchhoff current and voltage laws
Joseph Henry	Scottish-American	Henry	Unit of inductance (H)
James Prescott Joule	British	Joule	Unit of energy (J)
Michael Faraday	British	Faraday	Unit of capacitance (F)

Electric circuit: A closed loop of pathway with electric current flowing through it.

Requirements of a basic circuit

- Power supply (power source): A device that supplies electrical energy to a load.
- Load: A device that is connected to the output terminal of a circuit, and consumes electrical energy.
- Wires: Wires connect the power supply unit and load together, and carry current flowing through the circuit.

Schematic: A simplified circuit diagram that shows the interconnection of circuit components, and is represented by circuit symbols.

Circuit symbols: The idealization and approximation of the actual circuit components.

Electric current (*I*): A flow of electric charges through an electric circuit:

$$I = \frac{Q}{t} \qquad \left(\text{or } I = \frac{dq}{dt}\right)$$

Current direction

- Conventional current flow version: A flow of positive charge (proton) from the positive terminal of a power supply to its negative terminal.
- Electron flow version: A flow of negative charge (electron) from the negative terminal of a power supply to its positive terminal.

Ammeter: An instrument used for measuring current, represented by the symbol Ⓐ. It should be connected in series in the circuit.

Electromotive force (EMF): An electric pressure or force supplied by a voltage source causing current to flow in a circuit.

Voltage (*V*) or potential difference: The amount of energy or work that would be required to move electrons between two points.

$$V = \frac{W}{Q} \qquad \left(\text{or } v = \frac{dw}{dt}\right)$$

Source voltage or applied voltage (*E* or *V*s): EMF can be called source voltage or applied voltage. The EMF is supplied by a voltage source and applied to the load in a circuit.

Load voltage (*V*): Voltage across two terminals of the load.

- Voltage drop: Voltage across a component when current flows from a higher potential point to a lower potential point in a circuit.
- Voltage rise: Voltage across a component when current flows from a lower point to a higher point in a circuit.

Voltmeter: An instrument used for measuring voltage. Its symbol is Ⓥ and it should be connected in parallel with the component.

Resistor (R): A two-terminal component of a circuit that limits the flow of current.

Resistance (*R*): Measure of a material's opposition to the flow of current.

Factors affecting the resistance: $R = \rho\frac{\ell}{A}$, where cross-sectional area is (A), length is (ℓ), temperature is (T), and resistivity is (ρ).

Ohmmeter: An instrument used for measuring resistance. Its symbol is (Ω) and the resistor must be removed from the circuit to measure the resistance.

Conductance (*G*): It is the reciprocal of resistance: $G = \frac{1}{R}$

Ohm's law: It expresses the relationship between current I, voltage V, and resistance R.

$$I = \frac{V}{R} \quad \text{or} \quad I = \frac{E}{R}$$

Conductance form of Ohm's law: $I = GV$.

Reference direction of current: Assuming an arbitrarily chosen current direction as the reference direction of current:

- If $I > 0$, actual current direction is consistent with the reference current direction.
- If $I < 0$, actual current direction is opposite to the reference current direction.

Reference polarity of voltage: Assuming an arbitrarily chosen voltage polarity as the reference polarity of voltage:

- If $V > 0$: actual voltage polarity is consistent with the reference voltage polarity.
- If $V < 0$: actual voltage polarity is opposite to the reference voltage polarity.

Mutually related polarity of voltage and current: If the reference current direction is assigned by an arrow pointing from + to − voltage of the component, then the reference current direction and reference voltage polarity is consistent.

Symbols and units of electrical quantities

Quantity	**Quantity symbol**	**Unit**	**Unit symbol**
Charge	Q	Coulombs	C
EMF	E	Volt	V
Work (energy)	W	Joule	J
Resistance	R	Ohm	Ω
Resistivity	ρ	Ohm · meters	Ω · m
Conductance	G	Siemens or Mho	S or ℧
Current	I	Ampere	A
Voltage	V or E	Volt	V

Practice problems

1.1

1. Write a short essay titled "I am looking forward to my electrical/electronics career."
2. List the electric and electronic products that you know.

3. Classroom discussion: The electrical history that I know.
4. Listed the physicists who were named for the following units of electricity:
   - (a) Resistance
   - (b) Current
   - (c) Voltage
   - (d) Power

5. (   ) was named for the unit of charge.
6. (   ) was named for the unit of frequency.
7. What is the unit of energy?
8. (   ) was named for the unit of inductance.
9. Maxwell is the unit of (   ), 1 Web = 10 (   ) Maxwell.
10. What is the unit of capacitance?

1.2

11. What are three basic requirements for a circuit?
12. A simplified circuit diagram that shows the interconnection of circuit components, and is represented by the (ideal) circuit symbols is (   ).
13. Draw the following circuit components using circuit symbols: fixed resistor, variable resistor, capacitor, inductor, fuse, wire connection, and grounding.

1.3

14. (   ) is measured by the amount of charge (Q) that flows past a given point at a certain time.
15. (   ) is an instrument used for measuring current; its symbol is (   ).
16. The instrument used for measuring current should be connected to (   ) in the circuit.
17. If 14 C charge through a specific point in seven seconds, the current equals to (   ).
18. How long can a 5 A current make 10 C charge through a particular point?
19. The conventional direction of current is a flow of positive charge from the (   ) terminal of a power supply to its (   ) terminal.

1.4

20. Electromotive force (EMF) is also called (   ); its symbol is (   ).
21. Voltage also called (   ); its symbol is (   ).
22. An instrument used for measuring voltage is called (   ), and it should be connected in (   ) with the component.
23. The voltage that across two terminals of a lamp is called the (   ) voltage.
24. (   ) is an electric pressure or force that is supplied by a voltage source, which causes electric current to flow in a circuit.

1.5

25. An instrument used for measuring resistance is called ( ); its symbol is ( ).
26. The instrument used for measuring resistance should be connected in ( ) with the resistor.
27. Factors affecting resistance are A, ρ, ℓ, and ( ).
28. The cross-section of a 100 m copper wire is 0.13 cm^2. Determine the resistance of this copper wire.
29. If $V = 20$ V and $R = 100\ \Omega$, calculate the current.
30. If $E = 12$ V and $I = 0.1$ A calculate the resistance and conductance.
31. Plot the I–V characteristics of problem 30.

1.6

32. If $I < 0$, the actual current direction is ( ) with the reference current direction.
33. If $V > 0$, the actual voltage polarity is ( ) with the reference voltage polarity.
34. If the reference current direction is assigned by an arrow pointing from ( ) to ( ) of the voltage of the component, it is called ( ) polarity of the voltage and current.

Chapter 2

Basic laws of electric circuits

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2.1 Power and energy

2.1.1 Work

Work (W)

- Work is the result when a force acts on an object and causes it to move a certain distance. It is the product of the force (F) and the displacement (S) in the direction of the motion (Figure 2.1).

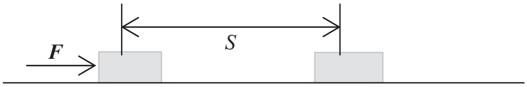


Figure 2.1 Force and displacement

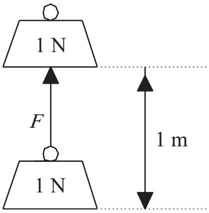


Figure 2.2 Work

- Work depends on:
  - The amount of force (F) applied to an object.
  - The distance or displacement (S) that the object moves.
- Work (W): Work is done when a *force* is applied to an object over a *distance* (or displacement).

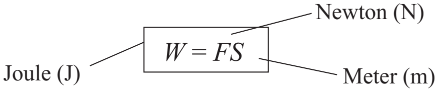
Example: – To push a door open does some work.
– To push on the wall does not (the distance = 0).

The meaning of the word "work" in everyday life is not the same as in physics.

Example: Getting a good mark in physics takes a lot of hard *work*.

Calculating work

- $\boxed{\text{Work} = \text{Force} \times \text{Displacement} \quad W = FS}$
- Units:



- If using a force of 1 N to lift of an object to 1 m, 1 J of work is done in overcoming the downward force of gravity as shown in Figure 2.2.
- When the force (F) and displacement (S) do not point in the same direction, the formula to calculate work will be: $\boxed{W = (F\cos\theta)S}$
  - where the angle θ is the angle between force (F) and displacement (S),
  - when θ is 0 degree, $\cos 0° = 1$, $W = (F\cos\theta)S = FS$.
- It is the same in an electric circuit: Work is done after the electrons or charges are moved to a certain distance in a circuit as a result of applying an electric field force from the power supply.

2.1.2 Energy

Energy and work

- Energy: The capacity to do work (the physical or mental strength that allows one to do work) is called energy. It is not work itself, but a transfer of energy.

- Work is a transfer of energy; when an object does work on another object, some of its energy is transferred to that object.

  Even though you cannot ever really see it, you use energy to do work every day. For example, after you eat and sleep, your body converts the stored energy to keep you doing daily work, such as walking, running, reading, and writing.

- An object has the capacity to do work producing energy. Such energy means that
  - light bulbs can glow
  - wind can blow
  - machines can work
  - airplanes can fly
  - … …

The main types of energy

Energy	**Definition**	**Example**
Light energy	The energy that comes from light. It is the radiant energy which can be seen by the human eye	Light bulb
Heat or thermal energy	The energy (heat) generated by the movement of particles within an object	Stove
Sound energy	The energy generated by vibrating sound waves (move back and forth quickly)	Music
Chemical energy	The energy stored in the internal structure of an atom or molecule (particle)	Gasoline
Electrical energy	Energy generated by the flow of electric charge (charged particles) through a conductor (wire)	Toaster
Nuclear energy	Energy stored in the nucleus (core) of an atom	Nuclear power plant
Gravitational potential energy	Energy stored in an object's height	A book on a table
Kinetic energy	Energy in motion	A moving car
Mechanical energy	Potential energy + kinetic energy	Windmill

The law of conservation of energy

- The law of conservation of energy is one of most important rules in natural science.
- The law of conservation of energy states that energy cannot be created or destroyed, but it can be changed (transferred) from one form to another (the total energy remains constant).

  Examples: A moving car hits a parked car: Energy is transferred from the moving car to the parked car and causes the parked car to move.

“Converted” means “never disappeared” in physics terms. For example:

- Electrical generator: mechanical energy → electrical energy
- Lamp: electrical energy → light energy
- Battery: chemical energy → electrical energy
- Solar panel: light energy → electrical energy

2.1.3 Power

Power refers to the speed of energy conversion or consumption; it is a measure of how fast energy is transforming or being used.

1 N object lifted to 1 m may have different time rates depending on the amount of power applied (refer to the example in Figure 2.2).

- If a higher power is applied to the object (an adult is lifting it), it will take a shorter period of time to lift it.
- If a lower power is applied to the object (a kid is lifting it), it will take a longer period of time to lift it.
- Power is defined as the rate of doing work, or the amount of work done per unit of time.
  Note:
  - Our daily consumption of electricity is electrical energy, and not electrical power.
  - The hydro bill that you receive is for electrical power—the amount of electrical energy consumed in 1 or 2 months.

Energy, work, and power

Energy	– Energy is the capacity to do work.
Work	– Work is a transfer of energy.
Power	– Power is the speed of energy conversion, or work done per unit of time: $P = \frac{W}{t}$

Calculating power

- $\boxed{\text{Power} = \frac{\text{Work}}{\text{Time}}}$ or $\boxed{P = \frac{W}{t}}$
- Units: $$\text{Watt (W)} \;—\; \boxed{P = \frac{W}{t}} \;\begin{matrix}\text{— Joule (J)}\\ \text{— Second (s)}\end{matrix}$$

  or $$\text{Watt (W)} \;—\; \boxed{P = \frac{W}{t}} \;\begin{matrix}\text{— Kilowatt-hour (kWh)}\\ \text{— Hour (h)}\end{matrix}$$

2.1.4 Electric power

Electric power is the speed of electrical energy conversion or consumption in an electric circuit, and it is a measure of how fast electrons or charges are moving in a circuit.

Calculating electric power

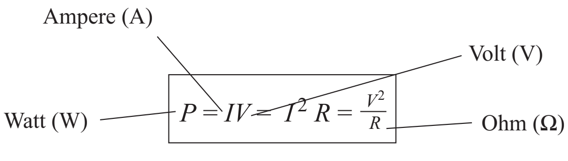
- $$\boxed{\text{Power} = \text{Current} \times \text{Voltage} = \text{Current}^2 \times \text{Resistance} = \frac{\text{Voltage}^2}{\text{Resistance}}}$$

  $$\boxed{P = IV = I^2R = \frac{V^2}{R}}$$

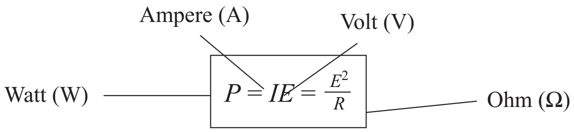
- or $$\boxed{\text{Power} = \text{Current} \times \text{Source voltage} = \frac{(\text{Source voltage})^2}{\text{Resistance}}} \quad \boxed{P = IE = \frac{E^2}{R}}$$

Electric power (P)	$P = IV = I^2R = \frac{V^2}{R} \quad \left(\text{or } P = IE = \frac{E^2}{R}\right)$

- Units:



or



Memory aid: The above power equations can be illustrated in Figure 2.3 as the memory aid for power equations. By covering power in any diagram, the correct equation is obtained to calculate the unknown power.

Example 2.1: In a circuit, voltage $V = 10$ V, current $I = 1$ A, and resistance $R = 10\ \Omega$, calculate the power in this circuit by using three power equations, respectively.

Solution:

$$P = IV = (1\text{ A})(10\text{ V}) = \boxed{10\text{ W}}$$

$$P = I^2R = (1\text{ A})^2(10\ \Omega) = \boxed{10\text{ W}}$$

$$P = \frac{V^2}{R} = \frac{(10\text{ V})^2}{10\ \Omega} = \boxed{10\text{ W}}$$

This example proved that the three power equations are equivalent since each equation leads to the same value of power at 10 W.

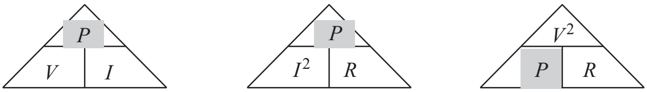


Figure 2.3 Memory aid for power equations

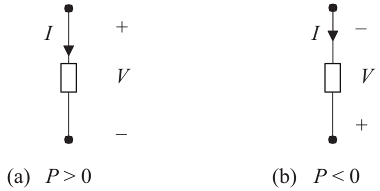


Figure 2.4 The reference direction of power

2.1.5 The reference direction of power

The concept of the reference direction of power

- When a component in a circuit has mutually related reference polarity of current and voltage (refer to Chapters 1–6), power is positive, i.e., $P > 0$, meaning the component absorption (or consumption) of energy.
- When a component in a circuit has non-mutually related reference polarity of current and voltage, power is negative, i.e., $P < 0$, meaning the component releasing (or providing) of energy.

The concept of the reference direction of power can be illustrated in Figure 2.4.

The reference direction of power	– If a circuit has mutually related reference polarity of current and voltage: $P > 0$ (absorption energy). – If a circuit has non-mutually related reference polarity of current and voltage: $P < 0$ (releasing energy).

Example 2.2: Determine the reference direction of power in Figure 2.5(a) and (b).

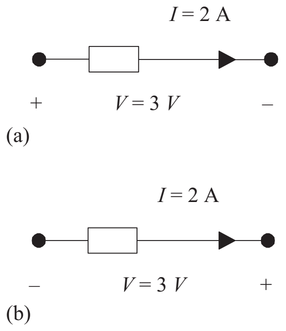


Figure 2.5 Figure for Example 2.2

Solution: (a) $P = IV = (2\text{ A})(3\text{ V}) = 6\text{ W}$

$\boxed{P > 0}$, the resistor absorbs energy.

(b) $P = I(-V) = (2\text{ A})(-3\text{ V}) = -6\text{ W}$

$\boxed{P < 0}$, the resistor releases energy.

Example 2.3: $I = 2$ A, $V_1 = 6$ V, $V_2 = 14$ V, and $E = 20$ V in a circuit as shown in Figure 2.6. Determine the powers dissipated on the resistors R_1, R_2, and R_1 and R_2 in series (a to c): in this figure.

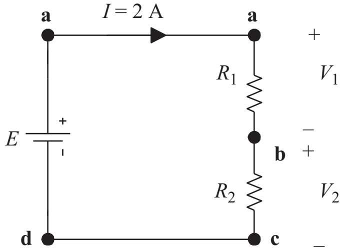


Figure 2.6 Figure for Example 2.3

Solution:

- Power for R_1:
  $P_1 = V_1 I = (6\text{ V})(2\text{ A}) = \boxed{12\text{ W}}$ — $P > 0$, the resistor absorbs energy.
- Power for R_2:
  $P_2 = V_2 I = (14\text{ V})(2\text{ A}) = \boxed{28\text{ W}}$ — $P > 0$, the resistor absorbs energy.
- Power for R_1 and R_2 (a to c):
  $P_3 = (-E)I = (-20\text{ V})(2\text{ A}) = \boxed{-40\text{ W}}$ — $P < 0$, the resistor releases energy.
- $P_1 + P_2 + P_3 = 12\text{ W} + 28\text{ W} + (-40\text{W}) = \boxed{0\text{ W}}$ — Energy conservation.

Formula for current: If power is given in a circuit, using mathematical formulation to manipulate the power equation and solving for current I, we can express current I as follows:

$$\text{Since}\quad P = I^2R \qquad \text{or} \qquad I^2 = \frac{P}{R}, \qquad \text{so} \qquad \boxed{I = \sqrt{\frac{P}{R}}}$$

Formula for voltage: If power is given in a circuit, using mathematical formulation to manipulate the power equation and solving for voltage V, we can express voltage V as follows:

$$\text{Since}\quad P = \frac{V^2}{R} \qquad \text{or} \qquad V^2 = PR, \qquad \text{so} \qquad \boxed{V = \sqrt{PR}}$$

Example 2.4: If power consumed on a 2.5 Ω resistor is 10 W in a circuit, calculate the current flowing through and voltage across this resistor.

Solution:

- $I = \sqrt{\frac{P}{R}} = \sqrt{\frac{10\ \text{W}}{2.5\ \Omega}} = \boxed{2\ \text{A}}$
- $V = \sqrt{PR} = \sqrt{(10\ \text{W})(2.5\ \Omega)} = \boxed{5\ \text{V}}$

2.2 Kirchhoff's voltage law

2.2.1 Closed-loop circuit

Kirchhoff's laws

- Kirchhoff's laws are the most important fundamental circuit laws for analyzing and calculating electric circuits after Ohm's law.
- Physics Professor Kirchhoff: In 1847, a German physicist, physics professor Kirchhoff (Gustav Kirchhoff, 1824–1887) at the Berlin University developed the two laws that established the relationship between voltage and current in an electric circuit.

Closed-loop circuit

- A closed-loop circuit is a conducting path in a circuit that has the same starting and ending points.
- If the current flowing through a circuit from any point returns current to the same starting point, it would be a closed-loop circuit.
- As current flows through a closed-loop circuit, it is the same as having a round trip, so the starting and ending points are the same, and they have the same potential positions.

For example, Figure 2.7 is a closed-loop circuit, since current *I* starts at point a, passes through points b, c, d, and returns to the starting point a.

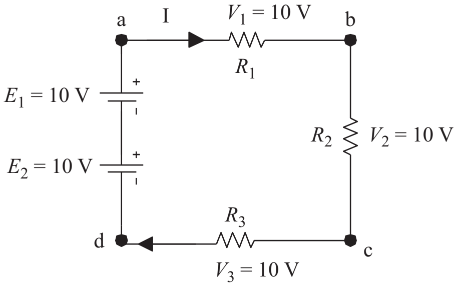


Figure 2.7 A closed-loop circuit

2.2.2 Kirchhoff's voltage law (KVL) #1

KVL #1

- Kirchhoff's voltage law (KVL) #1: The algebraic sum of the voltage or potential difference along a closed-loop circuit is always equal to zero at any moment, or the sum of voltages in a closed loop is always equal to zero.

  i.e., $\boxed{\Sigma V = 0}$ Σ is the Greek letter Sigma, meaning sum.

- The voltage in KVL includes voltage rising from the voltage sources (E) and voltage dropping on circuit elements or loads.

Signs of the voltage in the $\Sigma V = 0$:
The algebraic sum used in KVL #1 means that there are voltage polarities existing in a closed-loop circuit. It requires assigning a loop direction and it could be in either clockwise or counter-clockwise directions (usually choose clockwise).

- Assign a positive sign (+) for voltage (V or E) in the equation $\Sigma V = 0$ if the voltage reference polarity and the loop direction are the same, i.e., if the voltage reference polarity is from positive (+) to negative (–) and the loop direction is clockwise.
- Assign a negative sign (–) for voltage (V or E) in the equation $\Sigma V = 0$ if the voltage reference polarity and the loop direction are opposite, i.e., if the voltage reference polarity is from negative (–) to positive (+), and the loop direction is clockwise.

Example 2.5: Verify KVL #1 for the circuit of Figure 2.8.

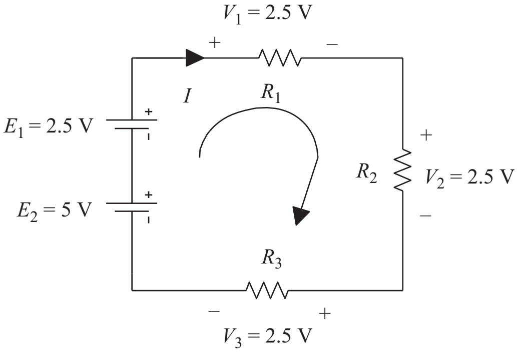


Figure 2.8 Figure for Example 2.5

Solution: Applying $\Sigma V = 0$ in Figure 2.8: $V_1 + V_2 + V_3 - E_2 - E_1 = 0$

$$(2.5 + 2.5 + 2.5 - 5 - 2.5)\text{ V} = \boxed{0}$$

KVL #1 $\boxed{\Sigma V = 0}$	– Assign a (+) sign for *V* or *E* if its reference polarity (+ to –) and loop direction (clockwise) are the same; – Assign a (–) for *V* or *E* if its reference polarity (– to +) and loop direction (clockwise) are opposite.

2.2.3 Kirchhoff's voltage law (KVL) #2

KVL #2

Kirchhoff's voltage law (KVL) #2: Kirchhoff's voltage law (KVL) can also be expressed in another way: the sum of the voltage drops (V) around a closed-loop must be equal to the sum of the voltage rises or voltage sources in a closed-loop circuit, i.e., $\boxed{\Sigma V = \Sigma E}$

Signs of the *V* in the $\Sigma V = \Sigma E$

- Assign a positive sign (+) for ***V*** if its reference polarity and loop directions are the same.

  (i.e., if the voltage reference polarity is from positive (+) to negative (–) and the loop direction is clockwise.)
- Assign a negative sign (–) for ***V*** if its reference polarity and the loop directions are opposite.

  (i.e., if the voltage reference polarity is from negative (–) to positive (+), and the loop direction is clockwise.)

Signs of the *E* in the $\Sigma V = \Sigma E$

- Assign a negative sign (–) for the voltage source ***E*** in the equation if its reference polarity and the loop direction are the same.

  (i.e., if its polarity is from positive (+) to negative (–) and the loop direction is clockwise.)
- Assign a positive sign (+) for the voltage source ***E*** in the equation, if its reference polarity and loop direction are opposite.

  (i.e., if its polarity is from negative (–) to positive (+) and the loop direction is clockwise.)

Example 2.6: Verify KVL #2 for the circuit of Figure 2.8.

Solution: Applying $\Sigma V = \Sigma E$ in Figure 2.8:

$$V_1 + V_2 + V_3 = E_1 + E_2$$

$$(2.5 + 2.5 + 2.5)\ \text{V} = (2.5 + 5)\ \text{V}$$

$$\boxed{7.5\ \text{V}} = \boxed{7.5\ \text{V}}$$

KVL #2 $\boxed{\Sigma V = \Sigma E}$	– Assign a (+) sign for *V* if its reference polarity and loop direction are the same; assign a (–) for *V* if its reference direction and loop direction are opposite. – Assign a (–) for *E* if its reference polarity and loop direction are the same; assign a **(**+**)** for *E* if its polarity and loop direction are opposite.

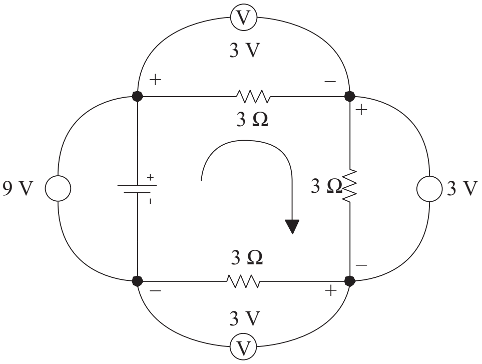


Figure 2.9 Experimental circuit of KVL

2.2.4 Experimental circuit of KVL

Kirchhoff's voltage law can be approved by an experimental circuit in Figure 2.9. If using a multimeter (voltmeter function) to measure voltages on all resistors and power supply in the circuit of Figure 2.9, the total voltage drops on all the resistors should be equal to voltage for the DC power supply.

Experimental circuit of KVL

- KVL #1, $\Sigma V = 0$: $(3 + 3 + 3 - 9)\ \text{V} = 0$
- KVL #2, $\Sigma V = \Sigma E$: $(3 + 3 + 3)\ \text{V} = 9\ \text{V}$

Example 2.7: Determine resistance R_3 in the circuit of Figure 2.10.

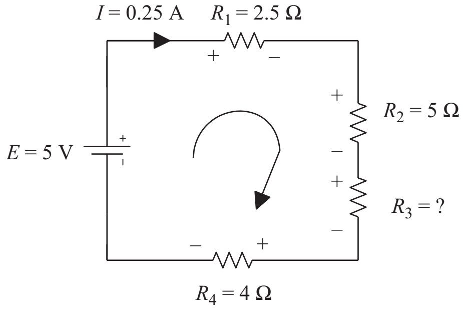


Figure 2.10 Circuit for Example 2.7

Solution: $R_3 = \dfrac{V_3}{I} \rightarrow V_3 = ?$

Applying KVL #1, $\Sigma V = 0:\ V_1 + V_2 + V_3 + V_4 - E = 0$

There
$$V_1 = IR_1 = (0.25\ \text{A})(2.5\ \Omega) = 0.625\ \text{V}$$
$$V_2 = IR_2 = (0.25\ \text{A})(5\ \Omega) = 1.25\ \text{V}$$
$$V_4 = IR_4 = (0.25\ \text{A})(4\ \Omega) = 1\ \text{V}$$

Solve for V_3 from $V_1 + V_2 + V_3 + V_4 - E = 0$:

$$V_3 = E - V_1 - V_2 - V_4 = (5 - 0.625 - 1.25 - 1)\ \text{V} = 2.125\ \text{V}$$

Therefore $$R_3 = \frac{V_3}{I} = \frac{2.125\ \text{V}}{0.25\ \text{A}} = \boxed{8.5\ \Omega}$$

2.2.5 KVL extension

Kirchhoff's voltage law (KVL) can be expended from a closed-loop circuit to any scenario loop in a circuit, because voltage or potential difference in the circuit can exist between any two points in a circuit.

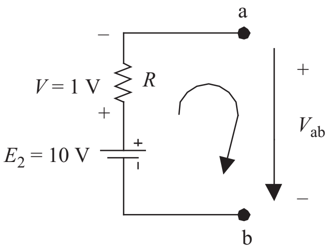


Figure 2.11 KVL extension

V_{ab} in the circuit of Figure 2.11 can be calculated using KVL #2 as follows:

$$\Sigma V = \Sigma E:\ V + V_{ab} = E_2$$
$$V_{ab} = E_2 - V$$
$$= (10 - 1)\ \text{V} = \boxed{9\ \text{V}}$$

Example 2.8: Determine the voltage across points a to b (V_{ab}) in the circuit of Figure 2.12.

Solution: V_{ab} can be solved in two methods as follows:

- Method 1: $\Sigma V = 0$: $V_1 + V_{ab} + V_4 - E = 0$
  where $V_{ab} = E - V_1 - V_4 = (5 - 1.5\ \text{V} - 1)\ \text{V} = \boxed{2.5\ \text{V}}$
- Method 2: $\Sigma V = 0$: $V_2 + V_3 - V_{ab} = 0$
  where $V_{ab} = V_2 + V_3 = (2 + 0.5)\ \text{V} = \boxed{2.5\ \text{V}}$

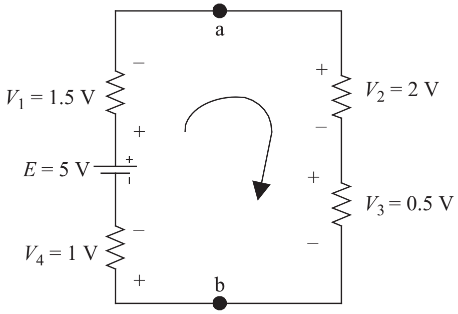


Figure 2.12 Circuit for Example 2.8

The physical property of KVL: The results from Example 2.8 show that voltage across two points a and b is the same, and it does not matter which path or branch is used to solve for voltage between these two points, the result should be the same. Therefore, the physical property of KVL is that *voltage does not depend on the path*.

2.3 Kirchhoff's current law

2.3.1 Kirchhoff's current law (KCL) #1

Node and branch

- A node (or junction) is the intersectional point of two or more current paths where current has several possible paths to flow.
- A branch is a current path between two nodes with one or more circuit components in series. For instance, point A is a node in Figure 2.13, and it has six branches. I_1, I_2, and I_3 are the currents flowing into the node A; I_4, I_5, and I_6 are the currents exiting the node A.

Kirchhoff's current law (KCL) #1: The algebraic sum of the total currents at entering and exiting a node (or junction) of the circuit is equal to zero, i.e., $\boxed{\Sigma I = 0}$.

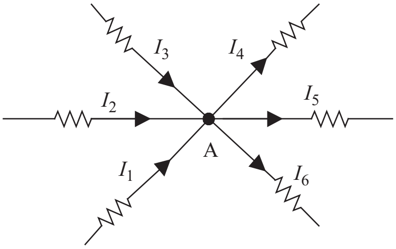


Figure 2.13 Nodes and branches

Signs of the I in the $\Sigma I = 0$

- Assign a positive sign (+) to the current in the equation (KCL #1) if current is entering the node.
- Assign a negative sign (−) to the current in the equation if current is exiting the node.

Example 2.9: Applying KCL #1 in Figure 2.13.

Solution: $\boxed{I_1 + I_2 + I_3 - I_4 - I_5 - I_6 = 0}$

KCL #1 **$\Sigma I = 0$**	– Assign a (+) sign for current in KCL if I is entering the node. – Assign a (−) for current in KCL if I is exiting the node.

Example 2.10: Determine current I_4 using KCL #1 of Figure 2.14.

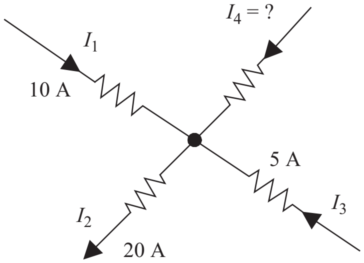


Figure 2.14 Figure for Example 2.10

Solution: $\Sigma I = 0$: $I_1 - I_2 + I_3 + I_4 = 0$

$$I_4 = I_2 - I_1 - I_3 = 20\text{ A} - 10\text{ A} - 5\text{ A} = \boxed{5\text{ A}}$$

2.3.2 *Kirchhoff's current law (KCL) #2*

Kirchhoff's current law (KCL) #2

Kirchhoff's current law can also be expressed in another way: The total current flowing into a node is equal to the total current flowing out of the node, i.e.,

$$\boxed{\Sigma I_{\text{in}} = \Sigma I_{\text{out}}}$$

Signs of the I_{in} in the $\Sigma I_{\text{in}} = \Sigma I_{\text{out}}$

- Assign a positive sign (+) to current I_{in} in the equation (KCL #2) if current is entering the node.
- Assign a negative sign (−) to I_{in} if current is exiting the node.

Signs of the I_{out} in the $\Sigma I_{in} = \Sigma I_{out}$

- Assign a positive sign (+) to current I_{out} in the equation (KCL #2) if current is exiting the node.
- Assign a negative sign (−) to I_{out} if current is entering the node.

Example 2.11: Verify KCL #2 and KCL #1 for the circuit of Figure 2.15.

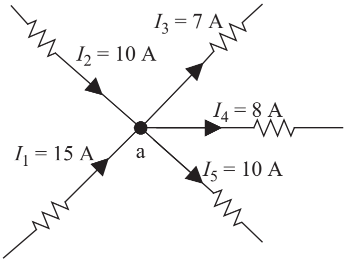


Figure 2.15 Figure for Example 2.11

Solution:

- KCL #2: $\Sigma I_{in} = \Sigma I_{out}$: $I_1 + I_2 = I_3 + I_4 + I_5$

  Substituting I with its respective values: $(15 + 10)\text{ A} = (7 + 8 + 10)\text{ A}$

  $25\text{ A} = 25\text{ A}$ (hence proved)
- KCL #1: $\Sigma I = 0$: $I_1 + I_2 - I_3 - I_4 - I_5 = 0$

  Substituting I with its respective values: $(10 + 15 - 7 - 8 - 10)\text{ A} = 0$

  $0\text{ A} = 0$ (hence proved)

KCL #2 $\boxed{\Sigma I_{in} = \Sigma I_{out}}$	– Assign a (+) sign for I_{in} if current is entering the node; assign a (–) sign for I_{in} if current is exiting the node. – Assign a (+) for I_{out} if current is exiting the node; assign a (–) sign for I_{out} if current is entering the node.

Example 2.12: Determine the current I_1 at node A and B in Figure 2.16.

Solution:

- Node A: $\Sigma I = 0$: $I_1 - I_2 - I_3 - I_4 = 0$

  $\Sigma I_{in} = \Sigma I_{out}$: $\boxed{I_1 = I_2 + I_3 + I_4}$

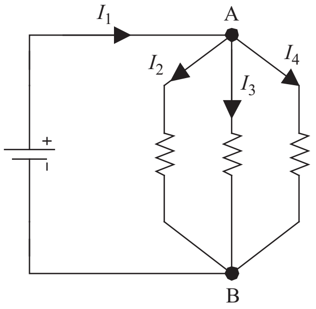


Figure 2.16 Circuit for Example 2.12

- Node B: $\Sigma I = 0$: $\quad I_2 + I_3 + I_4 - I_1 = 0$

  $\Sigma I_{\text{in}} = \Sigma I_{\text{out}}$: $\quad I_2 + I_3 + I_4 = I_1$

  or $\quad \boxed{I_1 = I_2 + I_3 + I_4}$

Experimental circuit of KCL

KCL can be proved by an experimental circuit in Figure 2.17:

- Measure branch currents I_1 and I_2 (entering) using two multimeters (ammeter function).
- I_1 and I_2 are equal to the source branch current I_3 (exiting). $I_3 = I_1 + I_2 = 0.25$ A

2.3.3 *Physical property of KCL*

Water flow analogy to electric current

- Water flowing in a pipe can be analogized as current flowing in a conducting wire with KCL.
- Water flowing into a pipe should be equal to the water flowing out of the pipe.
- In Figure 2.18, water flows in the three upstream creeks A, B, and C merging together to a converging point and forms the main water flow out of the converging point to the downstream creek.

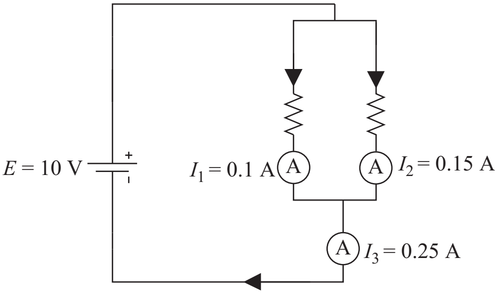


Figure 2.17 Experimental circuit for KCL

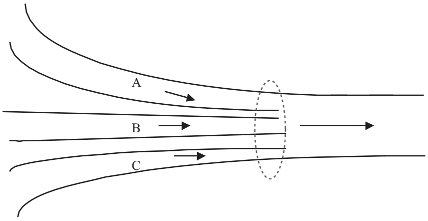


Figure 2.18 Creeks

Example 2.13: Determine current I_4 using KCL #2 of Figure 2.19.

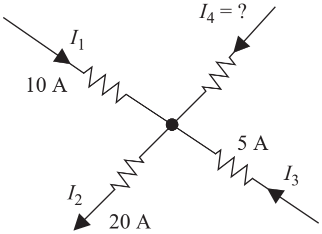


Figure 2.19 Circuit for Example 2.13

Solution:

$\Sigma I_{in} = \Sigma I_{out}$: $I_1 + I_3 + I_4 = I_2$

$I_4 = I_2 - I_3 - I_1$ Solve for I_4.

$I_4 = 20\,\text{A} - 10\,\text{A} - 5\,\text{A} = \boxed{5\,\text{A}}$ Substituting I with its respective values.

10 A + 5 A + 5 A = 20 A, ($\Sigma I_{in} = \Sigma I_{out}$) 20 A = 20 A (Proved)

Physical property of KCL: The physical property of KVL is that charges cannot accumulate in a node; what arrives at a node is what leaves that node.

- This results from the conservation of charges, i.e., charges can neither be created nor be destroyed or the amount of charges that enter the node equals the amount of charges that exit the node.
- Another property of KCL is the continuity of current (or charges), which is similar to the continuity of flowing water, i.e., the water or current will never discontinue at any moment in a pipe or conductor.

2.3.4 Procedure to solve a complicated problem

Steps to solve a complicated problem

1. Start from the unknown value in the problem and find the right equation that can solve this unknown.
2. Determine the new unknown of the equation in step 1 and find the equation to solve this unknown.

3. Repeat steps 1 and 2 until there are no more unknowns in the equation.
4. Substitute the solution from the last step into the previous equation, and solve the unknown. Repeat until the unknown in the original problem is solved.

It does not matter which field of natural science the problems are belonging to or how complicated they are, the procedure for analyzing and solving them are all similar.

Example 2.14: Determine the current I_1 of Figure 2.20.

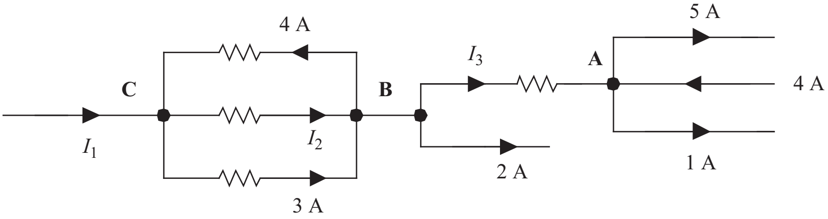


Figure 2.20 Circuit for Example 2.14

Solution:

The unknown in this problem is I_1. Find the right equation to solve I_1.

- At node C: $I_1 + 4\text{ A} = I_2 + 3\text{ A}$ (2.1) $I_2 = ?$ $\Sigma I_{\text{in}} = \Sigma I_{\text{out}}$

  (Besides I_1, the unknown in this equation is I_2.)
- Find the right equation to solve I_2.

  At node B: $I_2 + 3\text{ A} = 4\text{ A} + I_3 + 2\text{ A}$ (2.2) $I_3 = ?$ $\Sigma I_{\text{in}} = \Sigma I_{\text{out}}$

  (Besides I_2, the unknown in this equation is I_3.)
- Find the right equation to solve I_3.

  At node A: $I_3 + 4\text{ A} = (5 + 1)\text{ A}$, solve for I_3 : $I_3 = 2\text{ A}$ $\Sigma I_{\text{in}} = \Sigma I_{\text{out}}$

  (There are no more unknown elements in this equation except for I_3.)
- Substitute $I_3 = 2\text{A}$ into Equation (2.2) and solve for I_2:

$$I_2 + 3\text{ A} = (4 + 2 + 2)\text{ A}, \qquad \text{so} \qquad I_2 = 5\text{ A}$$

- Substitute $I_2 = 5\text{A}$ into (2.1) and solve for I_1:

$$I_1 + 4\text{ A} = (5 + 3)\text{ A}, \qquad \text{therefore,} \qquad I_1 = \boxed{4\text{ A}}$$

2.3.5 Supernode

Supernode: The concept of the node can be extended to a circuit that contains several nodes and branches, and this circuit can be treated as a supernode.

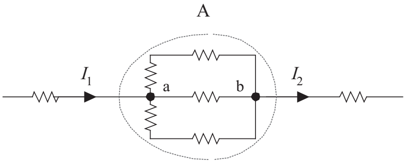


Figure 2.21 Supernode

- The circuit between nodes a and b in Figure 2.21 within the dashed circle can be treated as an extended node or supernode A.
- KCL can be applied to it: $\Sigma I_{\text{in}} = \Sigma I_{\text{out}}$ or $I_1 = I_2$.

Example 2.15: Determine the magnitudes and directions of I_3, I_4, and I_7 of Figure 2.22.

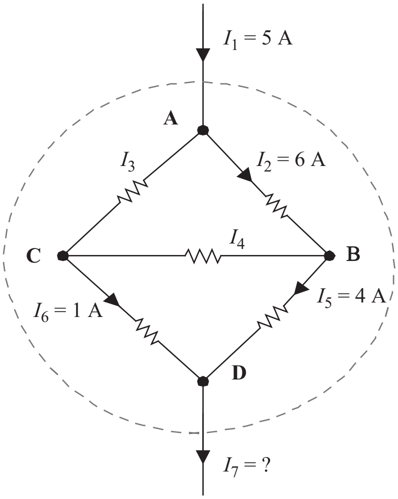


Figure 2.22 Circuit for Example 2.15

Solution:

- Treat the circuit between the nodes A and D (inside of the circle) as a supernode, and current entering the node A should be equal to current exiting the node D, therefore,

$$\boxed{I_7} = I_1 \boxed{= 5\text{A}}$$

- At node A: Since current entering node A is $I_1 = 5$ A,
  current leaving node A is $I_2 = 6$ A, so $I_2 > I_1$
  I_3 must be current entering node A to satisfy $\Sigma I_{\text{in}} = \Sigma I_{\text{out}}$
  i.e., $I_1 + I_3 = I_2$ or $5\text{A} + I_3 = 6\text{A}$, therefore, $\boxed{I_3 = 1\text{ A}}$

- At node B: Since current entering node B is $I_2 = 6$ A
  currents exiting node B is $I_5 = 4$ A, so $I_2 > I_5$
  I_4 must be current exiting node B to satisfy $\Sigma I_{in} = \Sigma I_{out}$
  i.e., $I_2 = I_4 + I_5$ or $6\text{A} = I_4 + 4\text{A}$, therefore, $\boxed{I_4 = 2\text{ A}}$
- Prove it at node C: $I_4 = I_3 + I_6$, 2 A = 1 A + 1 A, 2 A = 2 A (proved) $\Sigma I_{in} = \Sigma I_{out}$
  (I_3 is the current entering node A; I_4 is the current exiting node B.)

2.3.6 Some important circuit terminologies

Several important circuit terminologies

- Node: The intersectional point of two or more current paths where current has several possible paths to flow.
- Branch: A current path between two nodes where one or more circuit components is in series.
- Loop: A complete current path where current flows back to the start.
- Mesh: A loop in the circuit that does not contain any other loops (non-redundant loop).

Note

- A mesh is always a loop, but a loop is not necessarily a mesh.
- A mesh can be analogized as a windowpane, and a loop may include several such windowpanes.

Example 2.16: List the nodes, branches, meshes, and loops in Figure 2.23.

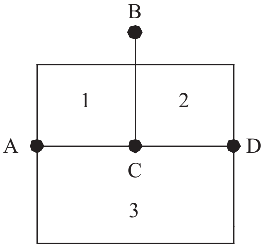


Figure 2.23 Illustration for Example 2.16

Solution:

- Node: four nodes—A, B, C, and D
- Branch: six branches—AB, BD, AC, BC, CD, and AD
- Mesh: 1, 2, and 3
- Loop: 1, 2, 3, A-B-D-C-A, A-B-D-A, etc.

2.4 Voltage source and current source

2.4.1 Ideal voltage source

Power supply

It is a circuit device that provides electrical energy to drive the system.

- A power supply is a source that can provide EMF (electromotive force) and current to operate the circuit.
- The power supply can be classified into two categories: voltage source and current source.

Ideal voltage source

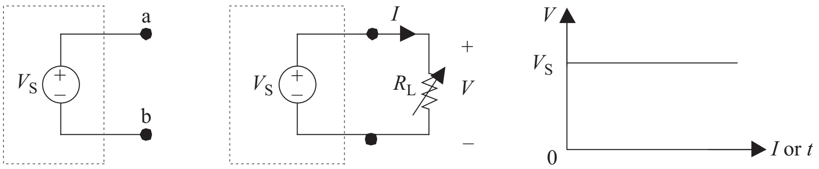
It is a two-terminal circuit device that can provide a constant output voltage V_{ab}, across its terminals, and is shown in Figure 2.24(a).

- Voltage of the ideal voltage source, V_S, will not change even if an external circuit such as a load R_L, is connected to it as shown in Figure 2.24(b), so it is an independent voltage source.
- The voltage of the ideal voltage source is independent of variations in its external circuit or load.
- The ideal voltage source has a zero internal resistance ($R_S = 0$), and it can provide maximum current to the load.

The characteristic curve of an ideal voltage source

- Current in the ideal voltage source is dependent on the variations in its external circuit.
- When the load resistance R_L changes, the current in the ideal voltage source also changes since $I = V/R_L$.
- The characteristic curve of an ideal voltage source is shown in Figure 2.24(c). The terminal voltage V_{ab} for an ideal voltage source is a constant, and same as the source voltage ($V_{ab} = V_S$), regardless of its load resistance R_L.

Ideal voltage source	– It can provide a constant terminal voltage that is independent of the variations in its external circuit, $V_{ab} = V_S$. – Its internal resistance, $R_S = 0$. – Its current depends on the variations in its external circuit.



(a) Ideal voltage source (b) Ideal voltage source with a load (c) Characteristic curve

Figure 2.24 Ideal voltage source

2.4.2 Real voltage source

Real voltage source (or voltage source)

- Usually a real-life application of a voltage source such as a battery, DC generator, or DC power supply will not reach a *perfect* constant output voltage after it is connected to an external circuit or load, since nothing is perfect.
- The real voltage sources all have a nonzero internal resistance R_S. $V_{ab} = V_S - IR_S$
- The real voltage source (or voltage source) can be represented as an ideal voltage source V_S in series with an internal resistor R_S as shown in Figure 2.25(a).
- Once a load resistor R_L is connected to the voltage source (Figure 2.25(b)), the terminal voltage of the source V_{ab} will change if the load resistance R_L changes.

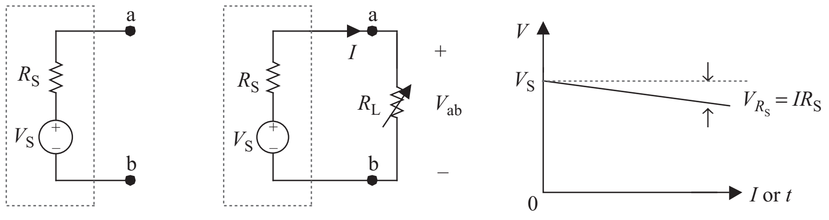
Small internal resistance R_S

- Since the internal resistance R_S is usually very small, V_{ab} will be a little bit lower than the source voltage V_S ($V_{ab} = V_S - IR_S$). $\boxed{I = \frac{V_S}{R_S + R_L}}$
- A smaller internal resistance can also provide a higher current through the external circuit of the real voltage source because $I \uparrow = \frac{V_S}{R_S \downarrow + R_L}$ (apply Ohm's law in Figure 2.25(b)).
- Once the load resistance R_L changes, current I in this circuit will change, and the terminal voltage V_{ab} also changes. This is why the terminal voltage of the real voltage source is not possible to keep at an ideal constant level ($V_{ab} \neq V_S$).
- The internal resistance of a real voltage source usually is much smaller than the load resistance, i.e., $R_S \ll R_L$, so the voltage drop on the internal resistance (IR_S) is also very small, and therefore, the terminal voltage of the real voltage source (V_{ab}) is approximately stable:

$$V_{ab} = V_S - IR_S \approx V_S$$

When a battery is used as a real voltage source, the older battery will have a higher internal resistance R_S and a lower terminal voltage V_{ab}.

Real voltage source (Voltage source)	– It has a series internal resistance R_S, and $R_S \ll R_L$ – The terminal voltage of the real voltage source is $V_{ab} = V_S - IR_S$



(a) Real voltage source (b) Real voltage source with a load (c) Characteristic curve

Figure 2.25 Real voltage source

Example 2.17: Determine the terminal voltages of the circuit in Figure 2.26(a) and (b).

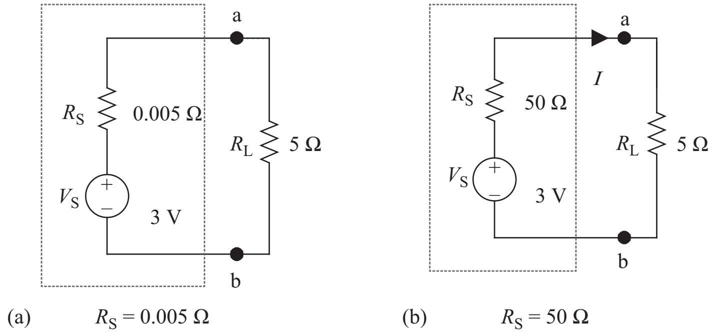


Figure 2.26 Circuit for Example 2.17

- When $R_S = 0.005\ \Omega$, $I = \dfrac{V_S}{R_S + R_L} = \dfrac{3\text{ V}}{(0.005 + 5)\ \Omega} \approx 0.5994\text{ A}$

  $$V_{ab} = IR_L = (0.5994\text{ A})(5\ \Omega) = \boxed{2.997\text{ V}}$$

- When $R_S = 50\ \Omega$, $I = \dfrac{V_S}{R_S + R_L} = \dfrac{3\text{ V}}{(50 + 5)\ \Omega} \approx 0.055\text{ A}$

  $$V_{ab} = IR_L = (0.055\text{ A})(5\ \Omega) = \boxed{0.275\text{ V}}$$

The internal resistance has a great impact on the terminal voltage and current

- Example 2.17 indicates that the internal resistance has a great impact on the terminal voltage and current of the voltage source.
- Only when the internal resistance is very small, can the terminal voltage of the source be kept approximately stable, such as in Example 2.17, when

  $$R_S = 0.005\ \Omega, \qquad V_{ab} = 2.997\text{ V}, \qquad V_S = 3\text{ V}$$

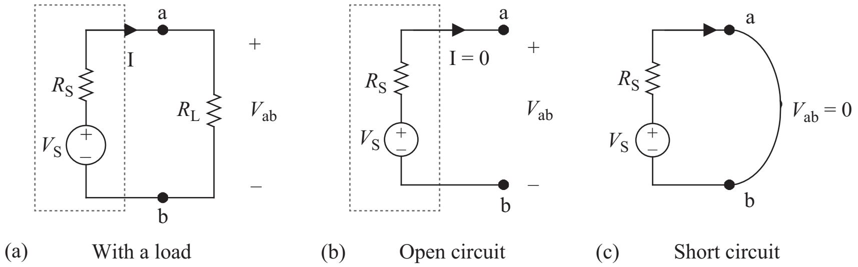
  In this case, the terminal voltage V_{ab} is very close to the source voltage V_S.
- But when $R_S = 50\ \Omega$, $V_{ab} = 0.275\text{ V} \ll V_S = 3\text{ V}$

  i.e., the terminal voltage V_{ab} is much less than the source voltage V_S.

A real voltage source has three possible working conditions

- When an external load R_L is connected to a voltage source (Figure 2.27(a)):

  $$V_{ab} = V_S - IR_S, \quad I = \frac{V_S}{R_S + R_L}$$



(a) With a load (b) Open circuit (c) Short circuit

Figure 2.27 Three states of a voltage source

- Open circuit: When there is no external load R_L connected to a voltage source (Figure 2.27(b)):

$$V_{ab} = V_S, \qquad I = 0$$

- Short circuit: When a jump wire is connected to the two terminals of a voltage source (Figure 2.27(c)):

$$V_{ab} = 0, \qquad I = V_S/R_S$$

2.4.3 *Ideal current source*

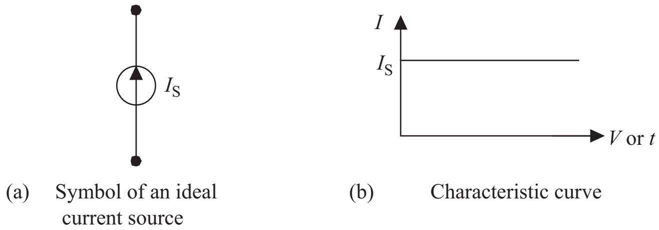
The current source is a circuit device that can provide a stable current to the external circuit. A transistor, an electronic element you may have heard, can be approximated as an example of a current source.

Ideal current source

- An ideal current source is a two-terminal circuit device that can provide a constant output current I_S through its external circuit.
- Current of the ideal current source will not change even an external circuit (load R_L) is connected to it, so it is an independent current source.
- The current of the ideal current source is independent of variations in its external circuit or load.
- Two-terminal voltage of the ideal current source is determined by the external circuit or load.
- The symbol of an ideal current source is shown in Figure 2.28(a), and its characteristic curve is shown in Figure 2.28(b).

The ideal current source has an infinite internal resistance ($R_S = \infty$), it can provide a maximum current to the load.

I_S represents the current for current source, and the direction of the arrow is the current direction of the source.



(a) Symbol of an ideal current source (b) Characteristic curve

Figure 2.28 Ideal current source

Ideal current source	– It can provide a constant output current I_S that does not depend on the variations in its external circuit. – Its internal resistance $R_S = \infty$. – Its voltage depends on variations in its external circuit. $V_{ab} = I_S R_L$

Example 2.18: The load resistances of R_L are 1,000 Ω and 50 Ω, respectively, in Figure 2.29. Determine the terminal voltage V_{ab} for the ideal current source in the circuit.

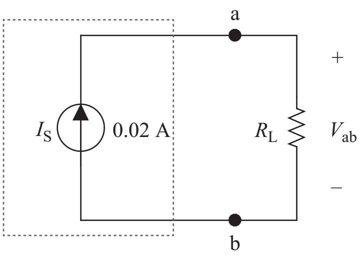


Figure 2.29 Circuit for Example 2.18

Solution:

- When $R_L = 1{,}000\ \Omega$,
$$\begin{aligned} V_{ab} &= I_S R_L \\ &= (0.02\ \mathrm{A})(1{,}000\ \Omega) \\ &= \boxed{20\ \mathrm{V}} \end{aligned}$$
- When $R_L = 50\ \Omega$,
$$\begin{aligned} V_{ab} &= I_S R_L \\ &= (0.02\ \mathrm{A})(50\ \Omega) \\ &= \boxed{1\ \mathrm{V}} \end{aligned}$$

The conditions of open circuit and short circuit of an ideal current source are as follows:

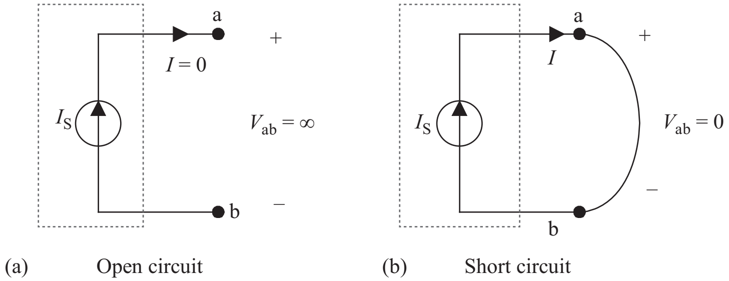
- Open circuit: $V_{ab} = \infty$, $I = 0$, as shown in Figure 2.30(a).
- Short circuit: $V_{ab} = 0$, $I = I_S$, as shown in Figure 2.30(b).

2.4.4 Real current source

Usually a real-life application of the current source will not reach a perfect constant output current after it is connected to an external circuit or load, as the real current sources all have a non-infinite internal resistance R_S.

Real current source (or current source)

- The real current source can be represented as an ideal current source I_S in parallel with an internal resistor R_S.
- Once a load resistor R_L is connected to the current source as shown in Figure 2.31, the current of the source will change if the load resistance R_L changes.
- Since the internal resistance R_S of the current source usually is very large, the load current I will be a little bit lower than the source current I_S.
- Once the load resistance R_L changes, the current in the load will also change. This is why the current of the real current source is not possible to keep at an ideal constant level.



(a) Open circuit (b) Short circuit

Figure 2.30 Open circuit and short circuit of an ideal current source

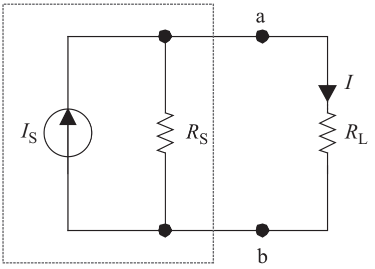


Figure 2.31 A real current source

Internal resistance

- A higher internal resistance R_S can provide a higher current through the external circuit of the real current source.
- The internal resistance of a real current source usually is much greater than the load resistance ($R_S \gg R_L$) and, therefore, the output current of the real current source is approximately stable.

Real current source (Current source)	– It has an internal resistance R_S ($R_S \gg R_L$). – R_S is in parallel with the current source.

Summary

Basic concepts

- Energy: the capacity to do work.
- Work: a transfer of energy.
- Power: the speed of energy conversion, or work done per unit of time.
- Electric power: the speed of electrical energy conversion or consumption in an electric circuit, and it is a measure of how fast electrons or charges are moving in a circuit.
- The reference direction of power:
  - If a circuit has mutually related reference polarity of current and voltage: $P > 0$ (absorption energy).
  - If a circuit has non-mutually related reference polarity of current and voltage: $P < 0$ (releasing energy).
- Branch: a current path between two nodes where one or more circuit components in series.
- Node: the intersectional point of two or more current paths where current has several possible paths to flow.
- Supernode: a part of the circuit that contains several nodes and branches.
- Loop: a complete current path where current flows back to the start.
- Mesh: a loop in the circuit that does not contain any other loops.
- Ideal voltage source: can provide a constant terminal voltage that does not depend on the variables in its external circuit. Its current depends on variables in its external circuit $V_{ab} = V_S$, $R_S = 0$.
- Real voltage source: with a series internal resistance R_S ($R_S \ll R_L$), the terminal voltage of the real voltage source is: $V_{ab} = V_S - IR_S$
- Ideal current source: can provide a constant output current I_S that does not depend on the variations in its external circuit, $R_S = \infty$. Its voltage depends on variations in its external circuit.
- Real current source: with an internal resistance R_S in parallel with the ideal current source, $R_S \gg R_L$.

Formulas

- Work: $W = FS$
- Power: $P = \frac{W}{t}$
- Electric power: $P = IV = I^2R = \frac{V^2}{R}$
- KVL #1: $\Sigma V = 0$
  - Assign a (+) sign for V or E if its reference polarity and loop direction are the same.
  - Assign a (−) sign for V or E if its reference polarity and loop direction are opposite.
- KVL #2: $\Sigma V = \Sigma E$
  - Assign a (+) sign for V if its reference polarity and loop direction are the same; assign a (−) sign for V if its reference direction and loop direction are opposite.
  - Assign a (−) sign for E if its reference polarity and loop direction are the same; assign a (+) for E if its polarity and loop direction are opposite.
- KCL #1: $\Sigma I_{\text{in}} = 0$
  - Assign a (+) sign for I if current is entering the node.
  - Assign a (−) sign for I if current is exiting the node.
- KCL #2: $\Sigma I_{\text{in}} = \Sigma I_{\text{out}}$
  - Assign a (+) sign for I_{in} if current is entering the node; assign a (−) sign for I_{in} if current is exiting the node.
  - Assign a (+) sign for I_{out} if current is exiting the node; assign a (−) sign for I_{out} if current is entering the node.

Practice problems

2.1

1. ( ) is the result when a force acts on an object and causes it to move a certain distance.
2. ( ) is the capacity to do ( ).
3. ( ) is a measure of how fast energy is transforming.
4. A device consumes 100 J energy in 5 s. Calculate its power.
5. The current flowing through a resistor is 1.5 A, and the power supply in this circuit is 6 V. Determine the power transferred from the source to the resistor.
6. The current flowing through a 220 Ω resistor is 0.01 A. Determine the power consumed by this resistor.
7. The power dissipated on a 100 Ω resistor is 0.1 W. Determine the current flowing through this resistor.

8. The power consumed by a 100 Ω resistor is 1 W. Determine the voltage of this resistor.
9. Calculate the power on the element according to the reference direction and the element values in the circuit of Figure 2.32. Determine if the resistor is absorbing or generating power.

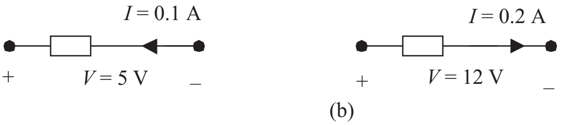


Figure 2.32

2.2

10. Four resistors are in series and connected to a 10 V voltage source. If the voltages across three resistors are $V_1 = 2.5$ V, $V_2 = 1.5$ V, and $V_4 = 3$ V, determine V_3.
11. Determine the unknown voltage V_{ab} in the circuit of Figure 2.33.

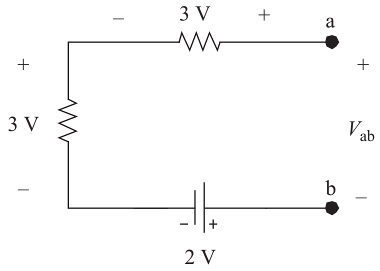


Figure 2.33

12. Determine the resistance R_2 and voltage across R_2 in the circuit of Figure 2.34.

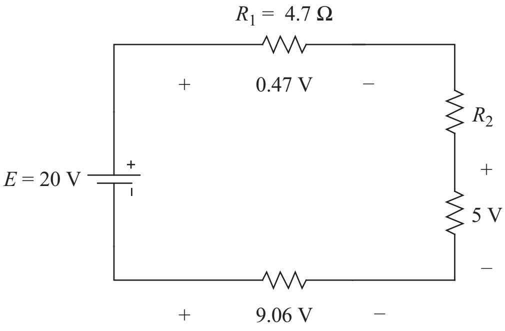


Figure 2.34

13. Calculate the power consumed by R_2 in the circuit of Figure 2.35.

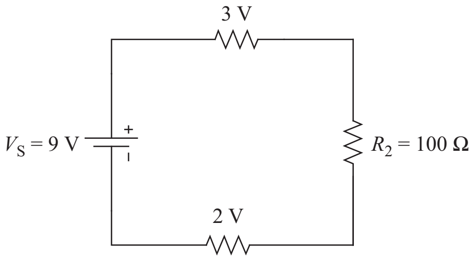

3 V
$V_S = 9$ V
+
−
$R_2 = 100\ \Omega$
2 V

Figure 2.35

2.3

14. Four resistors are in parallel. The current in the branch of the current source is 3 A, and other branch currents are $I_1 = 0.5$ A, $I_2 = 0.8$ A, and $I_4 = 0.05$ A, respectively, determine the branch current I_3.
15. Determine the unknown currents in the circuit of Figure 2.36.

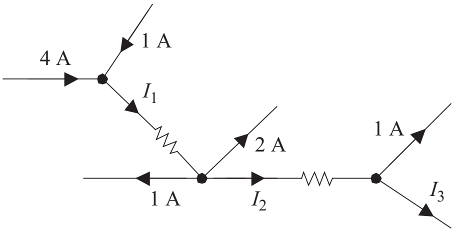

1 A
4 A
I_1
2 A
1 A
1 A
I_2
I_3

Figure 2.36

16. Determine the unknown currents in the circuit of Figure 2.37.

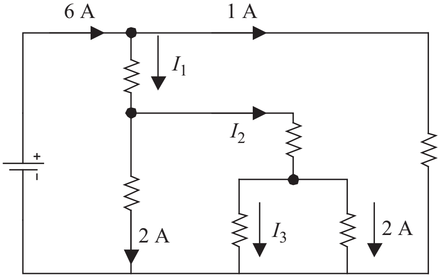

6 A
1 A
I_1
I_2
+
−
2 A
I_3
2 A

Figure 2.37

17. Determine the unknown currents in the circuit of Figure 2.38.

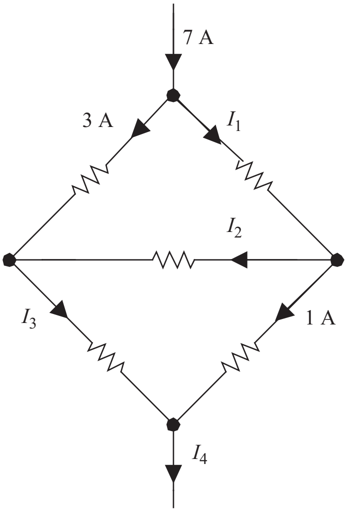

7 A
3 A
I_1
I_2
I_3
1 A
I_4

Figure 2.38

18. Determine the nodes, branches, meshes, and at least three loops in the circuit of Figure 2.39.

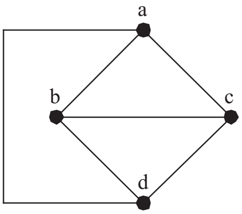


Figure 2.39

19. What is the relationship between voltage V_{AB} and current I in the circuit of Figure 2.40?

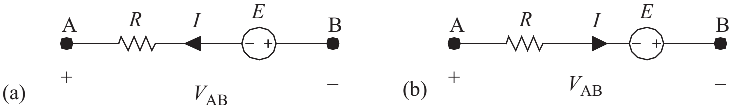

A
R
I
E
B
+
−
(a)
V_{AB}
A
R
I
E
B
+
−
(b)
V_{AB}

Figure 2.40

2.4

20. The difference between the ideal voltage source and the real voltage source is that the real voltage source has (   ), and its terminal voltage is (   ); the terminal voltage of an ideal voltage source is (   ).
21. The open-circuit voltage measured at the two terminals of two batteries in series is 14.2 V.
    (a) After the batteries are connected to a 100 Ω resistor, their terminal voltage decreased to 6.8 V; determine the internal resistance of the batteries.
    (b) Determine the terminal voltage of the batteries after the 100 Ω resistor is replaced by a 200 Ω resistor.
22. Determine voltage V_{ab} in the circuit of Figure 2.41.

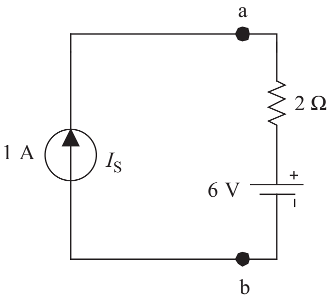


Figure 2.41

Chapter 3

Series–parallel resistive circuits

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3.1 Series resistive circuits and voltage divider rule

3.1.1 Series resistive circuits

Series circuit

There is only one path for current to flow.

- The components are connected one after the other.
- The current flow through each component is always the same.
- There is only one current path in a series circuit.

A series circuit has all its elements connected in one loop of wire (a closed circuit).

Example 3.1: An electrical circuit with three light bulbs (resistors) connected in series (Figure 3.1).

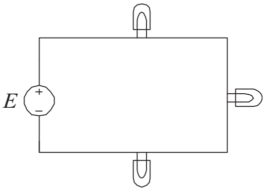


Figure 3.1 Series circuit

A series circuit can be analogized by water flowing in a series of tanks connected by a pipe.

- The water flows through the pipe from tank to tank. The same amount of water will flow in each tank.
- The same is true of an electrical circuit. There is only one pathway by which charges can travel in a series circuit. The same amount of charges will flow in each component of the circuit.

Schematic diagrams of series resistive circuits

- Figure 3.2(b) and (c) are also series circuits but drawn in different ways.
- If the circuit elements are connected one after the other, and there is only one current path for the circuit, it is said that they are connected in series.

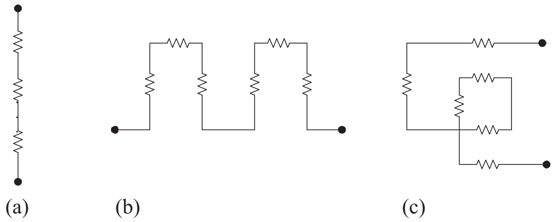


Figure 3.2 Series resistive circuits

3.1.2 Series voltage and resistance

Total series voltage (V_T or E)

- The voltage across the source or power supply (total voltage) is equal to the sum of the voltage that drops across each resistor in a series circuit (Figure 3.3).
- The terminal of the resistor connecting to the positive side (+) of the voltage source is positive.
- The terminal of the resistor connecting to the negative side (−) of the voltage source is negative.

Calculating the total voltage

- The total voltage of a series resistive circuit (*n* resistors):

$$\boxed{V_T = E = V_1 + V_2 + \cdots + V_n}$$

- The total voltage V_T of a series resistive circuit can be determined by Kirchhoff's voltage law (KVL) and Ohm's law.

$$\boxed{V_T = IR_1 + IR_2 + \cdots + IR_n = IR_T}$$

Total series resistance (R_T) or equivalent resistance (R_{eq})

- The total resistance (R_T) of a series resistive circuit is the sum of all resistances in the circuit.
- Calculating total or equivalent resistance of a series resistive circuit:

$$\boxed{R_T = R_{eq} = R_1 + R_2 + \cdots + R_n}$$

- The total resistance (R_T) is also called the equivalent resistance (R_{eq}) because this resistance is equivalent to the sum of all resistances when you look through the two terminals of the series resistive circuit.
- The total resistance of the series resistive circuit is always greater than the individual resistance in that circuit.

3.1.3 Series current and power

Series current (*I*)

- The current flowing through each element in a series circuit is always the same (there is only one current path in a series circuit).
- The current is always the same at any point in a series circuit.

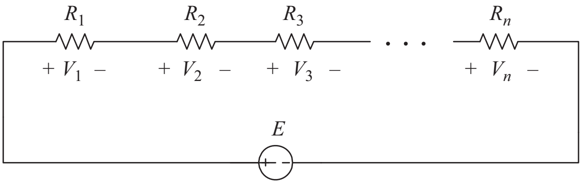


Figure 3.3 Series circuit

- Calculating the current of a series resistive circuit.

$$\boxed{I = \frac{V_T}{R_\mathrm{T}} = \frac{E}{R_T} = \frac{V_1}{R_1} = \frac{V_2}{R_2} = \ldots = \frac{V_n}{R_n}}$$

Total series power (P_T)

- Each of the resistors in a series circuit consumes power, which is dissipated in the form of heat.
- The total power (P_T) consumed by a series resistive circuit is the sum of power dissipated by the individual resistor.
- Since the total power must come from the source, it is actually the power supplied by the source.

Calculating the total power

- Total power: $P_\mathrm{T} = IE = IV_1 + IV_2 + \cdots + IV_\mathrm{n}$

$$\boxed{P_\mathrm{T} = P_1 + P_2 + \cdots + P_n}$$

or

$$\boxed{P_\mathrm{T} = IE = I^2 R_\mathrm{T} = \frac{E^2}{R_\mathrm{T}}}$$

- The power dissipated by the individual resistor in a series resistive circuit:

$$\boxed{P_1 = I^2 R_1 = IV_1 = \frac{V_1^2}{R_1}}$$

$$\boxed{P_2 = I^2 R_2 = IV_2 = \frac{V_2^2}{R_2}}$$

$\cdots \quad \cdots$

$$\boxed{P_n = I^2 R_n = IV_n = \frac{V_n^2}{R_n}}$$

3.1.4 An example of a series circuit

Example 3.2: A series resistive circuit is shown in Figure 3.4. Determine the following:

1. Total resistance R_T
2. Current I in the circuit
3. Voltage across the resistor R_1
4. Total voltage V_T
5. Total power P_T

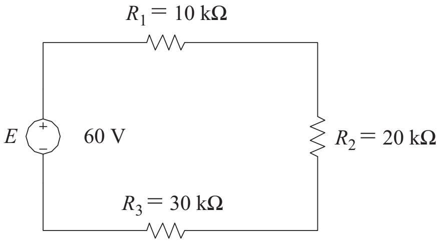

$R_1 = 10\ \mathrm{k\Omega}$
E
60 V
$R_2 = 20\ \mathrm{k\Omega}$
$R_3 = 30\ \mathrm{k\Omega}$

Figure 3.4 Circuit for Example 3.2

Solution:

1. $R_T = R_1 + R_2 + R_3 = (10 + 20 + 30)\ \mathrm{k\Omega} = \boxed{60\ \mathrm{k\Omega}}$
2. $I = \dfrac{E}{R_T} = \dfrac{60\ \mathrm{V}}{60\ \mathrm{k\Omega}} = \boxed{1\ \mathrm{mA}}$
3. $V_1 = IR_1 = (1\ \mathrm{mA})(10\ \mathrm{k\Omega}) = \boxed{10\ \mathrm{V}}$
4. $V_T = IR_T = (1\ \mathrm{mA})(60\ \mathrm{k\Omega}) = \boxed{60\ \mathrm{V}}$, $V_T = E = 60\ \mathrm{V}$ (Checked)
5. $P_T = IE = (1\ \mathrm{mA})(60\ \mathrm{V}) = \boxed{60\ \mathrm{mW}}$
   or $P_T = I^2R_T = (1\ \mathrm{mA})^2(60\ \mathrm{k\Omega}) = \boxed{60\ \mathrm{mW}}$ (Checked)

3.1.5 Voltage divider rule (VDR)

The VDR can be exhibited by using a potentiometer (or pot).

Pot: a variable resistor whose resistance across its terminals can be varied by turning a knob.

- A pot is connected to a voltage source, as shown in Figure 3.5(a).
- Using a voltmeter to measure the voltage across the pot, the voltage relative to the negative side of the 100 V voltage source is 1/2 $E = 50$ V when the arrow (knob) is at the middle of the pot.

  The circuit in Figure 3.5(a) is equivalent to Figure 3.5(b) since $R = R_1 + R_2 = 100\ \mathrm{k\Omega}$.
- The voltage will increase when the arrow moves up, and the voltage will decrease when the arrow moves down. This is the principle of the voltage divider.

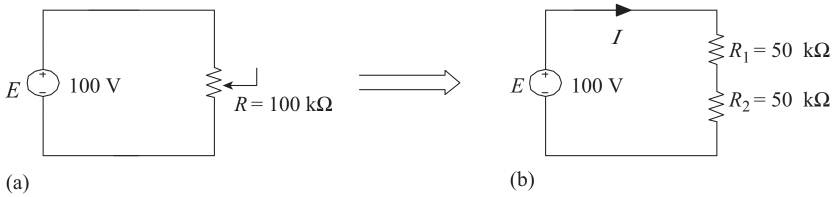

E
100 V
$R = 100\ \mathrm{k\Omega}$
(a)
E
100 V
I
$R_1 = 50\ \mathrm{k\Omega}$
$R_2 = 50\ \mathrm{k\Omega}$
(b)

Figure 3.5 Voltage divider

- The voltage divider is a design technique used to create different output voltages that is proportional to the input voltage.
- The voltage divider means that the source voltage E or total voltage V_T is divided according to the value of the resistors in the series circuit.

Voltage divider rule

- General form (when there are n resistors in series):

$$\boxed{V_X = V_T \frac{R_X}{R_T} \quad \text{or} \quad V_X = E\frac{R_X}{R_T} \quad (X = 1, 2, ..., n)}$$

  - **R_X and V_X** are the unknown resistance and voltage, respectively.
  - **R_T and V_T** are the total resistance and voltage in the series circuit, respectively.

- When there are only two resistors in series:

$$\boxed{V_1 = V_T \frac{R_1}{R_1 + R_2}, \quad V_2 = V_T \frac{R_2}{R_1 + R_2}}$$

Memory aid: The numerator of the VDR is always the unknown resistance.

Example 3.3: Use the VDR to determine the voltage drops across resistors R_2 and R_3 in the circuit of Figure 3.6.

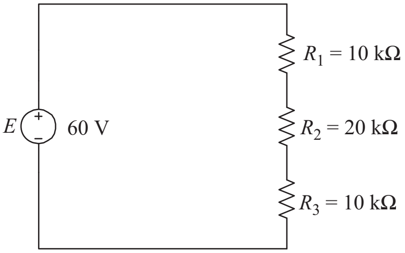


Figure 3.6 Circuit for Example 3.3

Solution: Use the general form of the VDR $V_X = E\frac{R_X}{R_T}$

$$V_2 = E\frac{R_2}{R_T} = E\frac{R_2}{R_1 + R_2 + R_3} = 60\text{ V}\frac{20\text{ k}\Omega}{(10 + 20 + 10)\text{ k}\Omega} = \boxed{30\text{ V}}$$

$$V_3 = E\frac{R_3}{R_T} = E\frac{R_3}{R_1 + R_2 + R_3} = 60\text{ V}\frac{10\text{ k}\Omega}{(10 + 20 + 10)\text{ k}\Omega} = \boxed{15\text{ V}}$$

The practical application of the voltage divider can be the volume control of an audio equipment. The knob of the pot in the circuit will eventually let you adjust the volume of the audio equipment.

3.1.6 Circuit ground

Electric circuit ground

- There is a ground for each electric circuit.
- A circuit ground is always at zero potential (0 V).
- A circuit ground provides a reference voltage level in which all other voltages in a circuit are measured.

There are two types of circuit grounds: one is the earth ground, and another is the common ground (or chassis ground).

The earth ground: connects to the earth ($V = 0$).

- The earth is always at zero potential (0 V) and measurements can be made by using earth as a reference.
- An equal number of negative and positive charges are distributed throughout the earth at any given time, the earth is an electrically neutral body.

An earth ground usually consists of a ground rod or a conductive pipe driven into the soil.

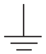
Common ground (or chassis ground): the common point for all elements in the circuit ($V = 0$).

- It is a connection to the main chassis of a piece of electronic or electrical equipment, such as a metal plate.
- All chassis grounds should lead to earth ground, so that it also provides a point that has zero voltage.

The neutral point in the alternating circuit (AC) is an example of the common ground.

The difference between the earth and chassis grounds

- Earth ground: connecting one terminal of the voltage source to the earth. The symbol for it is:



- Chassis ground or common ground: the common point for all elements in the circuit. All the common points are electrically connected together through metal plates or wires. The symbol for the common point is:

3.1.7 Voltage subscript notation

Subscript notation

- Single-subscript notation: the voltage from the subscript with respect to ground.
  In a circuit, the voltage with the single-subscript notation (such as V_b) is the voltage drop from the point b with respect to ground.
- Double-subscript notation: the voltage across the two subscripts.

The voltage with the double-subscripts notation (such as V_{bc}) is the voltage drop across the two points b and c (each point is represented by a subscript).

Example 3.4: Determine V_{bc}, V_{be}, and V_b in the circuit of Figure 3.7.

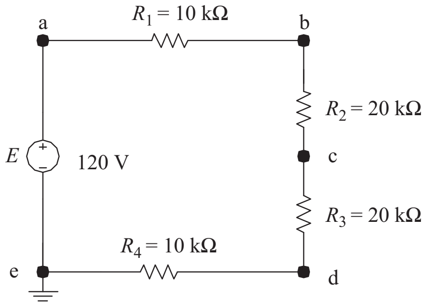


Figure 3.7 Circuit for Example 3.4

Solution:

$$V_{bc} = V_{R_2} = E\frac{R_2}{R_T} = E\frac{R_2}{R_1 + R_2 + R_3 + R_4}$$

$$= 120\ \text{V}\frac{20\ \text{k}\Omega}{(10 + 20 + 20 + 10)\text{k}\Omega} = 120\ \text{V}\frac{20\ \text{k}\Omega}{60\ \text{k}\Omega} = 40\ \text{V}$$

$$V_{be} = E\frac{R_2 + R_3 + R_4}{R_T} = 120\ \text{V}\frac{20\ \text{k}\Omega + 20\ \text{k}\Omega + 10\ \text{k}\Omega}{60\ \text{k}\Omega} = 100\ \text{V}$$

Use the general form of the VDR $V_X = E\frac{R_X}{R_T}$.

There the unknown voltage $V_x = V_{be}$, and the unknown resistance $R_X = R_2 + R_3 + R_4$.

$$V_b = V_{be} = \boxed{100\ \text{V}}$$

3.2 Parallel resistive circuits and current divider rule

3.2.1 Parallel resistive circuits

Parallel circuit: The components are connected end to end.

- There are at least two current paths in the circuit.
- The voltage across each component is the same.

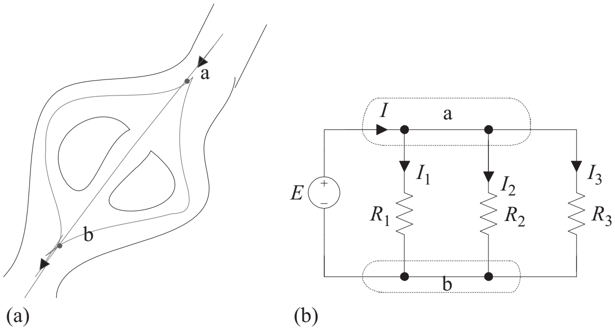


Figure 3.8 Parallel circuit

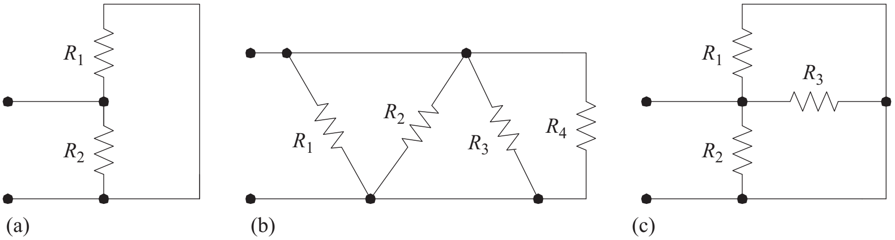


Figure 3.9 Parallel resistive circuits

Parallel circuits can be analogized by flowing water

- When water flowing in a river across small islands, the one water path will be divided by the islands and split into many more water paths (Figure 3.8(a)).
- When the water has passed the islands, it will become a single water path again.

Schematic diagrams of parallel resistive circuits

- Figures 3.9(a)–(c) are all parallel circuits but drawn in different ways.
- If the circuit elements are connected end to end and there are at least two current paths in the circuit, it is said that they are connected in parallel.

3.2.2 Parallel voltage and current

Parallel voltage

- All resistors in a parallel resistive circuit are connected between the two nodes, the voltage between these two nodes must be the same.
- The voltage drop across each resistor must be the same.
- The voltage drop across each resistor must equal the voltage of the source *E* in a parallel resistive circuit.
- Parallel voltage: $\boxed{V = E = V_1 = V_2 = \ldots = V_n}$

If all the resistors are light bulbs and have the same resistances as in Figure 3.10, they will glow at the same brightness as they each receive the same voltage.

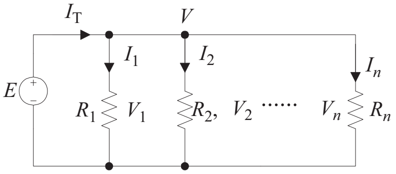


Figure 3.10 V and I in a parallel circuit

Parallel current

- If the parallel circuit was a river, the total volume of water in the river would be the sum of water in each branch (Figure 3.8(a)).
- This is the same with the current in the parallel resistive circuit. The total current is equal to the sum of currents in each resistive branch, and the total current entering and exiting parallel resistive circuit is the same.

Calculating parallel current

- Calculating the total parallel current:

$$\boxed{I_T = \frac{V}{R_{eq}} = I_1 + I_2 + \ldots I_n}$$

- If resistances are different in each branch of a parallel circuit, the branch currents will be different.
- Calculating the branch currents:

$$\boxed{I_1 = \frac{V}{R_1}}, \quad \boxed{I_2 = \frac{V}{R_2}}, \quad \ldots \quad \boxed{I_n = \frac{V}{R_n}}$$

3.2.3 Parallel resistance and power

Equivalent parallel resistance

- The amount of current flowing through each branch in the parallel resistive circuit depends on the amount of resistance in each branch.
- The total resistance of a set of resistors in a parallel resistive circuit is found by adding up the reciprocals of the resistance values and then taking the reciprocal of the total.

Calculating parallel resistance

- The equivalent parallel resistance for the parallel circuit:

$$\boxed{R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}}}$$

- Usually parallel can be expressed by a symbol of "// " such as: $R_1 // R_2 // \ldots // R_n$

$$\boxed{R_{eq} = R_1 // R_2 // \ldots // R_n}$$

- It will be more convenient to use the conductance (G) than the resistance in the parallel circuits. Since the conductance $G = \frac{1}{R}$, therefore,

$$\boxed{G_{\text{eq}} = \frac{1}{R_{\text{eq}}} = G_1 + G_2 + \ldots + G_n}$$

- When there are only two resistors in parallel:

$$\boxed{R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2} = R_1 // R_2} \qquad \text{when } n = 2$$

Note: The total resistance of the parallel resistive circuit is always less than the individual resistance.

So, usually for parallel circuits, the *equivalent* resistance is used instead of the *total* resistance.

Total parallel power

- The total power is the sum of the power dissipated by the individual resistors in a parallel resistive circuit.
- Calculating the total power in a parallel resistive circuit:

$$\boxed{P_{\text{T}} = P_1 + P_2 + \ldots + P_n}$$

or

$$\boxed{P_{\text{T}} = I_{\text{T}} V = I_{\text{T}}{}^2 R_{\text{eq}} = \frac{V^2}{R_{\text{eq}}}}$$

- The power consumed by each resistor in a parallel circuit:

$$\boxed{P_1 = I_1 V = I_1{}^2 R_1 = \frac{V^2}{R_1}, \quad P_2 = I_2 V = I_2{}^2 R_2 = \frac{V^2}{R_2}, \ldots, \quad P_n = I_n V = I_n{}^2 R_n = \frac{V^2}{R_n}}$$

3.2.4 An example of a parallel circuit

Example 3.5: A parallel circuit is shown in Figure 3.11. Determine (a) R_2, (b) I_{T}, and (c) P_3, given $R_{\text{eq}} = 1.25\ \text{k}\Omega$, $R_1 = 20\ \text{k}\Omega$, $R_3 = 2\ \text{k}\Omega$, and $I_3 = 18$ mA.

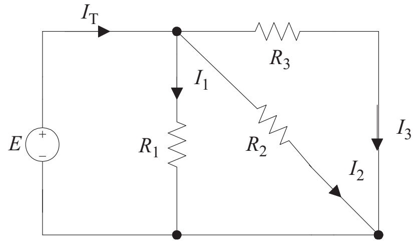


Figure 3.11 Circuit for Example 3.5

Solution:

(a) Since $R_2 = \frac{1}{G_2}$, determine G_2 first.

$$G_{\text{eq}} = G_1 + G_2 + G_3$$

$$G_2 = G_{\text{eq}} - G_1 - G_3$$

$$= \frac{1}{R_{\text{eq}}} - \frac{1}{R_1} - \frac{1}{R_3} = \frac{1}{1.25\ \text{k}\Omega} - \frac{1}{20\ \text{k}\Omega} - \frac{1}{2\ \text{k}\Omega} = 0.25\ \text{mS}$$

$$\therefore\ R_2 = \frac{1}{G_2} = \frac{1}{0.25\ \text{mS}} = 4\ \text{k}\Omega$$

$$R_{\text{eq}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{20\ \text{k}\Omega} + \frac{1}{4\ \text{k}\Omega} + \frac{1}{2\ \text{k}\Omega}} = 1.25\ \text{k}\Omega \quad \text{(Proved)}$$

(b) $I_{\text{T}} = \frac{E}{R_{\text{eq}}} = \frac{V_3}{R_{\text{eq}}} = \frac{I_3 R_3}{R_{\text{eq}}} = \frac{(18\ \text{mA})(2\ \text{k}\Omega)}{1.25\ \text{k}\Omega} = 28.8\ \text{mA}$

(c) $P_3 = I_3^2 R_3 = (18\ \text{mA})^2 (2\ \text{k}\Omega) = \boxed{648\ \text{mW}}$

3.2.5 Current divider rule (CDR)

Current divider rule (CDR)

- General form: $\boxed{I_{\text{x}} = I_{\text{T}} \frac{R_{\text{eq}}}{R_{\text{x}}}}$ or $\boxed{I_{\text{x}} = I_{\text{T}} \frac{G_{\text{x}}}{G_{\text{eq}}}}$
  - I_{x} and R_{x} are the unknown current and resistance, respectively.
  - I_{T} is the total current in the parallel resistive circuit.
- When there are two resistors in parallel:

$$\boxed{I_1 = I_{\text{T}} \frac{R_2}{R_1 + R_2}}$$

$$\boxed{I_2 = I_{\text{T}} \frac{R_1}{R_1 + R_2}}$$

- The VDR can be used for series circuits, and the CDR can be used for parallel circuits.

Memory aid

- The CDR is similar in form to the VDR. The difference is that the denominator (bottom) of the general form current divider is the unknown resistance.

- When there are two resistors in parallel, the numerator is the other resistance (other than the unknown resistance).

$$\text{Recall the VDR}: \quad V_x = V_T \frac{R_x}{R_T}, \quad V_1 = V_T \frac{R_1}{R_z + R_2}, \quad V_2 = V_T \frac{R_2}{R_1 + R_2}$$

Example 3.6: Determine the currents I_1, I_2, and I_3 in the circuit of Figure 3.12.

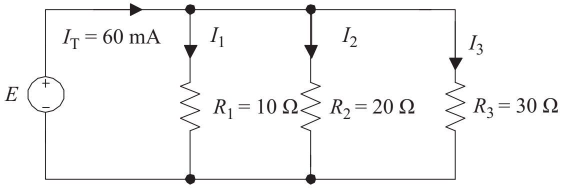


Figure 3.12 Circuit for Example 3.6

Solution: $R_{eq} = R_1//R_2//R_3 = \dfrac{1}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}} = \dfrac{1}{\dfrac{1}{10\,\Omega}+\dfrac{1}{20\,\Omega}+\dfrac{1}{30\,\Omega}} \approx 5.455\,\Omega$

$$I_1 = I_T \frac{R_{eq}}{R_1} = 60\text{ mA}\frac{5.455\,\Omega}{10\,\Omega} = \boxed{32.73\text{ mA}}$$

$$I_2 = I_T \frac{R_{eq}}{R_2} = 60\text{ mA}\frac{5.455\,\Omega}{20\,\Omega} = \boxed{16.37\text{ mA}}$$

$$I_3 = I_T \frac{R_{eq}}{R_3} = 60\text{ mA}\frac{5.455\,\Omega}{30\,\Omega} = \boxed{10.91\text{ mA}}$$

The conclusion that can be drawn from the above example is that the greater the branch resistance, the less the current flows through that branch, or the less the share of the total current.

Example 3.7: Determine the resistance R_2 for the circuit in Figure 3.13.

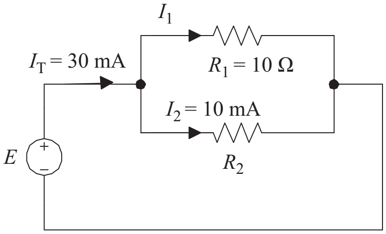


Figure 3.13 Circuit for Example 3.7

Solution: Solve R_2 from the current divider formula $I_2 = I_T \dfrac{R_1}{R_1 + R_2}$

$$I_2(R_1 + R_2) = I_T R_1 \quad I_2 R_2 = I_T R_1 - I_2 R_1$$

$$R_2 = \frac{R_1(I_T - I_2)}{I_2} = \frac{10\ \Omega(30\ \text{mA} - 10\ \text{mA})}{10\ \text{mA}} = 20\ \Omega$$

3.3 Series–parallel resistive circuits

3.3.1 Equivalent resistance of a series–parallel circuit

Series–parallel circuits

- The most practical electric circuits are not simple series or parallel configurations, but combinations of series and parallel circuits, or the series–parallel configurations.
- Many circuits have various combinations of series and parallel components, i.e., circuit elements are series-connected in some parts and parallel in others.
- Series–parallel circuit: the series–parallel circuit is a combination of series and parallel circuits.

  The series–parallel configurations have a variety of circuit forms, and some of them may be very complex. However, the same principles and rules or laws that have been introduced in the previous chapters are applied.

Equivalent resistance

- The key to solving series–parallel circuits is to identify which parts of the circuit are series and which parts are parallel and then simplify them to an equivalent circuit and find an equivalent resistance.
- Method for determining the equivalent resistance of series–parallel circuits:
  - Determine the equivalent resistance of the parallel part of the series–parallel circuits.
  - Determine the equivalent resistance of the series part of the series–parallel circuits.
  - Plot the equivalent circuit if necessary.
  - Repeat the above steps until the resistances in the circuit can be simplified to a single equivalent resistance R_{eq}.

Note: Determine R_{eq} step by step from the far end of the circuit to the terminals of the R_{eq}.

3.3.2 Analysis of the series–parallel circuits

Example 3.8: Analysis of the series–parallel circuit in Figure 3.14.

- In Figure 3.14(a), the resistor R_5 is in series with R_6 and in parallel with R_4 and R_3. This can be expressed by the equivalent circuit in Figure 3.14(b).

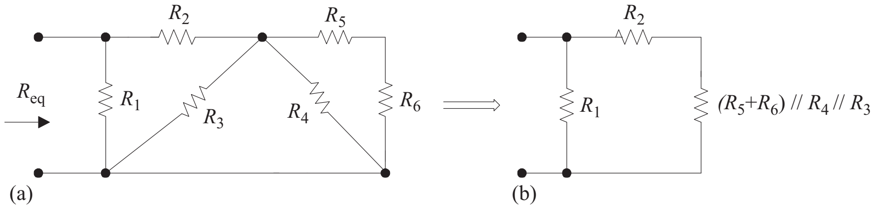

R_2
R_5
R_{eq}
R_1
R_3
R_4
R_6
(a)
R_2
R_1
(R_5+R_6) // R_4 // R_3
(b)

Figure 3.14 Circuit for Example 3.8

- R_2 is in series with $(R_5 + R_6)//R_4//R_3$ and in parallel with R_1. That is the equivalent resistance R_{eq} for the series–parallel circuit,

  i.e., $R_{eq} = \{[(R_5 + R_6)//R_4//R_3] + R_2\}//R_1$

Example 3.9: Determine the equivalent resistance R_{eq} (formula) for the circuit shown in Figure 3.15(a).

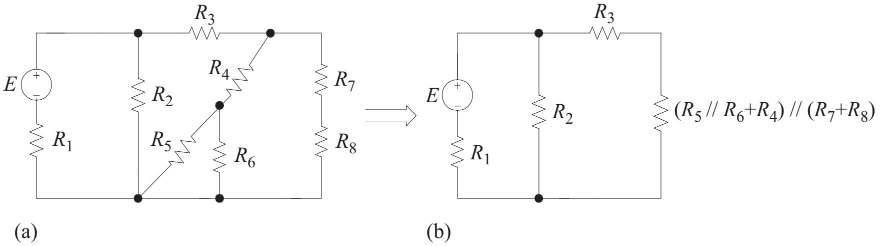

R_3
E
R_4
R_7
R_2
R_1
R_5
R_6
R_8
(a)
R_3
E
R_2
$(R_5$ // $R_6+R_4)$ // (R_7+R_8)
R_1
(b)

Figure 3.15 Circuit for Example 3.9

Solution: $R_{eq} = [(R_5 //R_6 + R_4)//(R_7 + R_8) + R_3]//R_2 + R_1$

Example 3.10: Determine the R_{eq} for the circuit shown in Figure 3.16(a).

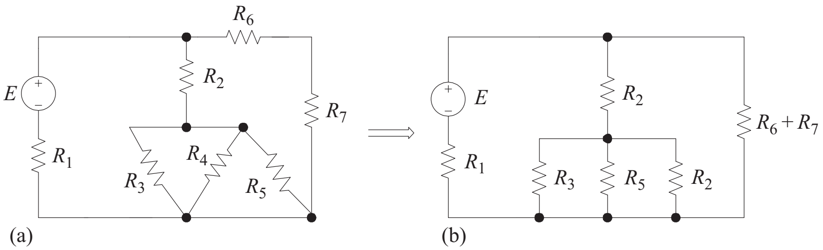

R_6
E
R_2
R_7
R_1
R_4
R_3
R_5
(a)
E
R_2
R_6+R_7
R_1
R_3
R_5
R_2
(b)

Figure 3.16 Circuits for Example 3.10

Solution: $R_{eq} = [(R_3//R_4//R_5) + R_2]//(R_6 + R_7) + R_1$

3.3.3 Currents and voltages of a series–parallel circuit

Determine currents and voltages

After determining the equivalent resistance of the series–parallel circuit, the total current as well as currents and voltages for each resistor can be determined by using the following steps:

- Total current: apply Ohm's law with the equivalent resistance solved from the previous section to determine the total current in the series–parallel circuit.

$$\boxed{I_T = \frac{E}{R_{eq}}}$$

- Unknown currents and voltages: apply the VDR, CDR, Ohm's law, KCL, and KVL to determine the unknown currents and voltages in the series–parallel circuit.

Example 3.11: Determine the currents and voltages for each resistor in the circuit of Figure 3.17.

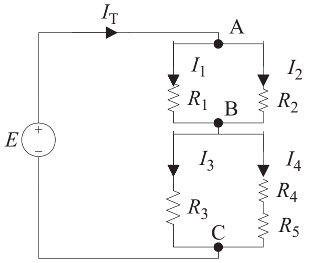


Figure 3.17 Circuit for Example 3.11

Solution:

- $R_{eq} = (R_1//R_2) + [(R_4 + R_5)//R_3]$

  $I_T = \dfrac{E}{R_{eq}}$

- $V_{R_1} = V_{R_2} = V_{AB} = I_T\ (R_1//R_2), \quad I_1 = \dfrac{V_{AB}}{R_1}, \quad I_2 = \dfrac{V_{AB}}{R_2}$

  or $\quad I_1 = I_T \dfrac{R_2}{R_1 + R_2}, \quad I_2 = I_T \dfrac{R_1}{R_1 + R_2}$ The current divider rule.

- $V_{R_3} = V_{R_4} + V_{R_5} = V_{BC} = I_T\ [(R_4 + R_5)//R_3]$

  $I_3 = \dfrac{V_{BC}}{R_3}, \quad I_{4,5} = \dfrac{V_{BC}}{R_4 + R_5}$

  Check: $I_T = I_1 + I_2$ or $I_T = I_3 + I_{4,5}$ KCL

  $V_{AB} + V_{BC} = E$ KVL

Example 3.12: Determine the current I_T (formula) in the circuit of Figure 3.18.

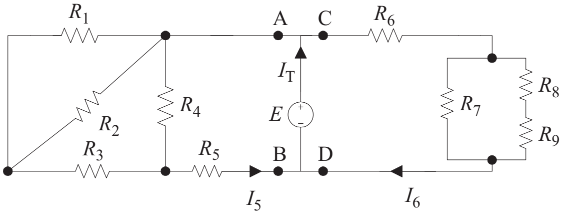


Figure 3.18 Circuit for Example 3.12

Solution: $I_T = I_{5(?)} + I_{6(?)}$

The method of analysis: $I_T = ?\ I_T = I_5 + I_6,\ I_5 = ?\ I_6 = ?$

$$I_5 = \frac{V_{AB}}{R_{AB(?)}} \qquad I_5 = \frac{V_{AB}}{R_{AB}}, \quad V_{AB} = E, \quad R_{AB} = ?$$

$$R_{AB} = [(R_1//R_2 + R_3)//R_4] + R_5$$

$$I_6 = \frac{V_{CD}}{R_{CD(?)}} \qquad I_6 = \frac{V_{CD}}{R_{CD}}, \quad V_{CD} = E, \quad R_{CD} = ?$$

$$R_{CD} = [(R_8 + R_9)//R_7] + R_6$$

3.4 Wye (Y) and delta (Δ) configurations and their equivalent conversions

3.4.1 Wye and delta configurations

Introduction to wye and delta configurations

- Sometimes, the circuit configurations will be neither in series nor in parallel, and the analysis method for series–parallel circuits described in previous chapters may not apply.
- For example, the configuration of three resistors R_a, R_b, and R_c in the circuit of Figure 3.19(a) are neither in series nor in parallel. So how do we determine the equivalent resistance R_{eq} for this circuit?
- If we convert this to the configuration of resistors R_1, R_2, and R_3 in the circuit of Figure 3.19(b), the problem can be easily solved,

  i.e., $R_{eq} = [(R_1 + R_d)//(R_2 + R_e)] + R_3$

Y and Δ configurations

- The resistors of R_a, R_b, and R_c in the circuit of Figure 3.19(a) are said to be in the delta (Δ) configuration.

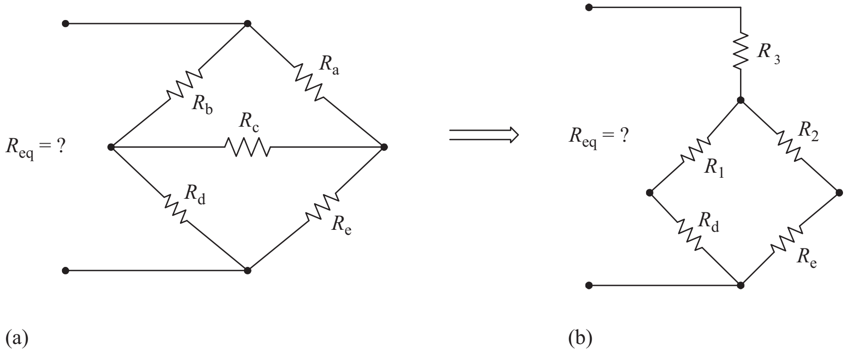

R_{eq} = ?
R_a
R_b
R_c
R_d
R_e
(a)
R_{eq} = ?
R_3
R_2
R_1
R_d
R_e
(b)

Figure 3.19 Delta (Δ) and wye (Y) configurations

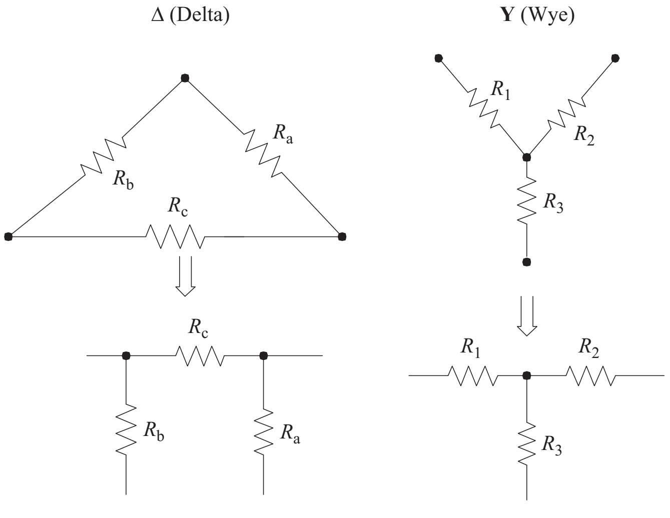

Δ (Delta)
R_a
R_b
R_c
R_c
R_b
R_a
π (Pi)
Y (Wye)
R_1
R_2
R_3
R_1
R_2
R_3
T (Tee)

Figure 3.20 π and T configurations

- R_1, R_2, and R_3 in the circuit of Figure 3.19(b) is called the wye (Y) configuration.

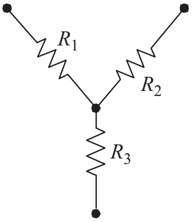
The delta and wye designations are from the fact that they look like a triangle Δ and the letter Y, respectively, in electrical drawings.

3.4.2 Tee (T) and pi (π) configurations

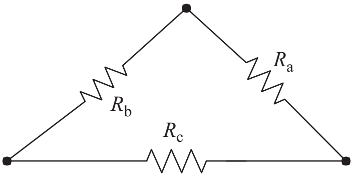
- The delta and wye designations are also referred to as tee (T) and pi (π) circuits as shown in Figure 3.20.

- Wye (Y) or tee (T) and delta (Δ) or pi (π) configurations:

Y or T configuration:



Δ or π configuration:



- Wye (Y) and delta (Δ) configurations are often used in three-phase AC circuits. They can also be used in the bridge circuit that will be discussed later.
- It is very important to know the conversion method of the two circuits and be able to convert back and forth between the wye (Y) and delta (Δ) configurations.

3.4.3 Delta to wye conversion (Δ→Y)

- There are three terminals in the delta (Δ) or wye (Y) configurations that can be connected to other circuits (a, b, and c as shown in Figure 3.21).
- The delta or wye conversion is used to establish equivalence for the circuits with three terminals, meaning that the resistors of the circuits between any two terminals must have the same values for both circuits as shown in Figure 3.21.

i.e., $R_{ac(Y)} = R_{ac(\Delta)} \quad R_{ab(Y)} = R_{ab(\Delta)} \quad R_{bc(Y)} = R_{bc(\Delta)}$

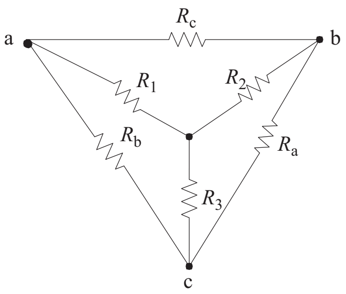


Figure 3.21 Delta and wye configurations

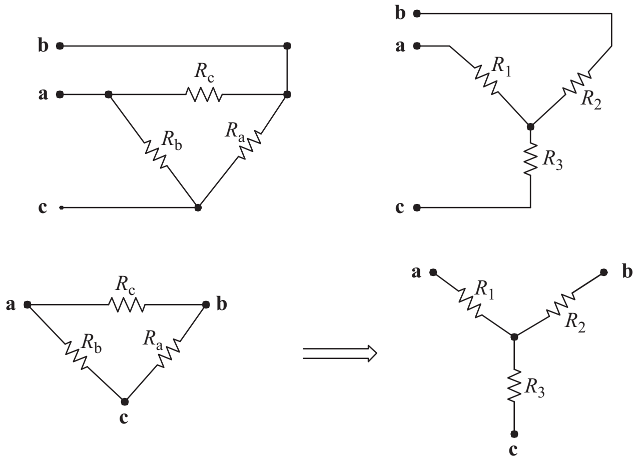


Figure 3.22 Delta converted to wye configuration

- Delta to wye conversion: the circuit in delta configuration is converted to wye configuration as shown in Figure 3.22.
- Equations for delta to wye (Δ →Y):

$$\boxed{R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \quad R_2 = \frac{R_a R_c}{R_a + R_b + R_c}, \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}}$$

3.4.4 *Wye to delta conversion (Y→Δ), R_Y, and R_Δ*

Equations for wye to delta (Y→Δ)

- The circuit in wye configuration is converted to delta as shown in Figure 3.23:
- Equations for R_a, R_b, and R_c (Y→Δ):

$$\boxed{R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}, \; R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}, \; R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}}$$

R_Y and R_Δ

- If all resistors in the wye (Y) configuration have the same values,

  i.e., $R_1 = R_2 = R_3 = R_Y$

  then all the resistances in the delta (Δ) configuration will also be the same,

  i.e., $R_a = R_b = R_c = R_\Delta$

- $\boxed{\text{If} \quad R_a = R_b = R_c = R_\Delta, \quad R_1 = R_2 = R_3 = R_Y}$

  the delta resistance R_Δ and wye resistance R_Y has the following relationship:

$$\boxed{R_Y = \frac{1}{3} R_\Delta \quad \text{or} \quad R_\Delta = 3R_Y}$$

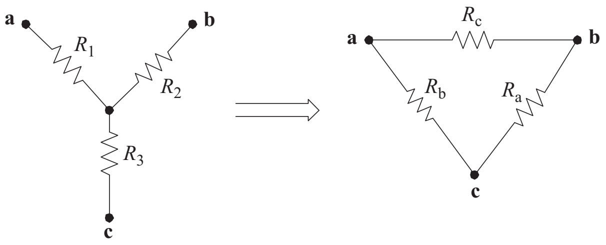

a
b
R_1
R_2
R_3
c
R_c
a
b
R_b
R_a
c

Figure 3.23 Wye converted to delta configuration

3.4.5 An example of wye and delta conversion

Example 3.13: Convert Δ to Y in the circuit of Figure 3.24, then Y to Δ to prove the accuracy of the equations. There the delta resistances $R_a = 30\ \Omega$, $R_b = 20\ \Omega$, and $R_c = 10\ \Omega$.

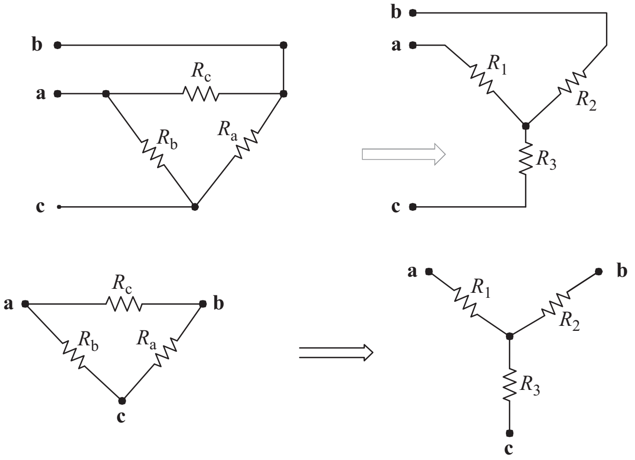

b
a
c
R_c
R_b
R_a
b
a
c
R_1
R_2
R_3
a
b
c
R_c
R_b
R_a
a
b
c
R_1
R_2
R_3

Figure 3.24 Circuit for Example 3.13

Solution: Δ →Y:

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{(30\ \Omega)(20\ \Omega)}{30\ \Omega + 20\ \Omega + 10\ \Omega} = 10\ \Omega$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c} = \frac{(30\ \Omega)(10\ \Omega)}{30\ \Omega + 20\ \Omega + 10\ \Omega} = 5\ \Omega$$

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{(20\ \Omega)(10\ \Omega)}{30\ \Omega + 20\ \Omega + 10\ \Omega} \approx 3.33\ \Omega$$

Y→ Δ:

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{[(3.33)(5) + (5)(10) + (10)(3.33)]\ \Omega^2}{3.33\ \Omega}$$

$$= \frac{99.95\ \Omega^2}{3.33\ \Omega} \approx 30\ \Omega$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{99.95\ \Omega^2}{5\ \Omega} \approx 20\ \Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{99.95\ \Omega^2}{10\ \Omega} \approx 10\ \Omega$$

The calculated delta resistances $R_a = 30\ \Omega$, $R_b = 20\ \Omega$, and $R_c = 10\ \Omega$ are the same with the resistances that were given (proved).

3.4.6 *Using Δ → Y conversion to simplify bridge circuits*

Wheatstone bridge

- The Wheatstone bridge circuit can be used to measure the unknown resistors.
- A basic Wheatstone bridge circuit is illustrated in Figure 3.25(a).
  - Sir **Charles Wheatstone** (1802–1875), a British physicist and an inventor, is most famous for the Wheatstone bridge circuit. He was the first person who implemented the bridge circuit when he "found" the description of the device.
  - **The bridge was invented by** Samuel Hunter Christie (1784–1865), a British scientist.

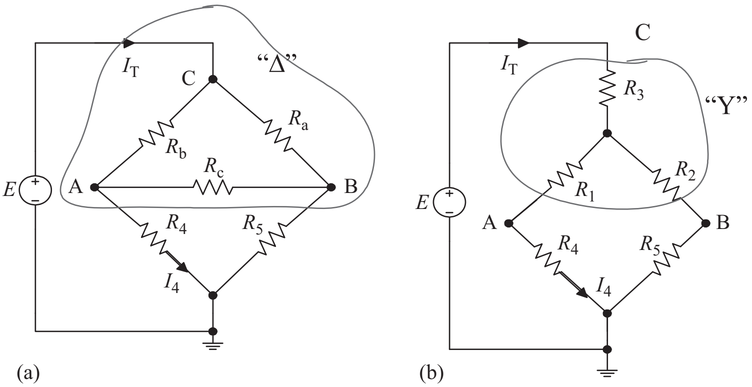


Figure 3.25 Wheatstone bridge circuit

Example 3.14: Determine the equations to calculate the total current I_T and branch current I_4 for the bridge circuit in Figure 3.25(a).

- Figure 3.25(a) can be converted to Figure 3.25(b) using the Δ →Y equivalent conversion.
- R_1, R_2, and R_3 in Figure 3.25(b) can be determined by the equations of Δ →Y conversion:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \quad R_2 = \frac{R_a R_c}{R_a + R_b + R_c}, \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

- The equivalent resistance R_{eq} of the bridge:

$$R_{eq} = R_3 + [(R_1 + R_4)//(R_2 + R_5)] \qquad R_2//R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

- The total current I_T: $I_T = \frac{E}{R_{eq}}$ $\qquad I = \frac{E}{R}$
- The branch current: $I_4 = I_T \frac{R_2 + R_5}{(R_1 + R_4) + (R_2 + R_5)}$ $\qquad I_2 = I_T \frac{R_1}{R_1 + R_2}$

If the wire between A and B in the circuit of Figure 3.25(a) is open, the R_{eq} will be

$$R_{eq} = (R_b + R_4)//(R_a + R_5)$$

3.4.7 Balanced bridge

Balanced bridge

- When the voltage across points A and B terminals in a bridge circuit shown in Figure 3.26 is zero, i.e., $V_{AB} = 0$, the Wheatstone bridge is said to be balanced.
- A balanced Wheatstone bridge circuit can accurately measure an unknown resistor.

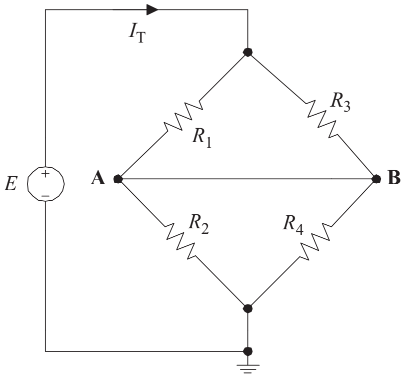


Figure 3.26 A balanced bridge

Determine the voltage V_{AB} in the points A and B

The voltage V_{AB} is the voltage from point A to ground (V_A) and then from ground to point B, i.e.,

$$V_{AB} = V_A + (-V_B)$$

$$V_{AB} = E\frac{R_2}{R_1 + R_2} - E\frac{R_4}{R_3 + R_4} \qquad V_2 = V_T\frac{R_2}{R_1 + R_2}$$

$$V_{AB} = E\frac{R_2(R_3 + R_4) - R_4(R_1 + R_2)}{(R_1 + R_2)(R_3 + R_4)} = E\frac{R_2R_3 - R_4R_1}{(R_1 + R_2)(R_3 + R_4)}$$

The balanced condition

- When $V_{AB} = 0$, or when the bridge is balanced, the numerator of the above equation will be zero. i.e.,

$$R_2R_3 - R_4R_1 = 0 \quad \text{this gives :} \quad R_2R_3 = R_4R_1$$

- Balanced bridge:

$$\boxed{\text{When } V_{AB} = 0, \quad R_2R_3 = R_4R_1}$$

3.4.8 Measure unknown resistors using the balanced bridge

The method of using the balanced bridge to measure an unknown resistor

- If the unknown resistor is in the position of R_4 in the circuit of Figure 3.26, using a variable (adjustable) resistor to replace R_2.
- Connecting a galvanometer in between terminals A and B can measure the small current I_G in terminals A and B as shown in Figure 3.27.

Galvanometer is a type of ammeter that can measure small current accurately.

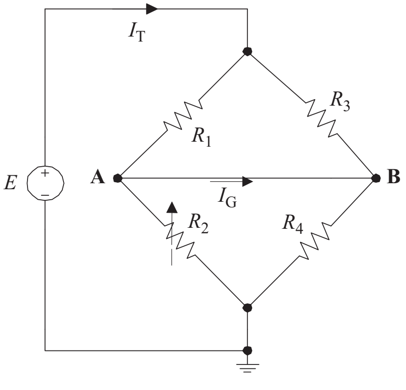


Figure 3.27 Measure an unknown R using a balanced bridge

- Adjust R_2 until the current I_G measured by the galvanometer or current in the A and B branch is zero ($I_G = 0$). This means $V_{AB} = 0$, or the bridge is balanced.
- Unknown resistor $R_x = R_4$ at this time can be determined by the equation of the balanced bridge as follows:

  From: $R_2\ R_3 = R_4\ R_1$

  Solving for unknown resistor R_x: $\boxed{R_x = R_4 = \dfrac{R_2R_3}{R_1}}$

Example 3.15: $R_1 = 100\ \Omega$, $R_2 = 330\ \Omega$, and $R_3 = 470\ \Omega$ in a balanced bridge circuit as shown in Figure 3.27. Determine the unknown resistance R_x.

Solution: From: $R_2\ R_3 = R_4\ R_1$

Solving for R_4: $$R_x = R_4 = \frac{R_2R_3}{R_1} = \frac{(330\ \Omega)(470\ \Omega)}{100\ \Omega} = \boxed{1.551\ \text{k}\Omega}$$

Summary

Series circuits

- Series circuit: All components are connected one after the other, there is only one circuit path, and the current flow through each component is always the same.
- Total series voltage:

$$V_T = E = V_1 + V_2 + \cdots + V_n = IR_T$$
$$V_T = IR_1 + IR_2 + \cdots + IR_n = IR_T$$

- Total series resistance (equivalent resistance R_{eq}): $R_T = R_1 + R_2 + \cdots + R_n$
- Series current: $I = \dfrac{V_T}{R_T} = \dfrac{E}{R_T} = \dfrac{V_1}{R_1} = \dfrac{V_2}{R_2} = \ldots = \dfrac{V_n}{R_n}$
- Total series power: $P_T = P_1 + P_2 + \cdots + P_n = IE = I^2R_T = \dfrac{E^2}{R_T}$
- The voltage divider rule (VDR):
  - General form: $V_x = V_T\dfrac{R_x}{R_T}$ or $V_x = E\dfrac{R_x}{R_T}$ $\qquad x = 1, 2, \ldots, n$
  - When there are only two resistors in series: $V_1 = V_T\dfrac{R_1}{R_1 + R_2}$, $V_2 = V_T\dfrac{R_2}{R_1 + R_2}$
- The earth ground: connects to the earth ($V = 0$).
- Common ground or chassis ground: the common point for all components in the circuit ($V = 0$).
- Single-subscript notation: the voltage from the subscript with respect to ground.
- Double-subscript notation: the voltage across the two subscripts.

Parallel circuits

- Parallel circuit: The components are connected end to end, there are at least two current paths in the circuit, and the voltage across each component is the same.
- Parallel voltage: $V = E = V_1 = V_2 = \cdots = V_n$
- Parallel currents: $I_1 = \frac{V}{R_1},\ I_2 = \frac{V}{R_2}, \cdots, I_n = \frac{V}{R_n},\ I_T = \frac{V}{R_{eq}} = I_1 + I_2 + \cdots + I_n$
- Equivalent parallel resistance: $R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}} = R_1 // R_2 // \cdots // R_n$

  When $n = 2$: $R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = R_1 // R_2$
- Equivalent parallel conductance: $G_{eq} = G_1 + G_2 + \cdots + G_n$
- Total parallel power: $P_T = P_1 + P_2 + \cdots + P_n = {I_T}^2 R_{eq} = \frac{V^2}{R_{eq}} = I_T V$
- The current divider rule (CDR):
  - General form: $I_x = I_T \frac{R_{eq}}{R_x}$ or $I_x = I_T \frac{G_x}{G_{eq}}$

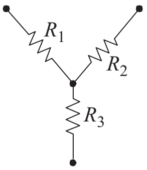
    I_x and R_x are unknown current and resistance, respectively.
  - When there are two resistors in parallel: $I_1 = I_T \frac{R_2}{R_1 + R_2},\quad I_2 = I_T \frac{R_1}{R_1 + R_2}$

Series–parallel circuits

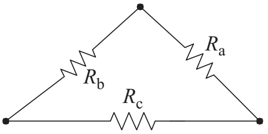
- Series–parallel circuits are a combination of series and parallel circuits.
- Method of determining the equivalent resistance of series–parallel circuits:
  - Determine the equivalent resistance of the parallel part of the series–parallel circuits.
  - Determine the equivalent resistance of the series part of the series–parallel circuits.
  - Plot the equivalent circuit if necessary.
  - Repeat the above steps until the resistance in the circuit can be simplified to a single equivalent resistance R_{eq}.
- Method for analyzing series–parallel circuits:
  - Apply Ohm's law to determine the total current: $I_T = \frac{E}{R_{eq}}$
  - Apply VDR, CDR, Ohm's law, KCL, and KVL to determine the unknown currents and voltages.

Wye and delta configurations and their conversions

- Y or T circuit:



- Δ or π circuit:



- Δ →Y:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \quad R_2 = \frac{R_a R_c}{R_a + R_b + R_c}, \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

- Y→ Δ:

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}, \; R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}, \; R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

- If $R_a = R_b = R_c = R_\Delta$ and $R_1 = R_2 = R_3 = R_Y$: $R_Y = \frac{1}{3}R_\Delta$ or $R_\Delta = 3R_Y$
- The balanced bridge: When $V_{AB} = 0$, $R_2\, R_3 = R_4\, R_1$

Practice problems

3.1

1. Connect each set of resistors in Figure 3.28(a)–(c) in series between terminals A and B (without changing the position of the resistors).

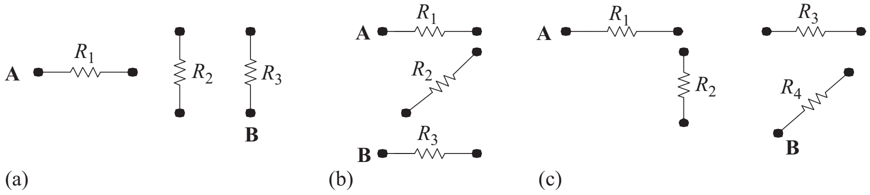


Figure 3.28

2. Determine the total (equivalent) resistance for each circuit in Figure 3.29.

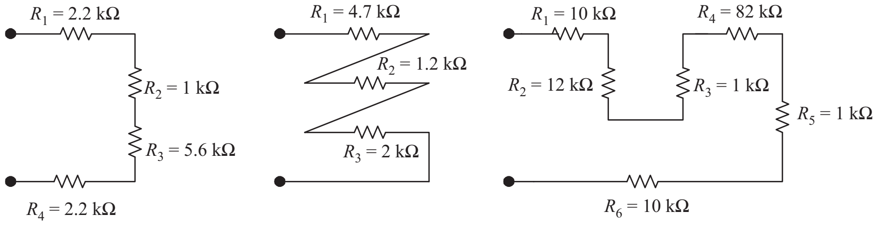

$R_1 = 2.2$ kΩ
$R_2 = 1$ kΩ
$R_3 = 5.6$ kΩ
$R_4 = 2.2$ kΩ
$R_1 = 4.7$ kΩ
$R_2 = 1.2$ kΩ
$R_3 = 2$ kΩ
$R_1 = 10$ kΩ
$R_4 = 82$ kΩ
$R_2 = 12$ kΩ
$R_3 = 1$ kΩ
$R_5 = 1$ kΩ
$R_6 = 10$ kΩ

Figure 3.29

3. Determine the following values in the circuit of Figure 3.30.
   (a) The current I;
   (b) The voltage V_{R_2} across the resistor R_2;
   (c) The total power P_T.

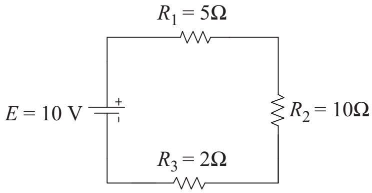

$R_1 = 5\Omega$
$E = 10$ V
+
$R_2 = 10\Omega$
$R_3 = 2\Omega$

Figure 3.30

4. Determine the source voltage E in the circuit of Figure 3.31.

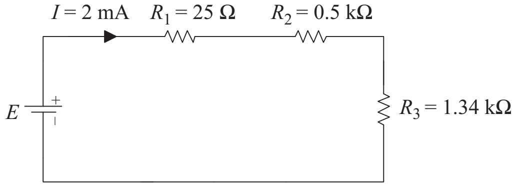

$I = 2$ mA
$R_1 = 25$ Ω
$R_2 = 0.5$ kΩ
E
+
$R_3 = 1.34$ kΩ

Figure 3.31

5. Determine the voltage across each resistor in the circuit of Figure 3.32 and check the results by using KCL.

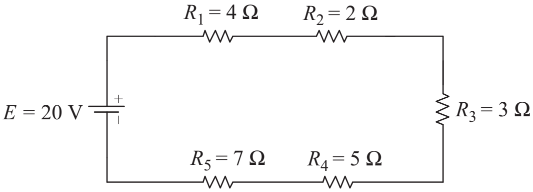

$R_1 = 4$ Ω
$R_2 = 2$ Ω
$E = 20$ V
+
$R_3 = 3$ Ω
$R_5 = 7$ Ω
$R_4 = 5$ Ω

Figure 3.32

6. Determine the unknown resistance in Figure 3.33 by using the VDR. $\left(\text{Hint: solve for } R_3 \text{ from } V_3 = E\dfrac{R_3}{(R_1+R_2)+R_3}\right)$

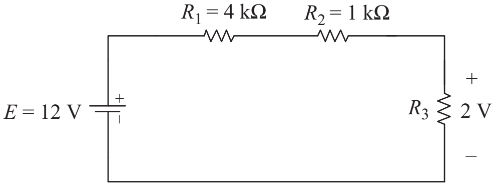


Figure 3.33

7. If the current $I = 1$ mA, the source voltage $E = 12$ V, design a two-resistor voltage divider circuit with $V_{R_1} = \frac{1}{3}V_{R_2}$ (determine R_1 and R_2).
   (Hint: $V_{R_1} = \frac{1}{3}V_{R_2}$, $V_{R_2} = 3V_{R_1}$)
8. Determine the voltages V_A and V_B in Figure 3.34.

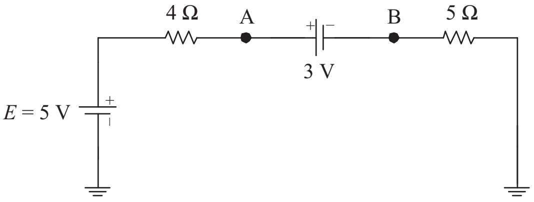


Figure 3.34

3.2

9. Connect each set of resistors in Figure 3.35(a) and (b) in parallel between terminals A and B (without changing the position of the resistors).

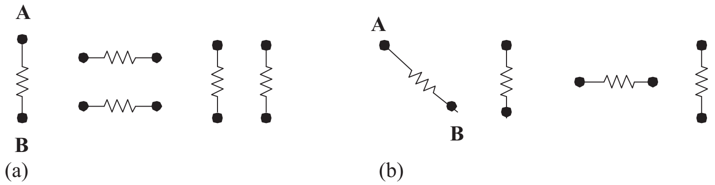


Figure 3.35

10. Determine the total (equivalent) resistance R_{eq}, and conductance G_{eq} in the circuit of Figure 3.36.
11. Determine the total current I_T, the branch current I_1 and I_3, and the total circuit power P_T in Figure 3.36(a), if the source voltage is 10 V.

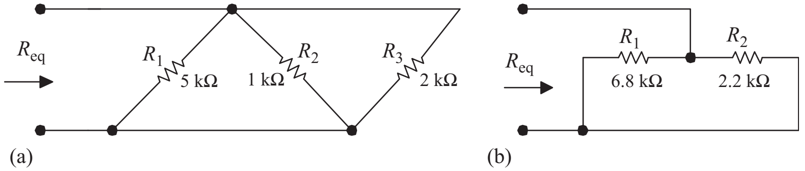

R_eq
R_1
5 kΩ
1 kΩ
R_2
R_3
2 kΩ
(a)
R_eq
R_1
R_2
6.8 kΩ
2.2 kΩ
(b)

Figure 3.36

12. Determine the branch current I_1 and I_2 in the circuit of Figure 3.37.

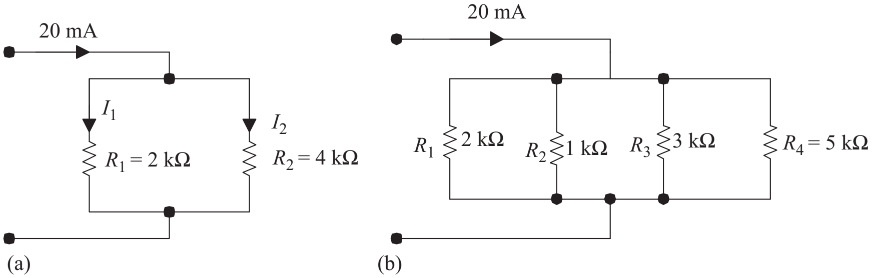

20 mA
I_1
R_1 = 2 kΩ
I_2
R_2 = 4 kΩ
(a)
20 mA
R_1
2 kΩ
R_2
1 kΩ
R_3
3 kΩ
R_4 = 5 kΩ
(b)

Figure 3.37

13. Determine the unknown resistances in the circuit of Figure 3.38.

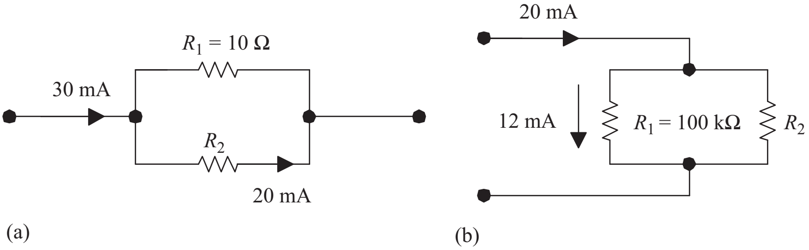

R_1 = 10 Ω
30 mA
R_2
20 mA
(a)
20 mA
12 mA
R_1 = 100 kΩ
R_2
(b)

Figure 3.38

3.3

14. Plot the series–parallel circuits described as follows:
    - (a) The parallel combination of R_1 and R_2 is in series with the series combination of R_3 and R_4.
    - (b) The series combination of R_1 and R_2 is in parallel with the series combination of R_3 and R_4.
15. Plot a series–parallel circuit described as follows: a series combination of two parallel circuits with each parallel circuit having three resistors.
16. Write the expression of the equivalent resistance for the circuit in Figure 3.39.

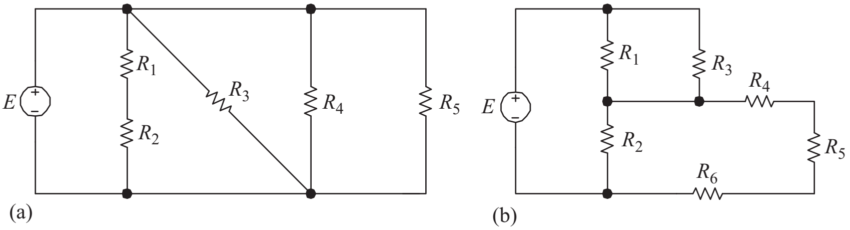

E
R_1
R_2
R_3
R_4
R_5
(a)
E
R_1
R_2
R_3
R_4
R_5
R_6
(b)

Figure 3.39

17. Write the expression of the equivalent resistance for the circuit in Figure 3.40.

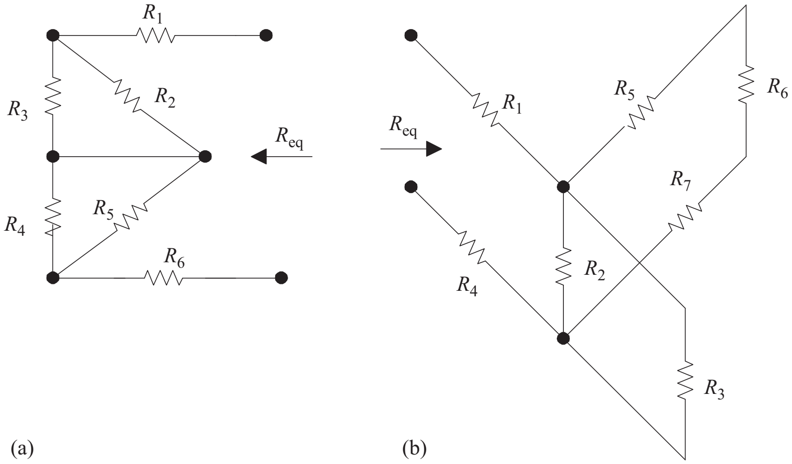

R_1
R_2
R_3
R_4
R_5
R_6
R_{eq}
(a)
R_{eq}
R_1
R_2
R_3
R_4
R_5
R_6
R_7
(b)

Figure 3.40

18. Calculate the branch currents I_{R_2} and I_{R_3} for the circuit in Figure 3.41 (Hint: use the CDR).

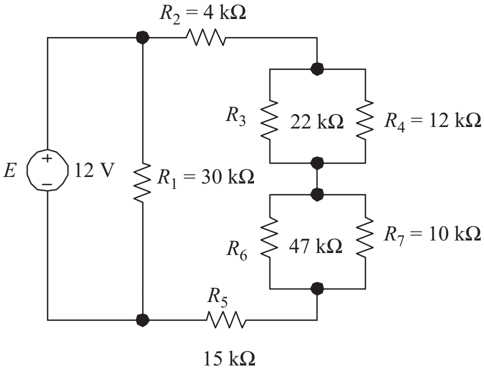
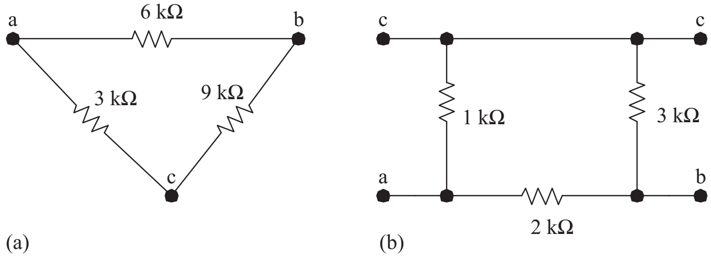

$R_2 = 4$ kΩ
R_3
22 kΩ
$R_4 = 12$ kΩ
E
12 V
$R_1 = 30$ kΩ
R_6
47 kΩ
$R_7 = 10$ kΩ
R_5
15 kΩ

Figure 3.41

19. Calculate the voltage across the resistors R_4 and R_5 for the circuit in Figure 3.41.

3.4

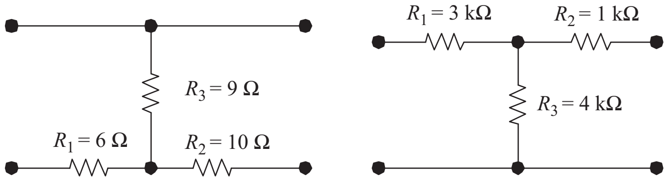
20. Convert the delta circuits in Figure 3.42 to wye circuits.


a
6 kΩ
b
3 kΩ
9 kΩ
c
c
c
1 kΩ
3 kΩ
a
b
2 kΩ

(a) (b)

Figure 3.42

21. Convert the wye circuits in Figure 3.43 to delta circuits.


R3 = 9 Ω
R1 = 6 Ω
R2 = 10 Ω
R1 = 3 kΩ
R2 = 1 kΩ
R3 = 4 kΩ

(a) (b)

Figure 3.43

22. Calculate I_b and I_e in the circuit of Figure 3.44 by using the method of delta–wye conversion.

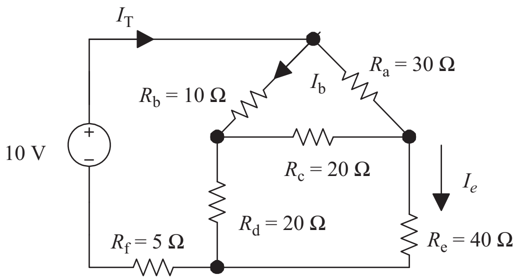

IT
Ra = 30 Ω
Ib
Rb = 10 Ω
10 V
+
–
Rc = 20 Ω
Ie
Rd = 20 Ω
Rf = 5 Ω
Re = 40 Ω

Figure 3.44

23. Determine the unknown resistance R_X of the balanced bridge circuit in Figure 3.45.

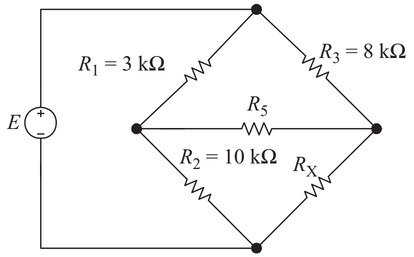

$R_1 = 3\ \text{k}\Omega$
$R_3 = 8\ \text{k}\Omega$
E
+
−
R_5
$R_2 = 10\ \text{k}\Omega$
R_X

Figure 3.45

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Chapter 4

Methods of DC circuit analysis

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4.1 Voltage source, current source, and their equivalent conversions

4.1.1 Source equivalent conversion

It is sometimes easier to convert a current source to an equivalent voltage source or vice versa to analyze and calculate the circuits.

Source equivalent conversion

- The source equivalent conversion means that if loads are connected to both the terminals of the two sources after conversion, the load voltage V_L and current I_L of the two sources should be the same (Figure 4.1).
- So, the source equivalent conversion actually means that the source terminals are equivalent, though the internal characteristics of each source circuit are not equivalent.

Conversion conditions

- If the internal resistance R_S in Figure 4.1(a) and (b) is equal, and the source voltage is $E = I_S R_S$ in Figure 4.1(a), and the source current is $I_S = \frac{E}{R_s}$ in Figure 4.1(b), then the current source and voltage source can be equivalently converted, i.e.,

$$\boxed{E = I_S R_S}, \qquad \boxed{R_S = R_S}, \qquad \boxed{I_S = \frac{E}{R_S}}$$

- When performing the source equivalent conversion, the reference polarities of voltage and current of the sources should be the same before and after the conversion as shown in Figures 4.1 and 4.2 (notice the polarities of sources E and I_S in the two figures).

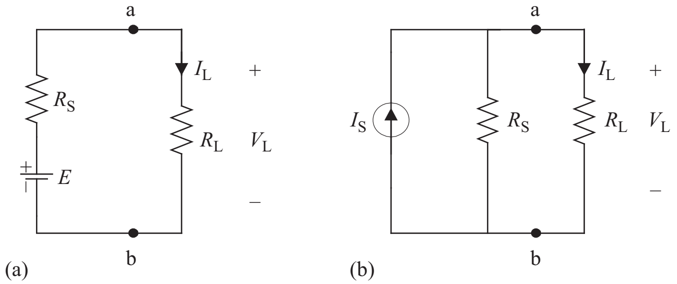


Figure 4.1 Sources equivalent conversion

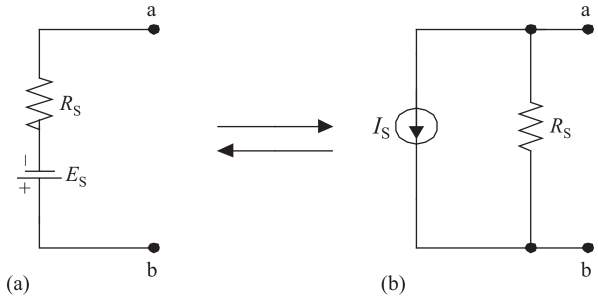


Figure 4.2 Polarity of conversion

4.1.2 Verification of source conversion

Verification of the source equivalent conversion in Figure 4.1

The following procedure can verify that the load voltage V_L and load current I_L in two circuits of Figure 4.1(a) and (b) are equal after connecting a load resistor R_L to the two terminals of these circuits.

- The voltage source in Figure 4.1(a):

$$\boxed{I_L = \frac{E}{R_S + R_L}} \qquad \text{Apply Ohm's law: } I = \frac{E}{R}$$

$$V_L = E\frac{R_L}{R_S + R_L} \qquad \text{Apply the voltage divider rule: } V_2 = V_T\frac{R_2}{R_1 + R_2}$$

$$\boxed{V_L = I_S R_S \frac{R_L}{R_S + R_L}} \qquad E = I_S R_S$$

- The current source in Figure 4.1(b):

$$I_L = I_S\frac{R_S}{R_S + R_L} \qquad \text{Apply the current divider rule: } I_2 = I_T\frac{R_1}{R_1 + R_2}$$

$$\boxed{I_L = \frac{E}{R_S + R_L}} \qquad E = I_S R_S$$

$$V_L = I_L R_L \qquad \text{Apply Ohm's law: } V = IR$$

$$= \left(I_S\frac{R_S}{R_S + R_L}\right)R_L \qquad I_L = I_S\frac{R_S}{R_S + R_L}$$

$$\boxed{V_L = I_S R_S \frac{R_L}{R_S + R_L}}$$

So, the load voltages V_L and currents I_L in two circuits of Figure 4.1(a) and (b) are the same, and the source conversion equations have been proved.

Source conversion summary

Source equivalent conversion	– Voltage source → Current source: $R_S = R_S, \qquad I_S = \frac{E}{R_S}$ – Current source → Voltage source: $R_S = R_S, \qquad E = I_S R_S$

4.1.3 Source conversion examples

Example 4.1: Convert the voltage source in Figure 4.3(a) to an equivalent current source and calculate the load current I_L for the circuit in Figure 4.3(a) and (b).

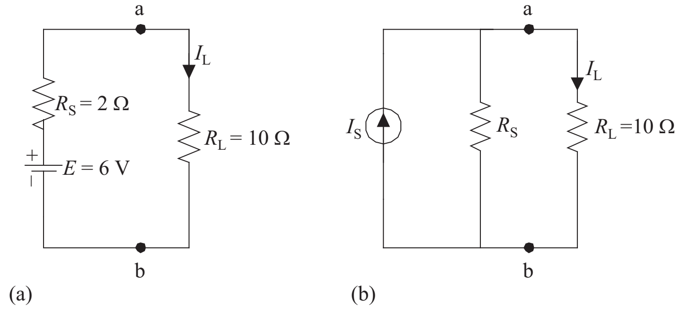

a
I_L
$R_S = 2\ \Omega$
$R_L = 10\ \Omega$
+
$E = 6$ V
b
(a)
a
I_L
I_S
R_S
$R_L = 10\ \Omega$
b
(b)

Figure 4.3 Figure for Example 4.1

Solution: The equivalent current source after the source conversion is shown in Figure 4.3(b); R_S is still 2 Ω in Figure 4.3(b).

- For Figure 4.3(b):

$$I_S = \frac{E}{R_S} = \frac{6\ \text{V}}{2\ \Omega} = 3\ \text{A} \qquad I = \frac{E}{R}$$

$$I_L = I_S \frac{R_S}{R_S + R_L} = 3\ \text{A}\frac{2\ \Omega}{(2+10)\ \Omega} = \boxed{0.5\ \text{A}} \qquad I_2 = I_T \frac{R_1}{R_1 + R_2}$$

- For Figure 4.3(a): $I_L = \dfrac{E}{R_S + R_L} = \dfrac{6\ \text{V}}{(2+10)\ \Omega} = \boxed{0.5\ \text{A}}$ $\qquad I = \dfrac{E}{R}$

Example 4.2: Convert the current source in Figure 4.4(a) to an equivalent voltage source, and determine the voltage source E_s and internal resistance R_S in Figure 4.4(b).

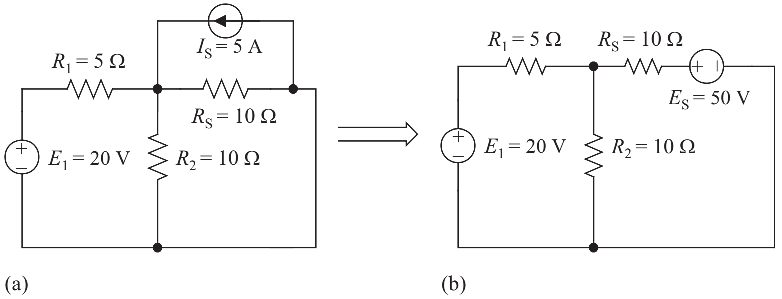

$R_1 = 5\ \Omega$
$I_S = 5$ A
$R_S = 10\ \Omega$
$E_1 = 20$ V
$R_2 = 10\ \Omega$
(a)
$R_1 = 5\ \Omega$
$R_S = 10\ \Omega$
$E_S = 50$ V
$E_1 = 20$ V
$R_2 = 10\ \Omega$
(b)

Figure 4.4 Figure for Example 4.2

Solution:

$$R_S = \boxed{10\ \Omega}$$

$$E_S = I_S R_S = (5\ \text{A})(10\ \Omega) = \boxed{50\ \text{V}}$$

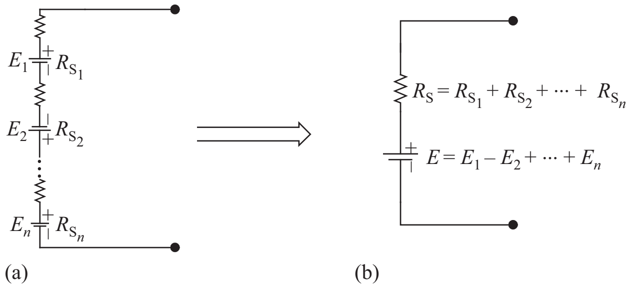

E_1
R_{S_1}
E_2
R_{S_2}
E_n
R_{S_n}
(a)
$R_S = R_{S_1} + R_{S_2} + \cdots + R_{S_n}$
$E = E_1 - E_2 + \cdots + E_n$
(b)

Figure 4.5 Voltage sources in series

4.1.4 Voltage sources in series

Voltage sources in series

- A circuit of voltage sources in series and its equivalent circuit are shown in Figure 4.5.
- Voltage sources connected in series are similar with the resistors connected in series.
- The equivalent internal resistance R_S for series voltage sources is the sum of the individual internal resistances:

$$\boxed{R_S = R_{S_1} + R_{S_2} + \cdots + R_{S_n}}$$

- The equivalent voltage E or V_S for series voltage sources is the algebraic sum of the individual voltage sources.

$$\boxed{E = E_1 + E_2 + \cdots + E_n}$$

or

$$\boxed{V_S = V_1 + V_2 + \cdots + V_n}$$

Signs of voltage sources in series

- Assign a positive sign (+) if the individual voltage has the same polarity as the equivalent voltage E (or V_S) as shown in Figure 4.5.
- Assign a negative sign (−) if the individual voltage has a different polarity from the equivalent voltage E (or V_S) as shown in Figure 4.5.

A flashlight is an example of voltage sources in series, where batteries are connected in series to increase the total equivalent voltage.

Voltage sources in series	– $R_S = R_{S_1} + R_{S_2} + \cdots + R_{S_n}$ $E = E_1 + E_2 + \cdots + E_n$ or $V_S = V_1 + V_2 + \cdots + V_n$ – Assign a (+) if E_n has the same polarity as E (or V_S). – Assign a (−) if E_n has a different polarity from E (or V_S).

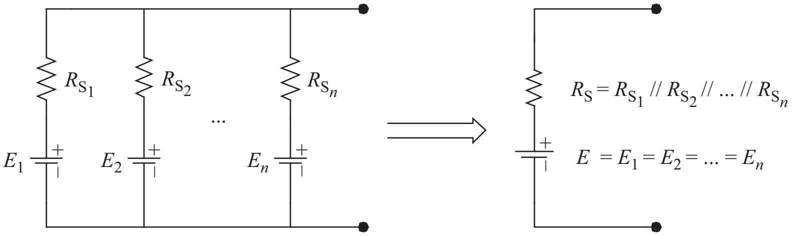

R_{S_1}
R_{S_2}
R_{S_n}
...
E_1
E_2
E_n
+
$R_S = R_{S_1} // R_{S_2} // ... // R_{S_n}$
$E = E_1 = E_2 = ... = E_n$

Figure 4.6 Voltage sources in parallel

4.1.5 Voltage sources in parallel

Voltage sources in parallel

- A circuit of voltage sources in parallel and its equivalent circuit are shown in Figure 4.6.
- The equivalent voltage E or V_S for the parallel voltage sources is the same as the voltage for each individual voltage source, i.e.,

$$\boxed{E = E_1 = E_2 = \cdots = E_n} \quad \text{or} \quad \boxed{V_S = V_{S_1} = V_{S_2} = \cdots = V_{S_n}}$$

- The equivalent internal resistance R_S is the individual internal resistances in parallel:

$$\boxed{R_S = R_{S_1}//R_{S_2}//\cdots//R_{S_n}}$$

Note:

- Only voltage sources that have the same values and polarities can be connected in parallel by using the method mentioned above.
- If the voltage sources having different values and polarities are connected in parallel, it can be solved by using Millman's theory that will be discussed in Chapter 5 (Section 5.4).

An application for voltage sources in parallel

- An example of application for voltage sources connected in parallel is for boosting (or jump starting) a "dead" vehicle.
- You may have experience using jumper cables by connecting the dead battery in parallel with a good car battery or with a booster (battery charger) to recharge that battery.
- It is the process of using the power from a charged battery to supplement the power of a discharged battery.
- It can provide twice the amount of current to the battery of the "dead" vehicle and successfully start the engine.

Voltage sources in parallel	– $R_S = R_{S_1}//R_{S_2}//\cdots//R_{S_n}$ – $E = E_1 = E_2 = \cdots = E_n$ or $V_S = V_{S_1} = V_{S_2} = \cdots = V_{S_n}$

Only voltage sources that have the same values and polarities can be in parallel.

4.1.6 Current sources in parallel

Current sources in parallel

- A circuit of current sources in parallel and its equivalent circuit are shown in Figure 4.7.
- Current sources connected in parallel can be replaced by a single equivalent resistance R_S in parallel with a single equivalent current I_S.
- The equivalent resistance R_S is the individual internal resistances in parallel:

$$R_S = R_{S_1} // R_{S_2} // \cdots // R_{S_n}$$

- The equivalent current I_S is the algebraic sum of the individual current sources:

$$\boxed{I_S = I_{S_1} + I_{S_2} + \cdots + I_{S_n}}$$

Signs of current sources in parallel

- Assign a positive sign (+) if the individual current is in the same direction as the equivalent current I_S.
- Assign a negative sign (−) if the individual current is in a different direction from the equivalent current I_S.

Current sources in parallel	– $R_S = R_{S_1} // R_{S_2} // \cdots // R_{S_n}$ $I_S = I_{S_1} + I_{S_2} + \cdots + I_{S_n}$ – Assign a (+) sign for I_{S_N} if it has the same polarity as I_S. – Assign a (−) sign for I_{S_N} if it has different polarity from I_S.

4.1.7 Current sources in series

Current sources in series

- Only current sources that have the same polarities and same values can be connected in series.
- There is only one current path in a series circuit, so there must be only one current flowing through it.
- This is the same concept as Kirchhoff's current law (KCL), otherwise if the current entering point **A** did not equal the current exiting point **A** in Figure 4.8, KCL would be violated at point **A**.

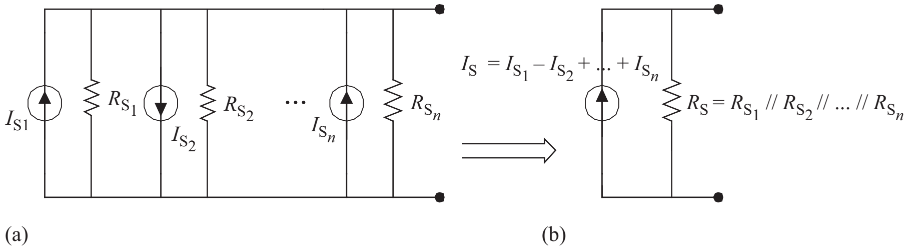

I_{S1}
R_{S_1}
I_{S_2}
R_{S_2}
…
I_{S_n}
R_{S_n}
$I_S = I_{S_1} - I_{S_2} + \ldots + I_{S_n}$
$R_S = R_{S_1} // R_{S_2} // \ldots // R_{S_n}$
(a)
(b)

Figure 4.7 Current sources in parallel

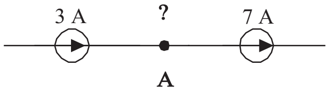


*Figure 4.8 KCL is violated at point **A***

Current sources in series	Only current sources that have same polarities and values can be connected in series.

Example 4.3: Determine the load voltage V_L in Figure 4.9.

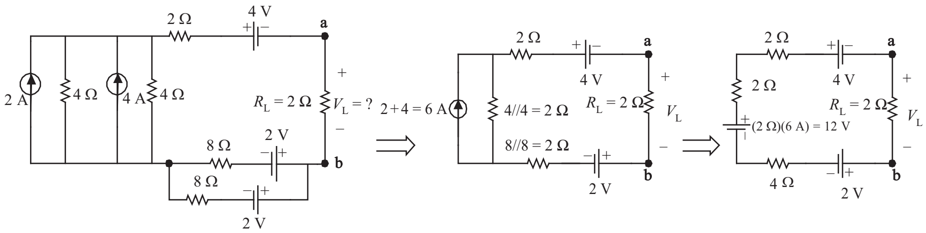


Figure 4.9 Figure for Example 4.3

Solution: The process of source equivalent conversion is shown in the circuit of Figure 4.9. Determine V_L by using the voltage divider rule as follows:

$$V_L = V_{ab} = IR_L = \frac{E}{R_T} R_L \qquad I = \frac{E}{R_T}$$

$$= \frac{(-4 - 2 + 12)\ \text{V}}{(2 + 2 + 4 + 2)\ \Omega}(2\ \Omega) = \boxed{1.2\ \text{V}} \qquad E = (-4 - 2 + 12)\ \text{V}$$

$$R_T = (2 + 2 + 4 + 2)\ \Omega$$

4.2 Branch current analysis

4.2.1 Branch current analysis

Circuit analysis techniques

- The methods of analysis stated in Chapter 3 are limited to an electric circuit that has a single power source.
- If an electric circuit or network has more than one source, it can be solved by the circuit analysis techniques that are discussed in Chapters 4 and 5.
- The branch current analysis is one of several basic methods for analyzing electric circuits.

Branch current analysis

- The branch current analysis is a circuit analysis method that writes and solves a system of equations in which the unknowns are the branch currents.
- This method applies Kirchhoff's laws and Ohm's law to the circuit and solves the branch currents from simultaneous equations.
- Once the branch currents have been solved, other circuit quantities such as voltages and powers can also be determined.
- The branch current analysis technique will use the terms node, branch, and independent loop (or mesh).

Review of some circuit terminologies

- Node: the intersectional point of two or more current paths where current has several possible paths to flow.
- Branch: a current path between two nodes where one or more circuit components is in series.
- Loop: a complete current path that current to flow back to the start.
- Mesh: a loop in the circuit that does not contain any other loops (it can be analyzed as a windowpane).

The circuits in Figure 4.10 have three meshes (or independent loops) and different number of nodes (the dark dots).

4.2.2 Procedure for applying the branch current analysis

Branch current analysis	– A circuit analysis method that writes and solves a system of KCL and KVL equations in which the unknowns are the branch currents. – It can be used for a circuit that has more than one source.

The steps of branch circuit analysis

1. Label the circuit:
    - – Label all the nodes.
    - – Assign an arbitrary reference direction for each branch current.
    - – Assign loop direction for each mesh (choose clockwise direction).
2. Apply KCL to numbers of independent nodes ($n - 1$), where n is the number of nodes.

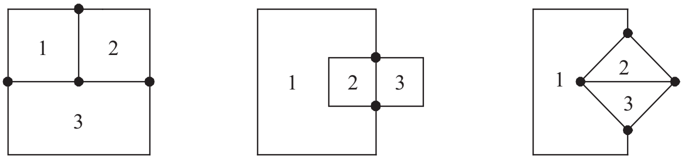


Figure 4.10 Nodes and meshes

3. Apply KVL to each mesh (or windowpane), and the number of KVL equations should be equal to the number of meshes, or $\boxed{\text{Equation \#} = \text{branch \#} - (\text{nodes \#} - 1)}$

   Note: If the circuit with a current source, source current will be the same with the mesh current, so the number of KVL equations can be reduced.
4. Solve the simultaneous equations resulting from steps 2 and 3, using the determinant or substitution methods to determine each branch current.
5. Calculate the other circuit unknowns from the branch currents in the problem if necessary.

The procedure of applying the branch current analysis method is demonstrated in the following example.

4.2.3 Branch current analysis examples

Example 4.4: Use the branch current analysis method to determine each branch current, power on resistor R_2, and also the voltage across the resistor R_1 in the circuit of Figure 4.11.

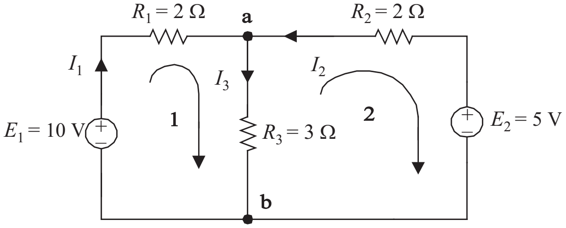


Figure 4.11 Circuit for Example 4.4

Solution: This circuit contains two voltage sources, and cannot be solved by using the methods we have learned in Chapter 3; let us try to use the branch current analysis method.

1. Label the circuit as shown in Figure 4.11:
   - Label the nodes a and b.
   - Assign an arbitrary reference direction for each branch current as shown in Figure 4.11.
   - Assign clockwise loop direction for each mesh as shown in Figure 4.11.
2. Apply KCL to $(n - 1) = (2 - 1) = 1$ number of independent nodes:

There are two nodes a and b, $\therefore\ n = 2$

$I_1 + I_2 = I_3$ $\qquad$ $\sum I = 0$

3. Apply KVL to each mesh.

The number of KVL equations should be equal to the number of meshes.
As there are two meshes in Figure 4.11, we should write two KVL equations.

or Equation # = branch # – (nodes # −1) = 3 – (2 – 1) = 2

– Mesh 1: $I_1R_1 + I_3R_3 - E_1 = 0$ $\quad \sum V = 0$

– Mesh 2: $-I_2R_2 - I_3R_3 + E_2 = 0$ $\quad \sum V = 0$

Recall $\Sigma V = 0$: Assign a positive sign (+) for E or $V = IR$ if its reference polarity and loop direction are the same; otherwise assign a negative sign (−).

4. Solve the simultaneous equations resulting from steps 2 and 3, and determine the branch currents I_1, I_2, and I_3. Three equations can solve three unknowns.

– Rewrite the above three equations in the standard form:

$I_1 + I_2 - I_3 = 0$ $\quad \sum I = 0$

$I_1R_1 + 0 + I_3R_3 = E_1$ $\quad \sum V = \sum E$, $R_2 = 0$ (R_2 is not in mesh 1).

$0 - I_2R_2 - I_3R_3 = -E_2$ $\quad \sum V = \sum E$, $R_1 = 0$ (R_1 is not in mesh 2).

– Substitute the values into equations:

$I_1 + I_2 - I_3 = 0$

$2I_1 + 0 + 3I_3 = 10\,\text{V}$ $\quad R_1 = 2\ \Omega,\ E_1 = 10\ \text{V},\ R_2 = 2\ \Omega$

$0 - 2I_2 - 3I_3 = -5\ \text{V}$ $\quad R_2 = 2\ \Omega,\ E_2 = 5\ \text{V},\ R_3 = 3\ \Omega$

– Solve simultaneous equations using the determinant method:

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 0 & -2 & -3 \end{vmatrix}$$

$$= (1)(0)(-3) + (2)(-2)(-1) + (0)(1)(3) - (-1)(0)(0)$$

$$-(3)(-2)(1) - (-3)(2)(1)$$

$$= 4 - (-6) - (-6) = 16$$

$$I_1 = \frac{\begin{vmatrix} 0 & 1 & -1 \\ 10 & 0 & 3 \\ -5 & -2 & -3 \end{vmatrix}}{\Delta} = \frac{(10)(-2)(-1) + (-5)(3)(1) - (-3)(10)(1)}{16} \approx 2.19\,\text{A}$$

$$I_2 = \frac{\begin{vmatrix} 1 & 0 & -1 \\ 2 & 10 & 3 \\ 0 & -5 & -3 \end{vmatrix}}{\Delta} = \frac{(1)(10)(-3) + (2)(-5)(-1) - (3)(-5)(1)}{16} \approx -0.31\,\text{A}$$

$$I_3 = \frac{\begin{vmatrix} 1 & 1 & 0 \\ 2 & 0 & 10 \\ 0 & -2 & -5 \end{vmatrix}}{\Delta} = \frac{-(10)(-2)(1) - (-5)(2)(1)}{16} \approx 1.88\ \text{A}$$

$$I_1 \approx \boxed{2.19\,\text{A}}, \qquad I_2 \approx \boxed{-0.31\,\text{A}}, \qquad I_3 \approx \boxed{1.88\,\text{A}}$$

Negative sign for I_2 indicates that the actual direction of I_2 is opposite with its assigned reference direction.

5. Calculate the other circuit unknowns from the branch currents:

$$P_2 = I_2^2 R_2 = (-0.31\ \mathrm{A})^2(2\ \Omega) \approx \boxed{0.19\ \mathrm{W}}$$

$$V_1 = I_1 R_1 = (2.19\ \mathrm{A})(2\ \Omega) = \boxed{4.38\ \mathrm{V}}$$

Example 4.5: Determine current I_2 in Figure 4.12 using the branch current analysis.

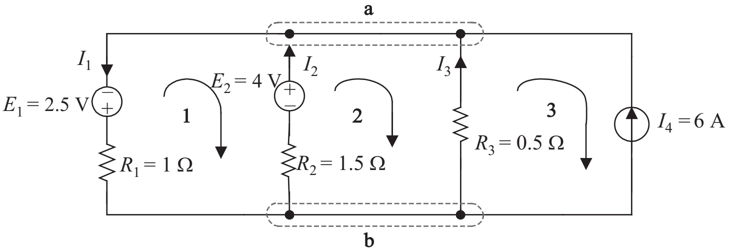


Figure 4.12 Circuit for Example 4.5

Solution:

1. Label the nodes, reference direction for branch currents and loop directions in the circuit as shown in Figure 4.12.
2. Apply KCL to $(n - 1) = (2 - 1) = 1$ number of independent nodes.

There are two nodes or supernodes a and b, so $n = 2$.

$$-I_1 + I_2 + I_3 + I_4 = 0 \qquad \sum I = 0 \text{ (node a)},\quad I_4 = 6\ \mathrm{A}$$

3. Apply KVL to each mesh.

There are three meshes in Figure 4.12, so we should write 3 KVL equations.

- Mesh 1: $-I_1R_1 - I_2R_2 = -E_1 - E_2$ $\qquad \sum V = \sum E$
- Mesh 2: $I_2R_2 - I_3R_3 = E_2$ $\qquad \sum V = \sum E$
- Mesh 3: There is no need to write KVL for mesh 3 since mesh 3 current is already known to be equal to the source current I_4 ($I_4 = 6$ A). Therefore, the numbers of loop equations can be reduced from 3 to 2.

4. Solve the simultaneous equations resulting from steps 2 and 3, and determine the branch current I_2.

$$-I_1 - 1.5\,I_2 = -2.5 - 4 \qquad -I_1 - 1.5\,I_2 + 0\,I_3 = -6.5$$

$$0\,I_1 + 1.5\,I_2 - 0.5\,I_3 = 4 \qquad 0\,I_1 + 1.5\,I_2 - 0.5\,I_3 = 4$$

$$-I_1 + I_2 + I_3 = -6 \qquad -I_1 + I_2 + I_3 = -6$$

Solve the above simultaneous equations using the determinant method:

$$\Delta = \begin{vmatrix} -1 & -1.5 & 0 \\ 0 & 1.5 & -0.5 \\ -1 & 1 & 1 \end{vmatrix}$$

$$= (-1)(1.5)(1) + (-1)(-0.5)(-1.5) - (-0.5)(1)(-1) = -2.75$$

$$I_2 = \frac{\begin{vmatrix} -1 & -6.5 & 0 \\ 0 & 4 & -0.5 \\ -1 & -6 & 1 \end{vmatrix}}{\Delta}$$

$$= \frac{(-1)(4)(1) + (-1)(-0.5)(-6.5) - (-0.5)(-6)(-1)}{-2.75} \approx 1.55\ \text{A}$$

So $I_2 \approx \boxed{1.55\ \text{A}}$

4.3 Mesh analysis

4.3.1 Mesh current analysis

Branch current analysis vs. mesh current analysis

- The branch current analysis in Section 4.2 is a circuit analysis method that writes and solves a system of KCL and KVL equations in which the ***unknowns*** are the ***branch currents.***
- Mesh current analysis is a circuit analysis method that writes and solves a system of KVL equations in which the ***unknowns*** are the ***mesh currents*** (a current that circulates in the mesh). It can be used for a circuit that has more than one source.
- The branch current analysis usually is a fundamental method for understanding mesh current analysis; mesh analysis is more practical and easier to use.

Mesh current analysis

- Mesh current analysis uses KVL and does not need to use KCL.
- Applying KVL to get the mesh equations and solve unknowns implies that will have less unknown variables, less simultaneous equations, and therefore less calculation than branch current analysis.
- After solving mesh currents, the branch currents of the circuit will be easily determined.

Mesh current analysis	– A circuit analysis method that writes and solves a system of KVL equations in which the *unknowns are the mesh currents*. – It can be used for a circuit that has more than one source.

4.3.2 Procedure for applying the mesh current analysis

The steps of mesh current analysis

1. Identify each mesh, and label all the nodes and reference directions for each mesh current (a current that circulates in the mesh) clockwise.
2. Apply KVL to each mesh of the circuit, and the number of KVL equations should be equal to the number of meshes (windowpanes).
   Or Equation # = branch # – (nodes # –1)

   Assign a positive sign (+) for each self-resistor voltage, and a negative sign (–) for each mutual-resistor voltage in KVL equations.
   - Self-resistor: a resistor that is located in a mesh where only one mesh current flows through it.
   - Mutual resistor: a resistor that is located in a boundary of two meshes and has two mesh currents flowing through it.
3. Solve the simultaneous equations resulting from step 2 using the determinant or substitution methods, and determine each mesh current.
4. Calculate the other circuit unknowns such as branch currents from the mesh currents in problem if necessary (choose the reference direction of branch currents first).

Note:

- Convert the current source to the voltage source first in the circuit, if there is any.
- If the circuit has a current source, the source current will be the same as the mesh current, so the number of KVL equations can be reduced.

The procedure for applying mesh current analysis method is demonstrated in the following examples.

4.3.3 Mesh current analysis examples

Example 4.6: Use the mesh current analysis method to determine each mesh current and branch currents I_{R_1}, I_{R_2}, and I_{R_3} in Figure 4.13.

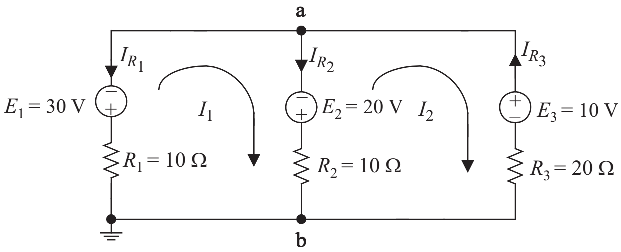


Figure 4.13 Circuit for Example 4.6

Solution:

1. Label all the reference directions for each mesh current I_1 and I_2 (clockwise) as shown in Figure 4.13.
2. Apply KVL around each mesh, and the number of KVL equations is equal to the number of meshes (there are two meshes in Figure 4.13).
   or Equation # = branch # – (nodes # – 1) = 3 – (2 – 1) = 2

   – Mesh 1: $(R_1 + R_2)I_1 - R_2 I_2 = -E_1 + E_2 \qquad \sum V = \sum E$

   – Mesh 2: $-R_2 I_1 + (R_2 + R_3)I_2 = -E_2 - E_3 \qquad \sum V = \sum E$

   Sign a (+) for each self-resistor voltage, and a (–) for each mutual-resistor voltage in KVL.

   Note: The above equations were written by ***inspection*** of the circuit (inspection method):

   First column I_1 Second column I_2 Source E

   – Mesh 1: (Self-resistor) I_1 – (Mutual-resistor) $I_2 = -E_1 + E_2$

   – Mesh 2: – (Mutual-resistor) I_1 + (Self-resistor) $I_2 = -E_2 - E_3$

3. Solve the simultaneous equations resulting from step 2, and determine the mesh current I_1 and I_2:

   – Mesh 1: $(10 + 10)I_1 - 10\,I_2 = -30 + 20$ i.e., $20\,I_1 - 10\,I_2 = -10$ (4.1)

   – Mesh 2: $-10\,I_1 + (10 + 20)I_2 = -20 - 10 \qquad -10\,I_1 + 30\,I_2 = -30$ (4.2)

   Solve for I_1 and I_2 using the substitution method as follows:
   – Solve for I_1 from (4.1):

   $$20\,I_1 = -10 + 10\,I_2, \qquad I_1 = -\frac{1}{2} + \frac{1}{2}I_2 \tag{4.3}$$

   Divide by 20 on both sides.

   – Substitute I_1 into (4.2) and solve for I_2:

   $$-10\left(-\frac{1}{2} + \frac{1}{2}I_2\right) + 30\,I_2 = -30, \qquad I_2 = \boxed{-1.4\ \mathrm{A}}$$

   – Substitute I_2 into (4.3) and solve for I_1:

   $$\mathrm{I}_1 = \frac{1}{2} + \frac{1}{2}(-1.4), \qquad I_1 = \boxed{-0.2\ \mathrm{A}}$$

4. Assuming the reference direction of unknown branch current I_{R_2} as shown in Figure 4.13, calculate I_{R_2} from the mesh currents by applying KCL at node **a**:

$$\sum I = 0: \qquad -I_{R_1} - I_{R_2} + I_{R_3} = 0 \quad \text{or} \quad I_1 - I_{R_2} - I_2 = 0$$

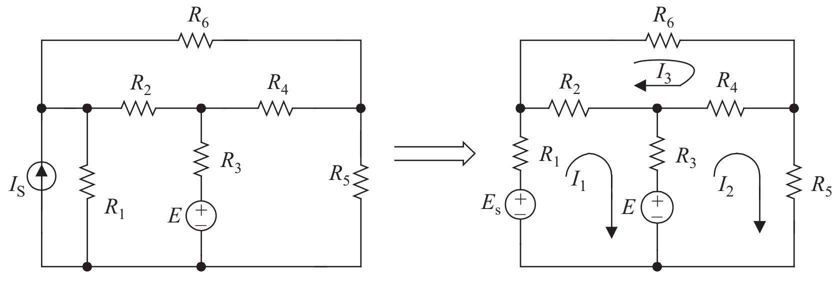
$$(\text{Since } \; I_1 = -I_{R_1} \; \text{ and } \; I_2 = -I_{R_3})$$

$$I_{R_2} = I_1 - I_2 = -0.2 - (-1.4) = \boxed{1.2\text{ A}}$$

$$I_{R_1} = -I_1 = \boxed{0.2\text{ A}}$$

$$I_{R_3} = I_2 = \boxed{1.4\text{A}}$$

Example 4.7: Write the mesh equations using the mesh current analysis method for the circuit in Figure 4.14.



(a)

(b) $(E_S = I_S R_1)$

Figure 4.14 Circuit for Example 4.7

Solution: Convert the current source to a voltage source as shown in Figure 4.14.

1. Label all the nodes and the reference directions for each mesh current (clockwise), as shown in Figure 4.14(b).
2. Apply KVL for each mesh, and the number of KVL equations is equal to the number of meshes (there are three meshes in Figure 4.14(b)).

Or Equation # = branch # – (nodes # – 1) = 6 – (4 – 1) = 3

- Mesh 1: $(R_1 + R_2 + R_3)I_1 - R_3 I_2 - R_2 I_3 = E_S - E \qquad \sum V = \sum E$
- Mesh 2: $-R_3 I_1 + (R_3 + R_4 + R_5)I_2 - R_4 I_3 = E \qquad \sum V = \sum E$
- Mesh 3: $-R_2 I_1 - R_4 I_2 + (R_2 + R_4 + R_6)I_3 = 0 \qquad \sum V = \sum E$

4.4 Nodal voltage analysis

4.4.1 Procedure for applying the node voltage analysis

Node voltage analysis

- The node voltage analysis is another method for analysis of an electric circuit with two or more sources.
- The node voltage analysis is a circuit analysis method that writes and solves a set of simultaneous KCL equations in which the ***unknowns*** are the ***node voltages***.
  - Recall that node is the intersectional point of two or more current paths.
  - Node voltage is voltage between a node and the reference node.

Node voltage analysis	– A circuit analysis method that writes and solves a set of simultaneous KCL equations in which the ***unknowns*** are the ***node voltages***. – It can be used for a circuit that has more than one source.

The steps of node voltage analysis

1. Label the circuit:
   - Label all the nodes and choose one of them to be the reference node.

     Usually ground or the node with the most branch connections should be chosen as the reference node (at which voltage is defined as zero).
   - Assign an arbitrary reference direction for each branch current (this step can be skipped if using the inspection method).
2. Apply KCL to all $n-1$ nodes except for the reference node (n is the number of nodes).
   - Method 1: Write KCL equations and apply Ohm's law to the equations; either resistance or conductance can be used.
     - Assign a positive sign (+) for the self-resistor or self-conductor voltage.
     - Assign a negative sign (−) for the mutual-resistor or mutual-conductor voltage.
   - Method 2: Convert voltage sources to current sources and write KCL equations using the inspection method.
3. Solve the simultaneous equations and determine each nodal voltage.
4. Calculate the other circuit unknowns such as branch currents from the nodal voltages in the problem, if necessary.

The procedure to apply node voltage analysis method is demonstrated in the following examples.

4.4.2 Node voltage analysis examples

Example 4.8: Write the node voltage equations for the circuit shown in Figure 4.15(a) using the node voltage analysis method.

Solution:

1. Label nodes a, b, and c, and choose ground c to be the reference node; assign the reference current directions for each branch as shown in Figure 4.15(a).

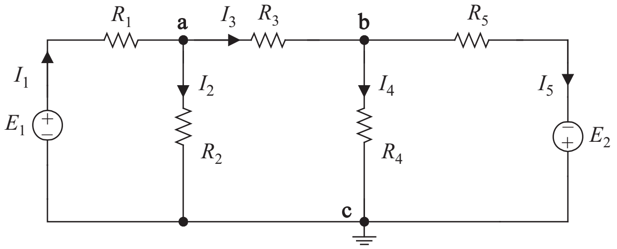


Figure 4.15(a) Circuit for Example 4.8

2. Apply KCL to $n - 1 = 3 - 1 = 2$ nodes (nodes a and b).
   - Method 1: Write KCL equations and apply Ohm's law to the equations.
     – Use resistance:

$$\text{Node a: } I_1 - I_2 - I_3 = 0, \quad \frac{E_1 - V_{\rm a}}{R_1} - \frac{V_{\rm a}}{R_2} - \frac{V_{\rm a} - V_{\rm b}}{R_3} = 0 \qquad \sum I = 0, \mathrm{I} = \frac{V}{R}$$

$$\text{Node b: } I_3 - I_4 - I_5 = 0, \quad \frac{V_{\rm a} - V_{\rm b}}{R_3} - \frac{V_{\rm b}}{R_4} - \frac{V_{\rm b} + E_2}{R_5} = 0 \qquad \sum I = 0, I = \frac{V}{R}$$

     – Use conductance:

$$\text{Node a: } I_1 - I_2 - I_3 = 0, \quad (E_1 - V_{\rm a})G_1 - V_{\rm a}G_2 - (V_{\rm a} - V_{\rm b})G_3 = 0 \qquad G = \frac{1}{R}$$

$$\text{Node b: } I_3 - I_4 - I_5 = 0, \quad (V_{\rm a} - V_{\rm b})G_3 - V_{\rm b}G_4 - (V_b + E_2)G_5 = 0$$

   - Method 2: Convert two voltage sources to current sources from Figure 4.15(a) to Figure 4.15(b), and write KCL equations by inspection.

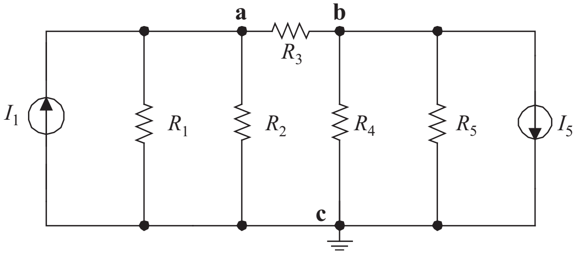


Figure 4.15(b) Circuit for method 2

– Use conductance:

First Column (V_a), second column (V_b), source I_S

Node a: $(G_1 + G_2 + G_3)V_a - G_3V_b = I_1$ $\qquad G = \frac{1}{R},\ I = GV,\ \sum I_{in} = \sum I_{out}$

Node b: $-G_3V_a + (G_3 + G_4 + G_5)V_b = -I_5$

– Use resistance:

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_a - \frac{1}{R_3}V_b = I_1 \qquad I = \frac{V}{R},\ \sum I_{in} = \sum I_{out}$$

$$-\frac{1}{R_3}V_a + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)V_b = -I_5$$

3. Two equations can solve two unknowns that are the node voltages V_a and V_b.

Note:

- The inspection method is similar with the one in mesh current analysis. The difference is that mesh current analysis uses mesh currents in each column, and node voltage analysis uses node voltage in each column.
- Assign a positive sign (+) for the self-resistor/conductor voltage and entering node current, and a negative sign (−) for the mutual-resistor/conductor voltage and exiting node current.

Example 4.9: Use the node voltage analysis to calculate currents I_1 and I_2 for the circuit shown in Figure 4.16(a) and (b).

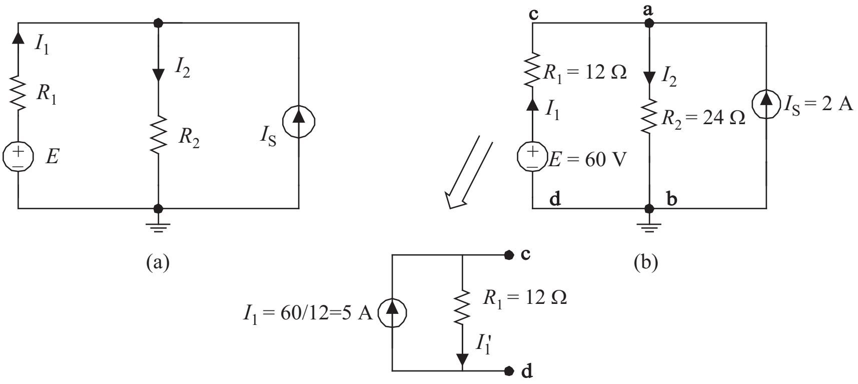


Figure 4.16 Circuit for Example 4.9

Solution:

1. Label nodes a and b, and choose b to be the reference node, and assign the reference current direction for each branch as shown in Figure 4.16(b).

2. Apply KCL to $n - 1 = 2 - 1 = 1$ node (node a):

   Use method 1: Write KCL equations and apply Ohm's law to the equations:

   - Use resistance:

$$I_1 - I_2 + I_S = 0, \qquad \frac{E - V_a}{R_1} - \frac{V_a}{R_2} + I_S = 0 \qquad\qquad \sum I = 0, \qquad I = \frac{V}{R}$$

   - Use conductance:

$$(E - V_a)G_1 - V_aG_2 + I_S = 0 \qquad\qquad \sum I = 0, \qquad G = \frac{1}{R}$$

3. Solve the above equation and determine the node voltage V_a:

$$\frac{E}{R_1} - \frac{V_a}{R_1} - \frac{V_a}{R_2} + I_S = 0$$

$$\frac{E}{R_1} + I_S = V_a\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad\qquad \text{Divide both sides by } \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$V_a = \frac{\dfrac{E}{R_1} + I_S}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{\left(\dfrac{60}{12} + 2\right)\text{ A}}{\left(\dfrac{1}{12} + \dfrac{1}{24}\right)\text{ S}} = \frac{7\text{ A}}{0.125\text{ S}} = \boxed{56\text{ V}}$$

4. Calculate the branch currents from the nodal voltages:

$$I_1 = \frac{E - V_a}{R_1} = \frac{(60 - 56)\text{V}}{12\,\Omega} \approx \boxed{0.33\text{ A}}$$

$$I_2 = -\frac{V_a}{R_2} = -\frac{56\text{ V}}{24\,\Omega} \approx \boxed{-2.33\text{ A}}$$

Use method 2:

- Convert voltage source to current source from the circuit of Figure 4.16(a) to the circuit of figure 4.16(c):

$$I_1 = \frac{E}{R_1} = \frac{60}{12} = 5\text{ A}, \qquad R_1//R_2 = 12//24 = 8\,\Omega$$

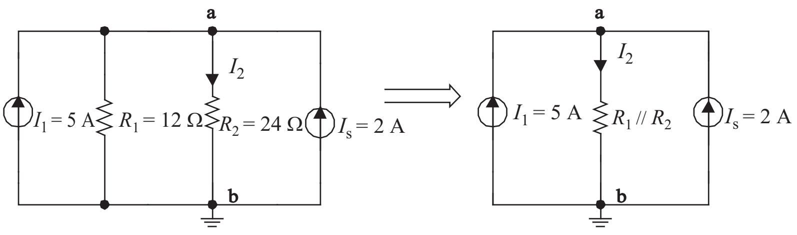


Figure 4.16(c) Circuit for Example 4.9 (cont.)

- Write KCL equation to node a using the inspection method:

$$\frac{V_a}{R_1//R_2} = I_1 + I_S \qquad\qquad I = \frac{V}{R},\ \sum I_{\text{in}} = \sum I_{\text{out}}$$

$$V_a = (I_1 + I_S)(R_1//R_2) = (5\,\text{A} + 2\,\text{A})(8\,\Omega) = \boxed{56\,\text{V}}. \qquad \text{Multiply both sides by } R_1//R_2.$$

(V_a is the same as that of method 1.)

Example 4.10: Write node voltage equations with resistances and conductances in the circuit of Figure 4.17 using the inspection method.

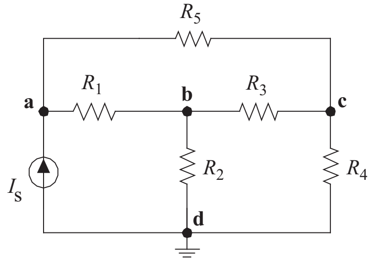


Figure 4.17 Circuit for Example 4.10

Solution:

1. Label all nodes a, b, c, and d ($n = 4$) in the circuit as shown in Figure 4.17, and choose d to be the reference node.

   (The step to assign each branch current with reference direction is skipped since this example is using the inspection method.)
2. Write KCL equations to $n - 1 = 4 - 1 = 3$ nodes using the inspection method.
   - Use resistance:

     First column (V_a), second column (V_b), third Column (V_c), source I_S

Node a: $$\left(\frac{1}{R_1} + \frac{1}{R_5}\right)V_a - \frac{1}{R_1}V_b - \frac{1}{R_5}V_c = I_S \qquad I = \frac{V}{R}$$

Node b: $$-\frac{1}{R_1}V_a + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_b - \frac{1}{R_3}V_C = 0$$

Node c: $$-\frac{1}{R_5}V_a - \frac{1}{R_3}V_b + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)V_C = 0$$

- Use conductance:

$$\begin{aligned}
&\text{Node a:} && (G_1 + G_5)V_a - G_1V_b - G_5V_c = I_S \\
&\text{Node b:} && -G_1V_a + (G_1 + G_2 + G_3)V_b - G_3V_c = 0 \\
&\text{Node c:} && -G_5V_a - G_3V_b + (G_3 + G_4 + G_5)V_c = 0
\end{aligned}$$

3. Three equations can solve three unknowns (node voltages V_a, V_b, and V_c).

4.4.3 Node voltage analysis vs. mesh current analysis

When to use the node voltage analysis or mesh current analysis?

The choice between the mesh current analysis and the node voltage analysis is often made on the basis of the circuit structure.

- The node voltage analysis is preferable for solving a circuit that is a parallel circuit, with current source(s), less nodes and more branches, and thus it is more convenient to solve circuit unknowns.
- The mesh current analysis is preferable for solving a circuit that has fewer meshes, more nodes, with voltage sources and requires solving circuit branch currents.

Summary

Source equivalent conversions and sources in series and parallel

- Voltage source → Current source: $R_S = R_S, \quad I_S = \dfrac{E}{R_S}$
- Current source → Voltage source: $R_S = R_S, \quad E = I_S R_S$
- Voltage sources in series:

$$R_S = R_{S_1} + R_{S_2} + \cdots + R_{S_n}$$

$$E = E_1 + E_2 + \cdots + E_n \qquad \text{or} \qquad V_S = V_{S_1} + V_{S_2} + \cdots + V_{S_n}$$

Assign a positive sign (+) if E_n has the same polarity as E (or V_S), otherwise assign a negative sign (−).

- Voltage sources in parallel:

$$R_S = R_{S_1} // R_{S_2} // \cdots // R_{S_n}$$

$$E = E_1 = E_2 = \cdots = E_n \qquad \text{or} \qquad V_S = V_{S_1} = V_{S_2} = \cdots = V_{S_n}$$

Only voltage sources that have the same values and polarities can be in parallel.

- Current sources in series: Only current sources that have the same polarities and values can be connected in series.

Branch current analysis

- Branch current analysis: A circuit analysis method that writes and solves a system of KCL and KVL equations in which the unknowns are the branch currents.
- Procedure for applying branch current analysis:
  1. Label the circuit:
     - Label all the nodes.
     - Assign an arbitrary reference direction for each branch current.
     - Assign loop direction for each mesh (choose clockwise direction).
  2. Apply KCL to numbers of independent nodes ($n - 1$), where n is the number of nodes.
  3. Apply KVL to each mesh, and the number of KVL equations should be equal to the number of meshes, or Equation # = branch # – (nodes # –1).
  4. Solve the simultaneous equations resulting from steps 2 and 3, and determine each branch current.
  5. Calculate the other circuit unknowns from the branch currents in the problem if necessary.

Mesh current analysis

- Mesh current analysis: A circuit analysis method that writes and solves a system of KVL equations in which the unknowns are the mesh currents.
- Procedure for applying mesh current analysis:
  1. Identify each mesh, and label all the reference directions for each mesh current (a current that circulates in the mesh) clockwise.
  2. Apply KVL to each mesh of the circuit, and the number of KVL equations should be equal to the number of meshes, or Equation # = branch # – (nodes # –1).

     Assign each self-resistor voltage as positive, and mutual-resistor voltage as negative in KVL equations.
     - Self-resistor/conductor: a resistor/conductor that only has one mesh current flowing through it.
     - Mutual-resistor/conductor: a resistor/conductor that has two mesh currents flowing through it.
  3. Solve the simultaneous equations resulting from step 2 and determine each mesh current.
  4. Calculate the other circuit unknowns such as branch currents from the mesh currents in the problem if necessary (choose the reference direction of branch currents first).

Nodal voltage analysis

- Nodal voltage analysis: A circuit analysis method that writes and solves a set of simultaneous of KCL equations in which the unknowns are the node voltages.

- Procedure for applying nodal voltage analysis
  1. Label the circuit:
     - Label all the nodes and choose one of them to be the reference node.
     - Assign an arbitrary reference direction for each branch current (this step can be skipped if using the inspection method).
  2. Apply KCL to all $n - 1$ nodes except for reference node (n is the number of nodes).
     - Method 1: Write KCL equations and apply Ohm's law to the equations; either resistance or conductance can be used. Assign a positive sign (+) for the self-resistor or self-conductor voltage and a negative sign (–) for the mutual-resistor or mutual-conductor voltage.
     - Method 2: Convert voltage sources to current sources and write KCL equations using the inspection method.
  3. Solve the simultaneous equations and determine each nodal voltage.
  4. Calculate the other circuit unknowns such as branch currents from the nodal voltages in problem if necessary.

Note: Branch current analysis, mesh current analysis, and node voltage analysis can be used for a circuit that has more than one source.

Practice problems

4.1

1. Convert a voltage source with $E = 18$ V and $R_S = 6\ \Omega$ to an equivalent current source.
2. Convert a current source with $I_S = 1.5$ A and $R_S = 3\ \Omega$ to an equivalent voltage source.
3. Calculate the load current I_L in the current source circuit of Figure 4.18. Then calculate the load current I_L again after converting the current source to an equivalent voltage source. Compare it with the I_L determined from the current source circuit.

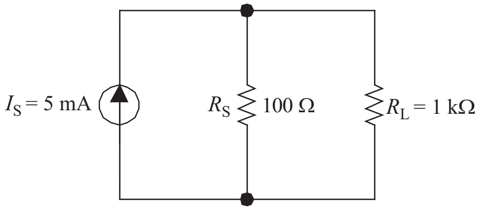


Figure 4.18

4. Determine the current I_L in the circuit of Figure 4.19.

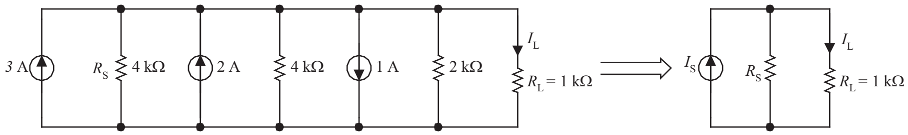

3 A
R_S
4 kΩ
2 A
4 kΩ
1 A
2 kΩ
I_L
$R_L = 1$ kΩ
I_S
R_S
I_L
$R_L = 1$ kΩ

Figure 4.19

4.2

5. List the number of nodes *n*, the number of branches, and the number of independent loops (meshes) for the circuit in Figure 4.20.

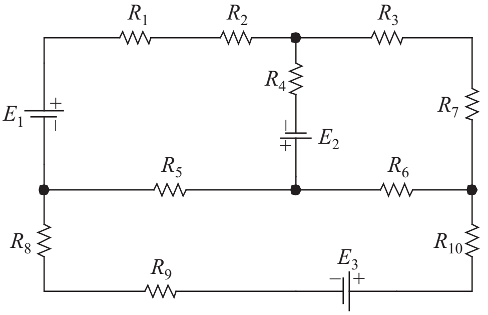

R_1
R_2
R_3
R_4
E_1
+
E_2
+
R_7
R_5
R_6
R_8
R_9
E_3
+
R_{10}

Figure 4.20

6. Determine the current in each branch of the circuit in Figure 4.21 using the branch current analysis method.

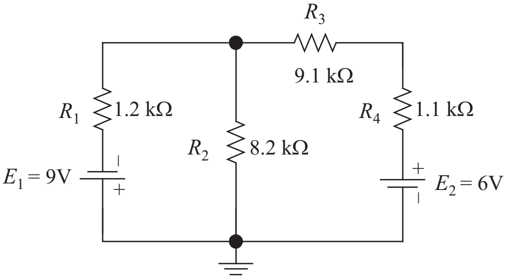

R_3
9.1 kΩ
R_1
1.2 kΩ
R_4
1.1 kΩ
R_2
8.2 kΩ
$E_1 = 9$V
+
+
$E_2 = 6$V

Figure 4.21

7. Solve for branch current *I* in Figure 4.22 using the branch current analysis method.

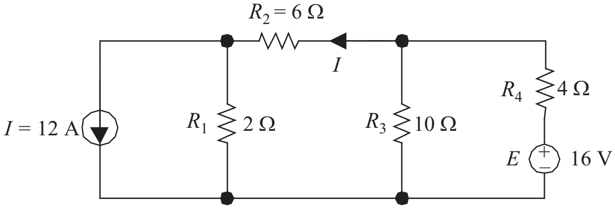
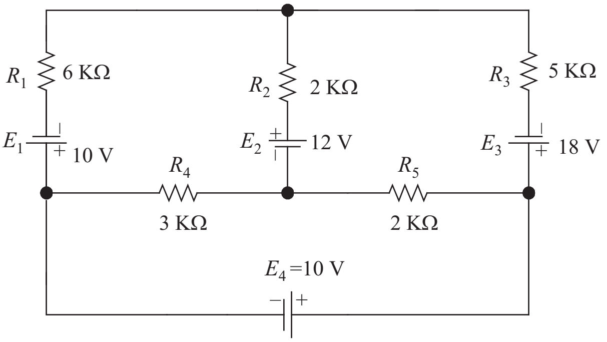

$R_2 = 6$ Ω
I
R_4
4 Ω
$I = 12$ A
R_1
2 Ω
R_3
10 Ω
E
16 V

Figure 4.22

4.3.

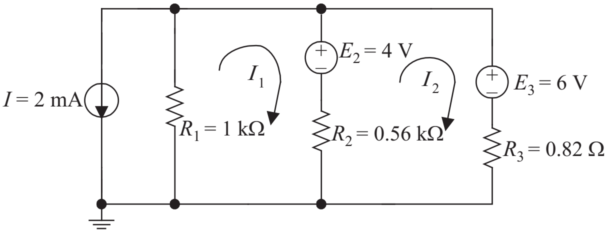
8. Write mesh equations for the circuit in Figure 4.23.



R_1
6 KΩ
E_1
10 V
R_2
2 KΩ
E_2
12 V
R_3
5 KΩ
E_3
18 V
R_4
3 KΩ
R_5
2 KΩ
E_4=10 V

Figure 4.23

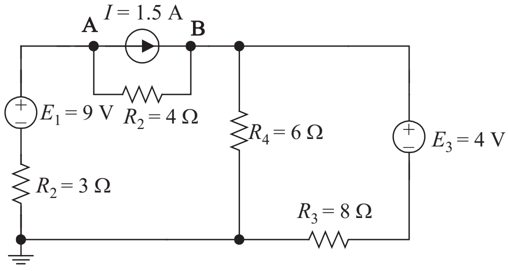
9. Determine the mesh current I_1, I_2, and voltage V_{R_2} in the circuit of Figure 4.24 using the mesh current analysis method. (Hint: Convert the current source to the equivalent voltage source first.)



$I = 2$ mA
$R_1 = 1$ kΩ
I_1
$E_2 = 4$ V
$R_2 = 0.56$ kΩ
I_2
$E_3 = 6$ V
$R_3 = 0.82$ Ω

Figure 4.24

10. Determine the mesh currents and voltage V_{AB} in the circuit of Figure 4.25 using the mesh current analysis method. (Hint: Convert the current source to the equivalent voltage source first.)



$I = 1.5$ A
A
B
$E_1 = 9$ V
$R_2 = 4$ Ω
$R_2 = 3$ Ω
$R_4 = 6$ Ω
$R_3 = 8$ Ω
$E_3 = 4$ V

Figure 4.25

4.4

11. Determine the branch currents I_a and I_b in the circuit of Figure 4.26 using the nodal voltage analysis method.

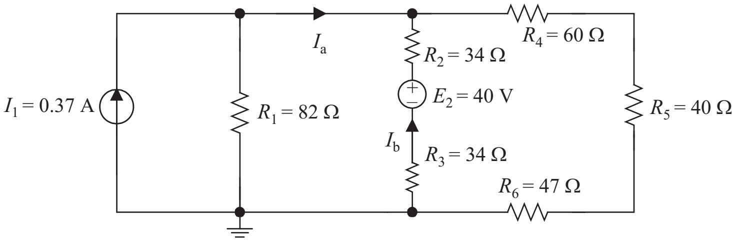

I_a
$R_4 = 60\ \Omega$
$R_2 = 34\ \Omega$
$E_2 = 40$ V
$I_1 = 0.37$ A
$R_1 = 82\ \Omega$
$R_5 = 40\ \Omega$
I_b
$R_3 = 34\ \Omega$
$R_6 = 47\ \Omega$

Figure 4.26

12. Determine the nodal voltage V_a in the circuit of Figure 4.27 using the nodal voltage analysis method.

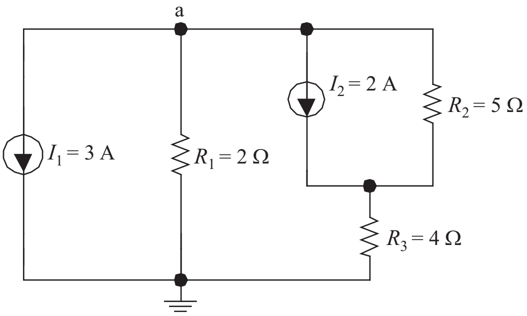

a
$I_2 = 2$ A
$R_2 = 5\ \Omega$
$I_1 = 3$ A
$R_1 = 2\ \Omega$
$R_3 = 4\ \Omega$

Figure 4.27

13. Determine the nodal voltage V_a in the circuit of Figure 4.28 using the nodal voltage analysis method.

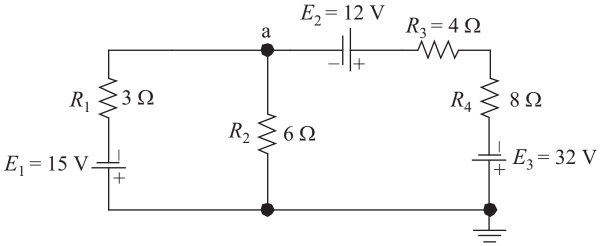

$E_2 = 12$ V
a
$R_3 = 4\ \Omega$
R_1
$3\ \Omega$
R_4
$8\ \Omega$
R_2
$6\ \Omega$
$E_1 = 15$ V
$E_3 = 32$ V

Figure 4.28

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Chapter 5

The network theorems

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Introduction to the network theorems

Limitations of DC circuit analysis methods

- The main methods for analyzing series and parallel circuits in Chapter 3 are Kirchhoff's laws.
- The branch current method, mesh, or loop analysis method and node voltage analysis method also use KCL and KVL as the main backbone.

- When the practical circuits are more and more complex, the applications of the above methods solving for currents and voltages can be quite complicated. This is because you need to solve the higher-order mathematic equations when using these methods.

Network theorems

- The scientists working in the field of the electrical engineering have developed more simplified theorems to analyze these kinds of complex circuits. (The complicated circuit is also called the network.)
- This chapter presents several theorems useful for analyzing such complex circuits or networks. These theorems include the superposition theorem, Thevenin's theorem, Norton's theorem, Millman's theorem, and the substitution theorem.
- In electric network analysis, the fundamental rules are still Ohm's law and Kirchhoff's laws.

Network	A network is a complicated circuit.

Linearity property

- The linearity property of a component describes a linear relationship between cause and effect.
- The pre-requirement of applying some of the above network theorems is that the analyzed network must be a linear circuit.
- A linear circuit has an output that is directly proportional to its input. The components of a linear circuit are the linear components.
- An example of linear component is a linear resistor. The voltage and current (input/output) of this linear resistor have a directly proportional (a straight line) relationship.

5.1 Superposition theorem

5.1.1 Steps to apply the superposition theorem

Superposition theorem

- When several power sources are applied to a single circuit or network at the same time, the superposition theorem can be used to separate the original network into several individual circuits for each power source working separately.
- Then, use series/parallel analysis to determine voltages and currents in the modified circuits.
- The actual unknown currents and voltages with all power sources can be determined by their algebraic sum; this is the meaning of the theorem's name—"superimposed."

Superposition theorem	The unknown voltages or currents in a network are the sum of the voltages or currents of the individual contributions from each single power supply, by setting the other inactive sources to zero.

Steps to apply the superposition theorem

1. Turn off all power sources except one.
   - Replace the voltage source with the short circuit (placing a jump wire).
   - Replace the current source with an open circuit.
   - Redraw the original circuit with a single source.
2. Analyze and calculate this circuit by using the single source series-parallel analysis method.
3. Repeat steps 1 and 2 for the other power sources in the circuit.
4. Determine the total contribution by calculating the algebraic sum of all contributions due to single sources.

Note:

- The result should be positive when the reference polarity of the unknown in the single source circuit is the same as the reference polarity of the unknown in the original circuit; otherwise, it should be negative.
- The superposition theorem can be applied to the linear network to determine only the unknown currents and voltages. It cannot calculate power, since power is a nonlinear variable. (Power can be calculated by the voltages and currents that have been determined by the superposition theorem.)

5.1.2 Superposition examples

Example 5.1: Determine the branch current I_c in the circuit of Figure 5.1(a) by using the superposition theorem.

Solution:

1. Choose E_1 to apply to the circuit first and use a jump wire (short circuit) to replace E_2 as shown in Figure 5.1(b).

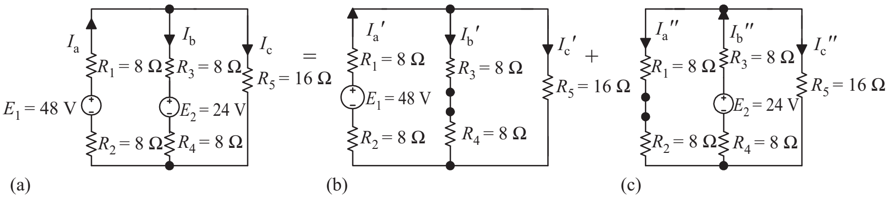


Figure 5.1 Circuit for Example 5.1

2. Calculate I_c' in the circuit of Figure 5.1(b):

$$R_{eq}' = R_5 // (R_3 + R_4) + (R_1 + R_2) = \left[\frac{16 \times (8+8)}{16 + (8+8)} + (8+8)\right] \Omega$$

$$= 24\ \Omega \qquad R_{eq} = \frac{R_1 R_1}{R_1 + R_2}$$

(view from the E_1 branch in the circuit of Figure 5.1(b) to determine R_{eq}')

$$I_a' = \frac{E_1}{R_{eq}'} = \frac{48\text{V}}{24\ \Omega} = 2\ \text{A} \qquad I = \frac{E}{R}$$

$$I_c' = I_a' \frac{R_3 + R_4}{(R_3 + R_4) + R_5} = 2\ \text{A} \frac{(8+8)\ \Omega}{(8+8+16)\ \Omega} = 1\text{A} \qquad I_2 = I_T \frac{R_1}{R_1 + R_2}$$

3. When E_2 is applied to the circuit, replace E_1 with a short circuit as shown in Figure 5.1(c), and calculate I_c'':

$$R_{eq}'' = R_5 // (R_1 + R_2) + (R_3 + R_4) = \left[\frac{16 \times (8+8)}{16 + (8+8)} + (8+8)\right] \Omega$$

$$= 24\ \Omega \qquad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

(view from the E_2 branch in the circuit of Figure 5.1(c) to determine R_{eq}'')

$$I_b'' = \frac{E_2}{R_{eq}''} = \frac{24\ \text{V}}{24\ \Omega} = 1\ \text{A} \qquad I = \frac{E}{R}$$

$$I_c'' = I_b'' \frac{R_1 + R_2}{(R_1 + R_2) + R_5} = (1\ \text{A}) \frac{(8+8)\Omega}{(8+8+16)\Omega} = 0.5\ \text{A} \qquad I_2 = I_T \frac{R_1}{R_1 + R_2}$$

4. Calculate the sum of currents I_c' and I_c'':

$$I_c = I_c' + I_c'' = (1 + 0.5)\ \text{A} = \boxed{1.5\ \text{A}}$$

Example 5.2: Determine the branch current I_2 and power P_2 of the circuit in Figure 5.2(a) by using the superposition theorem.

Solution:

1. When E is applied only to the circuit (using an open circuit to replace the current source I_1), calculate I_2' by assuming the reference direction of I_2' as shown in Figure 5.2(b):

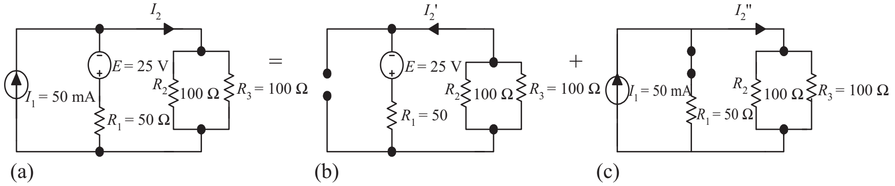


Figure 5.2 Circuit for Example 5.2

2. Calculate I_2' in the circuit of Figure 5.2(b):

$$I_2' = \frac{E}{(R_2//R_3)+R_1} = \frac{25\text{ V}}{\left(\dfrac{100\times 100}{100+100}+50\right)\Omega} = 0.25\text{ A} = 250\text{ mA} \qquad I=\frac{E}{R}$$

3. When the current source I_1 is applied only to the circuit only (the voltage source E is replaced by a jump wire), the circuit is as shown in Figure 5.2(c). Calculate I_2'' by assuming the reference direction of I_2'' as shown in the circuit of Figure 5.2(c):

$$I_2'' = I_1\frac{R_1}{R_1+R_2//R_3} = 50\text{ mA}\frac{50\,\Omega}{\left(50+\dfrac{100\times 100}{100+100}\right)\Omega} = 25\text{ mA} \qquad I_2 = I_T\frac{R_1}{R_1+R_2}$$

(Apply the current divider rule to the branches R_1 and R_2 // R_3.)

4. Calculate the sum of currents I_2' and I_2'':

$$I_2 = -I_2' + I_2'' = -250\text{ mA} + 25\text{ mA} = -225\text{ mA} = -\boxed{0.225\text{ A}}$$

- I_2' is negative as its reference direction in Figure 5.2(b) is opposite to that of I_2 in the original circuit of Figure 5.2(a).
- The negative I_2 implies that the actual direction of I_2 in Figure 5.2(a) is opposite to its reference direction.

5. Determine the power P_2:

$$P_2 = I_2^2 R_2 = (-0.225\text{ A})^2(100\,\Omega) \approx 5.06\text{ W} \qquad P=I^2R$$

Example 5.3: Determine the branch current I_3 in the circuit of Figure 5.3(a) using the superposition theorem.

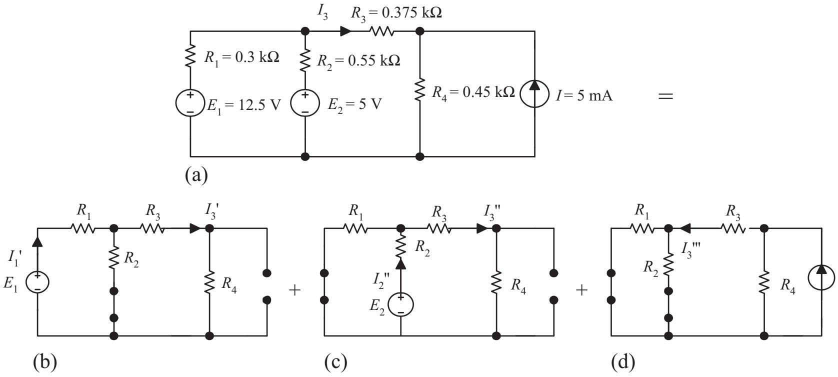


Figure 5.3 Circuit for Example 5.3

Solution:

1. Choose E_1 to apply to the circuit first and use a jump wire to replace E_2 and an open circuit to replace the current source I as shown in Figure 5.3(b).

2. Use the circuit in Figure 5.3(b) to determine $I_3{}'$

$$I_1{}' = \frac{E_1}{R_{\text{eq}}{}'} = \frac{E_1}{(R_3 + R_4)//R_2 + R_1}$$

$$= \frac{12.5\text{V}}{\left[\frac{(0.375 + 0.45) \times 0.55}{(0.375 + 0.45) + 0.55} + 0.3\right] \text{k}\Omega} \approx 19.84 \text{ mA}$$

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2}$$

$$I_3{}' = I_1{}' \frac{R_2}{R_2 + (R_3 + R_4)} = (19.84\,\text{mA}) \frac{0.55\,\text{k}\Omega}{0.55\,\text{k}\Omega + (0.375 + 0.45)\,\text{k}\Omega} \approx 7.94 \text{ mA}$$

(Apply the current divider rule to the branches R_2 and $(R_3 + R_4)$.)

$$I_1 = I_{\text{T}} \frac{R_2}{R_1 + R_2}$$

3.

- Use the circuit in Figure 5.3(c) to determine $I_3{}''$:

$$I_2{}'' = \frac{E_2}{R_{\text{eq}}{}''} = \frac{E_2}{(R_3 + R_4)//R_1 + R_2}$$

$$= \frac{5\text{V}}{\left[\frac{(0.375 + 0.45) \times (0.3)}{(0.375 + 0.45) + 0.3} + 0.55\right] \text{k}\Omega} \approx 6.49 \text{ mA}$$

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2}$$

$$I_3{}'' = I_2{}'' \frac{R_1}{R_1 + (R_3 + R_4)} = (6.49 \text{ mA}) \frac{0.3 \text{ k}\Omega}{[0.3 + (0.375 + 0.45)] \text{ k}\Omega} \approx 1.73 \text{ mA}$$

(Apply the current divider rule to the branches R_1 and $(R_3 + R_4)$.)

$$I_1 = I_T \frac{R_2}{R_1 + R_2}$$

- Use the circuit in Figure 5.3(d) to determine $I_3{}'''$:

$$I_3{}''' = I \frac{R_4}{(R_1//R_2 + R_3) + R_4} = (5 \text{ mA}) \frac{0.45 \text{ k}\Omega}{\left[\left(\frac{0.3 \times 0.55}{0.3 + 0.55} + 0.375\right) + 0.45\right] \text{k}\Omega}$$

$$\approx 2.21 \text{ mA}$$

(Apply the current divider rule to the branches R_4 and $(R_1//R_2 + R_3)$.)

$$I_2 = I_{\text{T}} \frac{R_1}{R_1 + R_2}$$

4. Calculate the sum of currents $I_3{}'$, $I_3{}''$, and $I_3{}'''$:

$$I_3 = I_3{}' + I_3{}'' - I_3{}''' = 7.94 \text{ mA} + 1.73 \text{ mA} - 2.21 \text{ mA} = \boxed{7.46 \text{ mA}}$$

$I_3{}'''$ is negative since its reference direction is opposite to that of I_3 in the original circuit of Figure 5.3(a).

5.2 Thevenin's and Norton's theorems

5.2.1 Introduction to Thevenin's and Norton's theorems

Background

- Thevenin's and Norton's theorems are two of the most widely used theorems to simplify the linear circuit for the ease of network analysis.
- In 1883, the French telegraph engineer M. L. Thevenin published his theorem of the network analysis method.
- Forty-three years later, an American engineer E. L. Norton in Bell Telephone laboratory published a similar theorem, but he used the current source to replace the voltage source in the equivalent circuit.

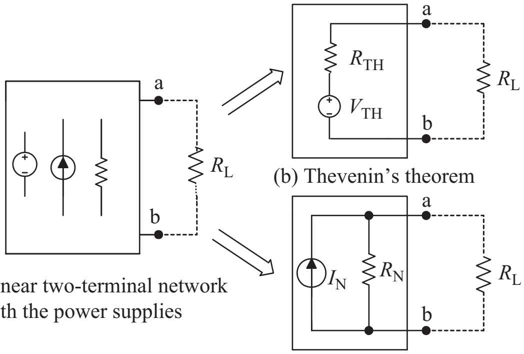
Introduction to Thevenin's and Norton's theorems

- These two theorems state that any complicated linear two-terminal network with power supplies can be simplified to an equivalent circuit that includes
  - an actual voltage source (Thevenin's theorem),
  - or an actual current source (Norton's theorem).
- The "linear two-terminal network with power supplies" means:
  - Network: the relatively complicated circuit.
  - Linear network: the circuits in the network are the linear circuits.
  - Two-terminal network: the network with two terminals that can be connected to the external circuits.
  - Network with the power supplies: network includes the power supplies.
- Any combination of power supplies and resistors with two terminals can be replaced by
  - a single voltage source and a single series resistor for Thevenin's theorem.
  - a single current source and a single parallel resistor for Norton's theorem.

5.2.2 Thevenin and Norton equivalent circuits

Thevenin and Norton equivalent circuits

- No matter how complex the inside construction of any two-terminal network with power supplies is, they can all be illustrated in Figure 5.4(a).
- According to Thevenin's and Norton's theorems, we can draw the following conclusion:
  - Any linear two-terminal network with power supplies can be replaced by an equivalent circuit as shown in Figure 5.4(b) or (c).
  - The equivalent means that any load resistor branch (or unknown current or voltage branch) connected between the terminals of Thevenin's or Norton's equivalent circuit will have the same current and voltage as if it was connected to the terminals of the original circuit.
  - Thevenin's and Norton's theorems allow for the analysis of the performance of a circuit from its terminal properties only.



(a) Linear two-terminal network with the power supplies

(b) Thevenin's theorem

(c) Norton's theorem

Figure 5.4 Thevenin's and Norton's Theorems

Thevenin's and Norton's theorems

Any linear two-terminal network with power supplies can be replaced by a simple equivalent circuit, which has a single power source and a single resistor.	
Thevenin's theorem	Thevenin's equivalent circuit is a *voltage source*—with an equivalent resistance R_{TH} in series with an equivalent voltage source V_{TH}.
Norton's theorem	Norton's equivalent circuit is a *current source*—with an equivalent resistance R_N in parallel with an equivalent current source I_N.

5.2.3 Equivalent resistance and voltage/current

R_{TH}, V_{TH}, R_N, and I_N

- Any combination of power supplies and resistors with two terminals can be replaced by a single voltage source and a single series resistor for Thevenin's theorem, and replaced by a single current source and a single parallel resistor for Norton's theorem.
- The key to applying Thevenin's and Norton's theorems is to determine the equivalent resistance R_{TH} and the equivalent voltage V_{TH} for Thevenin's equivalent circuit, and the equivalent resistance R_N and the equivalent current I_N for Norton's equivalent circuit.
- The value of R_N in Norton's equivalent circuit is the same as R_{TH} of Thevenin's equivalent circuit.

$$\boxed{R_N = R_{TH}}$$

  - The "TH" in V_{TH} and R_{TH} means Thevenin.
  - The "N" in I_N and R_N means Norton.

Note:

- Thevenin's and Norton's theorems are used very often, as it is often necessary to calculate the load (or a branch) current or voltage in practical applications.
- The load resistor can be varied sometimes (for instance, the wall-plug can connect to 60 W or 100 W lamps). Once the load is changed, the whole circuit has to be reanalyzed or recalculated.
- If Thevenin's and Norton's theorems are used, Thevenin's and Norton's equivalent circuits will not be changed except for their external load branches. The variation of the load can be more conveniently to determine by using Thevenin's or Norton's equivalent circuits.

5.2.4 Procedure for applying the Thevenin's and Norton's theorems

Steps to apply Thevenin's and Norton's theorems

1. Open and remove the load branch (or any unknown current or voltage branch) in the network, and mark the letters a and b on the two terminals.
2. Determine the equivalent resistance R_{TH} or R_{N}. It equals the equivalent resistance, looking at it from the a and b terminals when all sources are turned off or equal to zero in the network. That is, $\boxed{R_{\mathrm{TH}} = R_{\mathrm{N}} = R_{\mathrm{ab}}}$
    - A voltage source should be replaced by a short circuit.
    - A current source should be replaced by an open circuit.
3.
    - Determine Thevenin's equivalent voltage V_{TH}. It equals the open-circuit voltage from the original linear two-terminal network of a and b, i.e., $\boxed{V_{\mathrm{TH}} = V_{\mathrm{ab}}}$
    - Determine Norton's equivalent current I_{N}. It equals the short-circuit current from the original linear two-terminal network of a and b,

      i.e., $\boxed{I_{\mathrm{N}} = I_{\mathrm{sc}}}$ where "sc" means the short circuit.
4. Plot Thevenin's or Norton's equivalent circuit, and connect the load branch (or unknown current or voltage branch) to a and b terminals of the equivalent circuit. Then the load (or unknown) voltage or current can be determined.

The above procedure for analyzing circuits by using Thevenin's and Norton's theorems is illustrated in the circuits of Figure 5.5.

Thevenin and Norton equivalent circuits

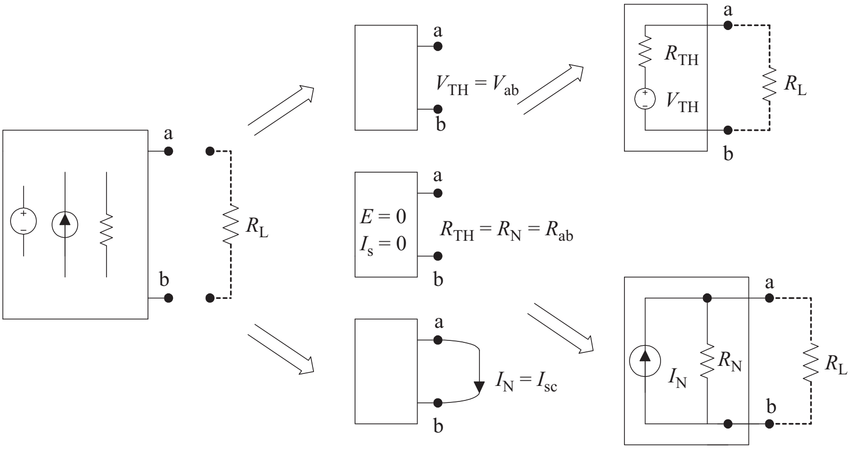

a
b
R_L
$V_{TH} = V_{ab}$
$E = 0$
$I_s = 0$
$R_{TH} = R_N = R_{ab}$
$I_N = I_{sc}$
R_{TH}
V_{TH}
I_N
R_N

Figure 5.5 The procedure for applying Thevenin's and Norton's theorems

5.2.5 Thevenin/Norton equivalent example

Example 5.4: Determine the load current I_L in the circuit of Figure 5.6(a) by using Thevenin's and Norton's theorems.

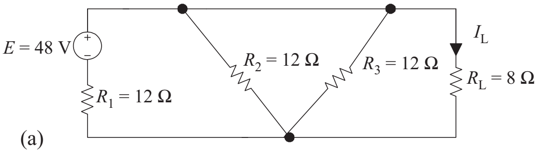

$E = 48$ V
$R_1 = 12\ \Omega$
$R_2 = 12\ \Omega$
$R_3 = 12\ \Omega$
I_L
$R_L = 8\ \Omega$
(a)

Figure 5.6(a) Circuit for Example 5.4

Solution:

1. Open and remove the load branch R_L, and mark a and b on the terminals of the load branch as shown in the circuit of Figure 5.6(b).

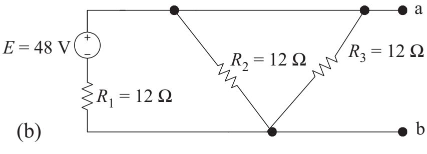

$E = 48$ V
$R_1 = 12\ \Omega$
$R_2 = 12\ \Omega$
$R_3 = 12\ \Omega$
a
b
(b)

Figure 5.6(b)

2. Determine Thevenin's and Norton's equivalent resistance R_{TH} and R_{N} (the voltage source is replaced by a short circuit) in the circuit of Figure 5.6(c).

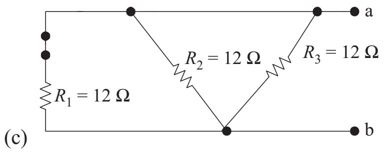

a
$R_2 = 12\ \Omega$
$R_3 = 12\ \Omega$
$R_1 = 12\ \Omega$
(c)
b

Figure 5.6(c)

$$R_{\mathrm{TH}} = R_{\mathrm{N}} = R_{\mathrm{ab}} = R_1 // R_2 // R_3 = (12//12//12)\ \Omega = 4\Omega \qquad \mathrm{R_{eq}} = \frac{R_1 R_2}{R_1 + R_2}$$

3.

- Determine Thevenin's equivalent voltage V_{TH}: Use the circuit in Figure 5.6(d) to calculate the open-circuit voltage across the terminals a and b.

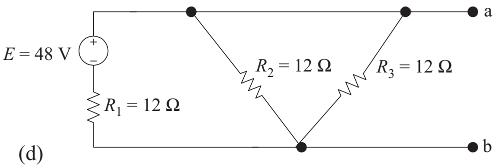

a
$E = 48$ V
+
−
$R_2 = 12\ \Omega$
$R_3 = 12\ \Omega$
$R_1 = 12\ \Omega$
(d)
b

Figure 5.6(d)

$$V_{\mathrm{TH}} = V_{\mathrm{ab}} = E\frac{R_2 // R_3}{R_1 + R_2 // R_3} = (48\ \mathrm{V})\frac{(12//12)\ \Omega}{(12 + 12//12)\ \Omega} = 16\ \mathrm{V} \qquad R_{\mathrm{eq}} = \frac{R_1 R_2}{R_1 + R_2}$$

Apply the voltage divider rule to the resistors $R_2//R_3$ and R_1

$$V_2 = V_{\mathrm{T}}\frac{R_2}{R_1 + R_2}$$

- Determine Norton's equivalent current I_{N}: Use the circuit in Figure 5.6(e) to calculate the short-circuit current in the terminals a and b.

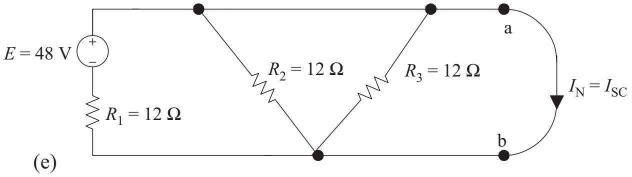
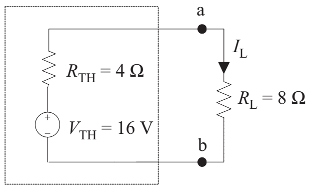

$E = 48$ V
+
−
a
$R_2 = 12\ \Omega$
$R_3 = 12\ \Omega$
$I_{\mathrm{N}} = I_{\mathrm{SC}}$
$R_1 = 12\ \Omega$
b
(e)

Figure 5.6(e)

$$I_{\mathrm{N}} = I_{\mathrm{SC}} = \frac{E}{R_1} = \frac{48\mathrm{V}}{12\Omega} = 4\mathrm{A} \qquad I = \frac{E}{R}$$

Since the current in the branch E and R_1 will go through a short circuit without resistance—through the branch a and b—and will not go through the branches R_2 and R_3 that have resistances, in this case $I_{\mathrm{N}} = \frac{E}{R_1}$.

4. Plot Thevenin's and Norton's equivalent circuits as shown in Figure 5.6(f) and (g). Connect the load R_L to a and b terminals of the equivalent circuits and determine the load current I_L.
   - Use Thevenin's equivalent circuit in Figure 5.6(f) to determine I_L.



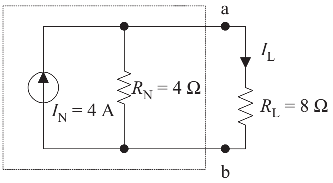
(f)

Figure 5.6(f)

$$I_L = \frac{V_{TH}}{R_{TH} + R_L} = \frac{16\ \text{V}}{(4+8)\ \Omega} \approx \boxed{1.33\ \text{A}}$$

$$I = \frac{V}{R}$$

- Use Norton's equivalent circuit in Figure 5.6(g) to calculate I_L.



(g)

Figure 5.6(g)

$$I_L = I_N \frac{R_N}{R_N + R_L} = 4\text{A} \frac{4\ \Omega}{(4+8)\ \Omega} \approx \boxed{1.33\ \text{A}}$$

(Apply the current divider rule to the resistors R_N and R_L.)

$$I_2 = I_T \frac{R_1}{R_1 + R_2}$$

5.2.6 *Viewpoints of Thevenin's and Norton's equivalent circuits*

Viewpoints

- One important way to apply Thevenin's and Norton's theorems for analyzing any network is to determine the viewpoints of Thevenin's and Norton's equivalent circuits.

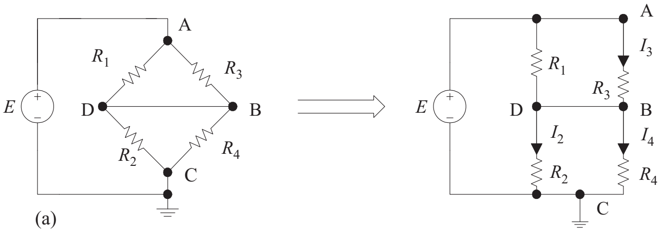


Figure 5.7(a) Viewpoints for the theorem

- The load branch (or any unknown current or voltage branch) belongs to the external circuit of the linear two-terminal network with power sources. The opening two terminals of the branch are the viewpoints for Thevenin's and Norton's equivalent circuits.
- There could be different viewpoints for the bridge circuit as shown in Figure 5.7(a).
  - If we want to determine the branch current I_3, we use A–B as viewpoints.
  - If we want to determine the branch current I_2, we use D–C as viewpoints, etc.
- Different equivalent circuits and results will be obtained from using different viewpoints.

Example 5.5: For the circuit in Figure 5.7(a):

(a) Plot Thevenin's equivalent circuit for calculating the current I_3.
(b) Determine Norton's equivalent circuit for the viewpoints B–C.
(c) Determine Thevenin's equivalent circuit for the viewpoints D–B.

Solution:

(a) **The viewpoints for calculating I_3** should be A–B (Figure 5.7(a)).

1. Open and remove R_3 in the branch A–B of Figure 5.7(a) and mark the letters a and b, as shown in the circuit of Figure 5.7(b).

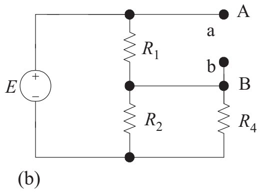


Figure 5.7 (b) Figure (b) for Example 5.5

2. Determine R_{TH} and R_{ab}. Replace the voltage source E with a short circuit, as shown in the circuit of Figure 5.7(c).

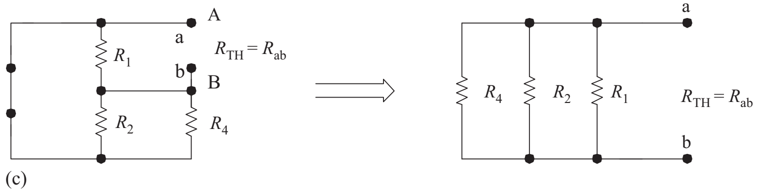

A
a
$R_{TH} = R_{ab}$
b
B
R_1
R_2
R_4
(c)
a
R_4
R_2
R_1
$R_{TH} = R_{ab}$
b

Figure 5.7(c)

$$R_{TH} = R_{ab} = (R_2 // R_4) // R_1 \qquad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

3. Determine V_{TH} using the circuit in Figure 5.7(d).

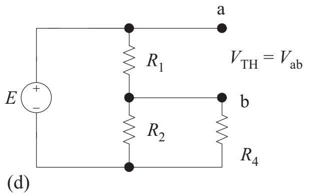

a
R_1
$V_{TH} = V_{ab}$
E
+
−
b
R_2
R_4
(d)

Figure 5.7(d)

$$V_{TH} = V_{ab} = E \frac{R_1}{R_1 + R_2 // R_4} \qquad V_1 = V_T \frac{R_1}{R_1 + R_2}$$

4. Plot Thevenin's equivalent circuit as shown in the circuit of Figure 5.7(e). Connect R_3 to the a and b terminals of the equivalent circuit and determine the current I_3:

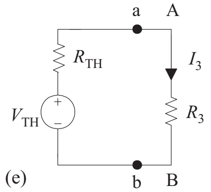

a
A
R_{TH}
I_3
V_{TH}
+
−
R_3
(e)
b
B

Figure 5.7(e)

$$I_3 = \frac{V_{TH}}{R_{TH} + R_3}$$

(b) **Norton's equivalent circuit for the viewpoints B–C.**

1. Open and remove R_4 in the branch B–C of Figure 5.7(a), and mark the letters a and b on the two terminals as shown in the circuit of Figure 5.7(f).

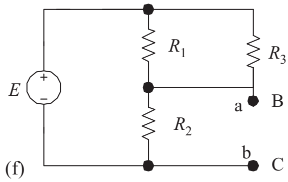


Figure 5.7(f)

2. Determine R_N. Replace the voltage source with a short circuit as shown in the circuit of Figure 5.7(g).

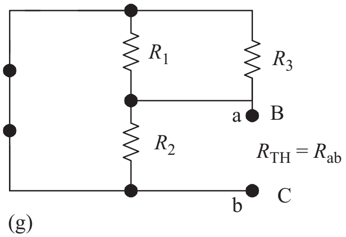


Figure 5.7(g)

3. Determine I_N using the circuit in Figure 5.7(h):

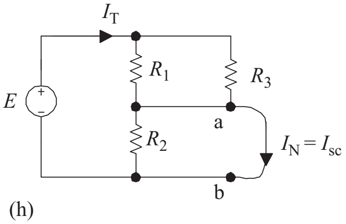


Figure 5.7(h)

$$I_N = I_T = \frac{E}{R_1 // R_3} \qquad I = \frac{E}{R}, \qquad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

(Since the current will go through the short circuit without resistance—the branch a and b—and will not go through the branch with resistance R_2, in this case $I_N = I_T$.)

4. Plot Norton's equivalent circuit as shown in the circuit of Figure 5.7(i).

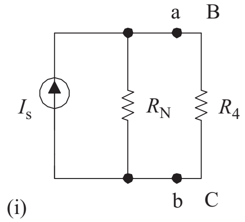


Figure 5.7(i)

(c) **Thevenin's equivalent circuit for the viewpoints D–B:**

1. Open branch D–B (Figure 5.7(a)) and mark the letters a and b on the two terminals as shown in the circuit of Figure 5.7(j).

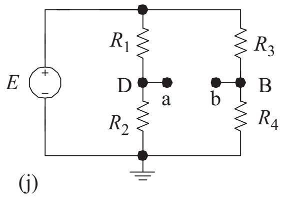


Figure 5.7(j)

2. Determine R_{TH}. Replace the voltage source with a short circuit as shown in the circuit of Figure 5.7(k).

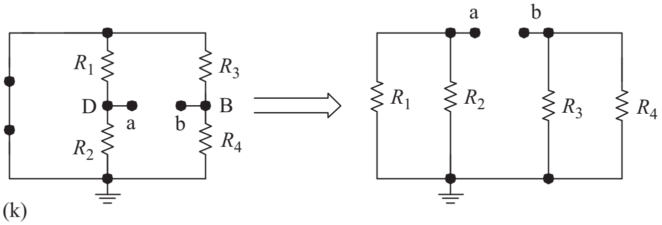


Figure 5.7(k)

$$R_{TH} = R_{ab} = (R_1 // R_2) + (R_3 // R_4) \qquad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

3. Determine V_{TH} using the circuit in Figure 5.7(j):

$$V_{TH} = V_{ab} = V_a + (-V_b) = E\frac{R_2}{R_1 + R_2} - E\frac{R_4}{R_3 + R_4} \qquad V_2 = V_T\frac{R_2}{R_1 + R_2}$$

4. Plot Thevenin's equivalent circuit as shown in the circuit of Figure 5.7(l).

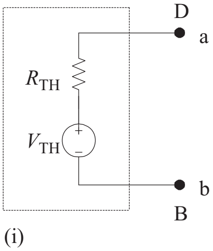

D
a
R_{TH}
V_{TH}
+
−
b
B
(i)

Figure 5.7(l)

5.2.7 Norton's theorem examples

Example 5.6: Determine current I_L in the circuit of Figure 5.8(a) by using Norton's theorem.

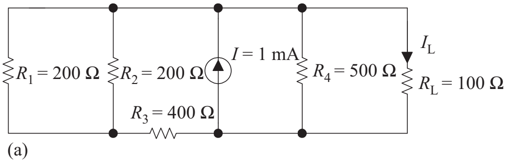

$R_1 = 200\ \Omega$
$R_2 = 200\ \Omega$
$I = 1$ mA
$R_4 = 500\ \Omega$
I_L
$R_L = 100\ \Omega$
$R_3 = 400\ \Omega$
(a)

Figure 5.8(a) Circuit for Example 5.6

Solution:

1. Open and remove R_L in the load branch (Figure 5.8(b)) and mark the letters a and b on its two terminals, as shown in the circuit of Figure 5.8(b).

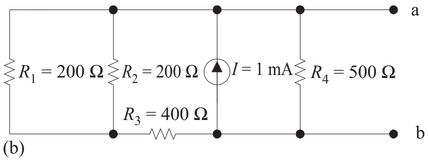

a
$R_1 = 200\ \Omega$
$R_2 = 200\ \Omega$
$I = 1$ mA
$R_4 = 500\ \Omega$
$R_3 = 400\ \Omega$
b
(b)

Figure 5.8(b)

2. Determine R_N. Replace the current source with an open circuit as shown in the circuit of Figure 5.8(c).

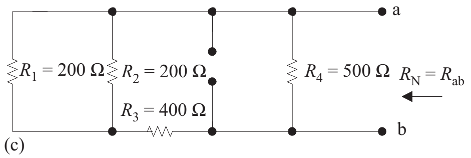

a
$R_1 = 200\ \Omega$
$R_2 = 200\ \Omega$
$R_3 = 400\ \Omega$
$R_4 = 500\ \Omega$
$R_N = R_{ab}$
b
(c)

Figure 5.8(c)

$$R_N = R_{ab} = (R_1 // R_2 + R_3) // R_4 = [(200 // 200 + 400) // 500]\Omega = 250\Omega \qquad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

3. Calculate I_N using the circuit of Figure 5.8(d).

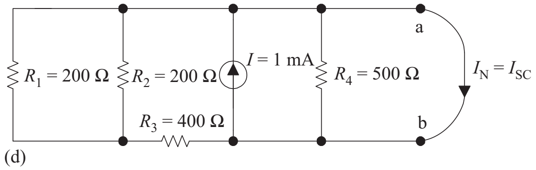

a
$R_1 = 200\ \Omega$
$R_2 = 200\ \Omega$
$I = 1$ mA
$R_4 = 500\ \Omega$
$I_N = I_{SC}$
$R_3 = 400\ \Omega$
b
(d)

Figure 5.8(d)

$$I_N = I = 1\text{mA}$$

Since the current I will flow through the short cut without resistance—the branch **a** and **b**—and will not go through the branch with resistance, $I_N = I$.

4. Plot Norton's equivalent circuit as shown in the circuit of Figure 5.8(e). Connect R_L to the a and b terminals of the equivalent circuit, and calculate the current I_L.

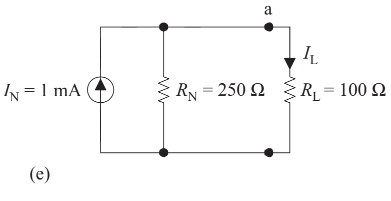
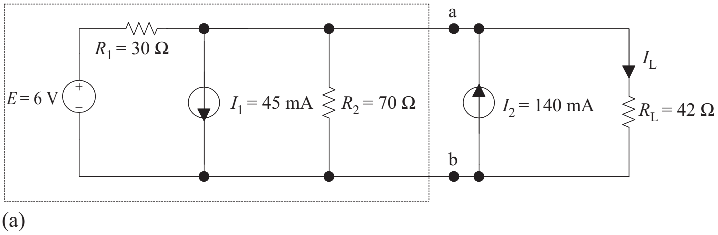

a
I_L
$I_N = 1$ mA
$R_N = 250\ \Omega$
$R_L = 100\ \Omega$
(e)

Figure 5.8(e)

$$I_L = I_N \frac{R_N}{R_L + R_N} = (1\text{ mA}) \frac{250\ \Omega}{(250 + 100)\ \Omega} \approx \boxed{0.71\text{ mA}} \qquad I_1 = I_T \frac{R_2}{R_1 + R_2}$$

When applying Thevenin's and Norton's theorems to analyze networks, it is often necessary to combine theorems that we have learned in the previous chapters. This is explained in the following example.

Example 5.7: Determine Norton's equivalent circuit for the left part of the terminals a and b in the circuit of Figure 5.9(a) and determine the current I_L.

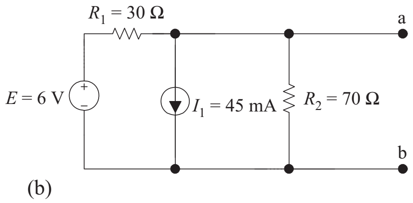


a
$R_1 = 30\ \Omega$
$E = 6$ V
$I_1 = 45$ mA
$R_2 = 70\ \Omega$
$I_2 = 140$ mA
I_L
$R_L = 42\ \Omega$
b
(a)

Figure 5.9(a) Circuit for Example 5.7

Solution:

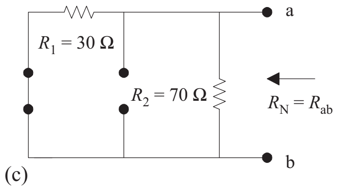
1. Open and remove the current source part on the right side of the circuit from the terminals a and b (Figure 5.9(a)), as shown in the circuit of Figure 5.9(b).



$R_1 = 30\ \Omega$
a
$E = 6$ V
$I_1 = 45$ mA
$R_2 = 70\ \Omega$
b
(b)

Figure 5.9(b)

2. Determine R_N. Replace the voltage source with a short circuit, and the current source with an open circuit, as shown in the circuit of Figure 5.9(c).



a
$R_1 = 30\ \Omega$
$R_2 = 70\ \Omega$
$R_N = R_{ab}$
b
(c)

Figure 5.9(c)

$$R_N = R_{ab} = R_1 // R_2 = \frac{(30\ \Omega)(70\ \Omega)}{30\ \Omega + 70\ \Omega} = 21\ \Omega \qquad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

3. Determine I_N using the circuit in Figure 5.9(d). Since there are two power supplies in this circuit, it is necessary to apply the network analyzing method for this complex circuit. Let us try to use the superposition theorem to determine I_N.

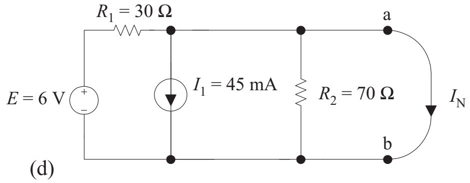

$R_1 = 30\ \Omega$
a
$E = 6$ V
+
−
$I_1 = 45$ mA
$R_2 = 70\ \Omega$
I_N
b
(d)

Figure 5.9(d)

- When the single voltage source E is applied to the circuit, the circuit is shown in Figure 5.9(e).

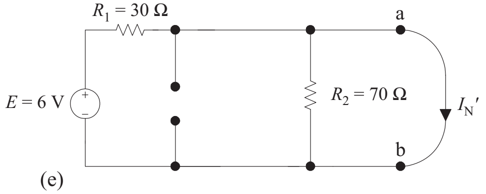

$R_1 = 30\ \Omega$
a
$E = 6$ V
+
−
$R_2 = 70\ \Omega$
I_N'
b
(e)

Figure 5.9(e)

Since R_2 is short-circuited by I_N' (recall that current always goes through the short circuit without resistance):

$$\therefore \quad I_N' = \frac{E}{R_1} = \frac{6\text{V}}{30\Omega} = 0.2\ \text{A} = 200\ \text{mA} \qquad I = \frac{E}{R}$$

- When the single current source I_1 is applied to the circuit, the circuit is shown in Figure 5.9(f).

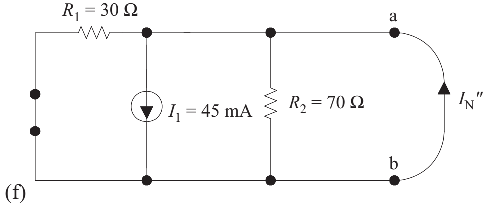

$R_1 = 30\ \Omega$
a
$I_1 = 45$ mA
$R_2 = 70\ \Omega$
I_N''
b
(f)

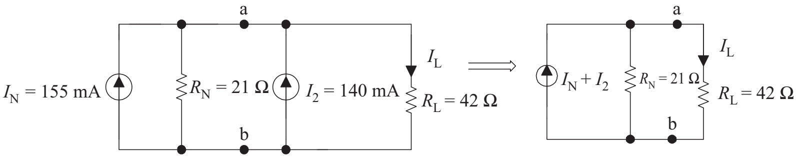
Figure 5.9(f)

Since R_1 and R_2 are short-circuited by the I_N'': $\qquad I_N'' = I_1 = 45$ mA

- Determine I_N:

$$I_N = I_N' - I_N'' = 200\text{ mA} - 45\text{ mA} = 155\text{ mA}$$

4. Plot Norton's equivalent circuit. Connect the right side of the a and b terminals of the current source (Figure 5.9(a)) to the a and b terminals of Norton's equivalent circuit, as shown in the circuits of Figure 5.9(g). Determine the current I_L.



(g)

Figure 5.9(g)

$$I_1 = I_T \frac{R_2}{R_1 + R_2}$$

$$\begin{aligned} I_L &= (I_N + I_2)\frac{R_N}{R_L + R_N} \\ &= (155 + 140)\text{ mA}\ \frac{21\ \Omega}{(42 + 21)\ \Omega} \approx \boxed{98.33\text{ mA}} \end{aligned}$$

5.3 Maximum power transfer

5.3.1 Maximum power transfer theorem

Transfer the maximum power from a source to a load

- Practical circuits are usually designed to provide power to the load.
- When working in electrical or electronic engineering fields, you are sometimes asked to design a circuit that will transfer the maximum power from a given source to a load.
- The maximum power transfer theorem can be used to solve this kind of problem.

Maximum power transfer theorem

- The maximum power transfer theorem states that when the load resistance is equal to the source's internal resistance, the maximum power will be transferred to the load.
- From the last section, we have learned that any linear two-terminal network with power supplies can be equally substituted by Thevenin's or Norton's equivalent circuits.

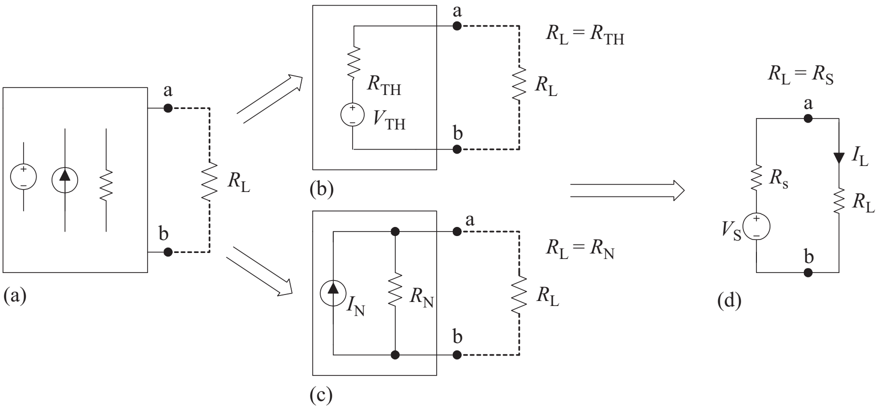


Figure 5.10 The maximum power transfer

- The maximum power transfer theorem implies that when the load resistance (R_L) of a circuit is equal to the internal resistance (R_S) of the source or the equivalent resistance of Thevenin's or Norton's equivalent circuits (R_{TH} or R_N), maximum power will be dissipated in the load.
- The maximum power transfer theorem is illustrated in the circuits of Figure 5.10.

5.3.2 Applications of maximum power transfer

Applications of the maximum power transfer theorem

- The maximum power transfer theorem is used very often in radios, stereos, TV, etc.
- If the load component is a speaker and the circuit that drives the speaker is a power amplifier, when the resistance of the speaker R_L is equal to the internal resistance R_S of the amplifier, the amplifier can transfer the maximum power to the speaker, i.e., the maximum volume can be delivered by the speaker.
- When the resistance of a TV receiver is equal to the internal resistance R_S of the antenna, the maximum signal from the antenna can be received.

Calculate the maximum load power

- Using the equivalent circuit in Figure 5.10(d) to calculate the power consumed by the load resistor R_L gives

$$P_L = I_L{}^2 R_L = \left(\frac{V_S}{R_S + R_L}\right)^2 R_L \qquad (5.1) \qquad P = I^2 R, \quad I = \frac{V}{R}$$

- When $R_L = R_S$, the maximum power that can be transferred to the load is

$$P_L = \frac{{V_S}^2}{(2R_S)^2} R_S = \frac{{V_S}^2}{4R_S}$$

The maximum load power	$P_L = \frac{{V_S}^2}{4R_S}$

If $V_S = 10$ V, $R_S = 30\ \Omega$, and $R_L = 30\ \Omega$

Then $P_L = \frac{{V_S}^2}{4R_S} = \frac{(10\text{ V})^2}{4(30\ \Omega)} \approx 0.833\text{ W} = \boxed{833\text{ mW}}$

Summary of the theorem

The maximum power transfer theorem	When the load resistance is equal to the internal resistance of the source ($R_L = R_S$); or when the load resistance is equal to the Thevenin's/Norton's equivalent resistance of the network ($R_L = R_{TH} = R_N$), the maximum power can be transferred to the load.

5.3.3 Proof of maximum power transfer theorem

An experiment circuit

- The maximum power transfer theorem can be proved by using an experiment circuit as shown in Figure 5.11.
- When the variable resistor R_L is adjusted, it will change the value of the load resistor. Replacing the load resistance R_L with different values in (5.1) gives different load power P_L, as shown in Table 5.1.

- When $R_L = 10\ \Omega$:

$$P_L = I^2 R_L = \left(\frac{V_S}{R_S + R_L}\right)^2 R_L = \left(\frac{10\text{ V}}{30\ \Omega + 10\ \Omega}\right)^2 (10\ \Omega) \approx \boxed{0.625\text{ W}}$$

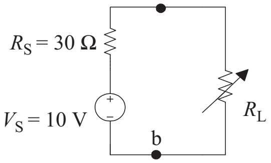


Figure 5.11 The experiment circuit

Table 5.1 The load power

R_L (Ω)	P_L (W)
10	0.625
20	0.800
30	0.833
40	0.816
50	0.781

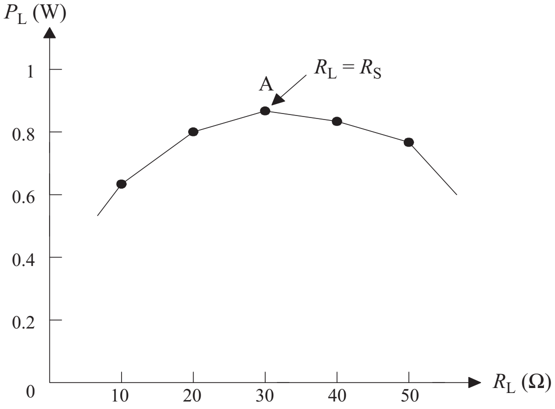


Figure 5.12 R_L–P_L curve

- When $R_L = 20\ \Omega$:

$$P_L = I^2 R_L = \left(\frac{V_S}{R_S + R_L}\right)^2 R_L = \left(\frac{10\ \text{V}}{30\ \Omega + 20\ \Omega}\right)^2 (20\ \Omega) \approx \boxed{0.8\text{W}}$$

- The R_L and P_L curves can be plotted from Table 5.1 as shown in Figure 5.12.

Table 5.1 and Figure 5.12 show that only when $R_L = R_S$ (30 Ω), the power for the resistor R_L reaches the maximum point A (0.833 W).

5.4 Millman's and substitution theorems

5.4.1 Millman's theorem

Introduction to Millman's theorem

- Millman's theorem is named after the Russian Electrical engineering professor Jacob Millman (1911–1991) who proved this theorem. A similar method, known as Tank's method, had already been used before Millman's proof.

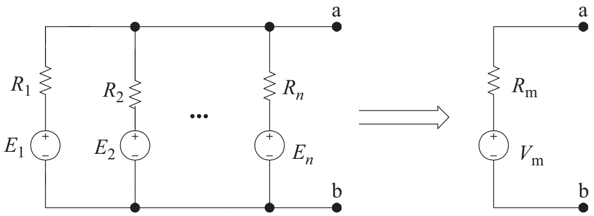


Figure 5.13 Millman's theorem

- The method using series—parallel power sources was stated in Chapter 4. However, the series—parallel method can only be used in power sources that have the same polarities and values.
- Millman's theorem in this chapter can be used to analyze circuits of parallel voltage sources that have different polarities and values. This can be shown in the circuit of Figure 5.13.

Millman's theorem

- Millman's theorem states that for a circuit of parallel branches, with each branch consisting of a resistor or a voltage source/current source, this circuit can be replaced by a single voltage source with voltage V_m in series with a resistor R_m as shown in Figure 5.13.
- Millman's Theorem, therefore, can determine the voltage across the parallel branches of a circuit.
- Calculating V_m and R_m:

$$\boxed{R_m = R_1 // R_2 // \ldots // R_n}$$

$$\boxed{V_m = R_m I_m = R_m \left(\frac{E_1}{R_1} + \frac{E_2}{R_2} + \ldots + \frac{E_n}{R_n} \right)} \qquad V = IR,\ I = \frac{V}{R}$$

Note:

- V_m is the algebraic sum for all the individual terms in the equation. It will be positive if E_n and V_m have the same polarities, otherwise it will be negative.
- The letter m in V_m and R_m means Millman.

5.4.2 *Millman's theorem example*

Example 5.8: Determine the load voltage V_L in the circuit of Figure 5.14 using Millman's theorem.

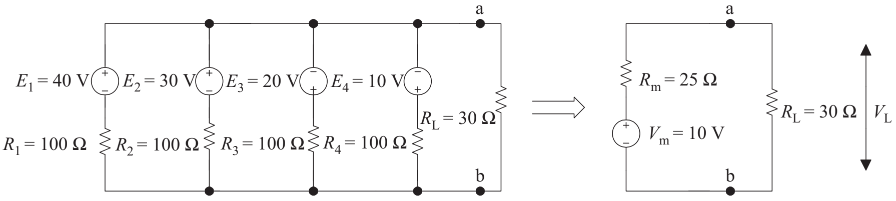

a
a
$E_1 = 40$ V
$E_2 = 30$ V
$E_3 = 20$ V
$E_4 = 10$ V
$R_1 = 100\ \Omega$
$R_2 = 100\ \Omega$
$R_3 = 100\ \Omega$
$R_4 = 100\ \Omega$
$R_L = 30\ \Omega$
$R_m = 25\ \Omega$
$V_m = 10$ V
$R_L = 30\ \Omega$
V_L
b
b

Figure 5.14 Circuit for Example 5.8

Solution:

- $$\begin{aligned} R_m &= R_1 // R_2 // R_3 // R_4 \\ &= (100 // 100 // 100 // 100)\ \Omega \\ &= \boxed{25\ \Omega} \end{aligned}$$
- $$V_m = R_m I_m = R_m \left(\frac{E_1}{R_1} + \frac{E_2}{R_2} - \frac{E_3}{R_3} - \frac{E_4}{R_4} \right)$$

E_3 and V_4 have different polarities with V_m.

$$\begin{aligned} &= (25\ \Omega) \left(\frac{40\text{ V}}{100\ \Omega} + \frac{30\text{ V}}{100\ \Omega} - \frac{20\text{ V}}{100\ \Omega} - \frac{10\text{ V}}{100\ \Omega} \right) \\ &= \boxed{10\text{ V}} \end{aligned}$$

- $$\begin{aligned} V_L &= V_m \frac{R_L}{R_L + R_m} \\ &= (10\text{ V}) \frac{30\ \Omega}{(30 + 25)\ \Omega} \\ &\approx \boxed{5.455\text{ V}} \end{aligned}$$

$V_2 = V_T \dfrac{R_2}{R_1 + R_2}$

Millman's theorem	When several voltage sources or branches consisting of a resistor are in parallel, they can be replaced by a single voltage source. $V_m = R_m I_m = R_m \left(\frac{E_1}{\mathrm{R}_1} + \frac{E_2}{R_2} + \ldots + \frac{E_n}{R_n} \right)$, $R_m = R_1 // R_2 // \ldots // R_n$ V_m will be positive if E_n and V_m have the same polarities, otherwise it will be negative.

5.4.3 Substitution theorem

Substitution theorem:

Substitution theorem	A branch in a network that consists of any component can be replaced by an equivalent branch that consists of any combination of components, as long as the currents and voltages on that branch do not change after the substitution.

Illustration of the substitution theorem

- The substitution theorem can be illustrated in the circuits of Figures 5.15 and 5.16.
- The current and voltage of branch a–b in the circuit of Figure 5.15 can be determined as follows:
  - The voltage across branch a–b:

$$V_2 = E\frac{R_2}{R_1 + R_2} = 20\ \text{V}\frac{6\ \text{k}\Omega}{(2+6)\ \text{k}\Omega} = 15\ \text{V}$$

$V_2 = V_\text{T}\dfrac{R_2}{R_1 + R_2}$

  - The current in the branch a–b: $I = \dfrac{E}{R_1 + R_2} = \dfrac{20\ \text{V}}{(2+6)\ \text{k}\Omega} = 2.5\ \text{mA}$

$I = \dfrac{E}{R}$

- According to the definition of the substitution theorem, any branch in the circuit of Figure 5.16 can replace the a–b branch in the circuit of Figure 5.15, since their voltages and currents are the same as the voltages and currents in the branch a–b in the circuit of Figure 5.15.

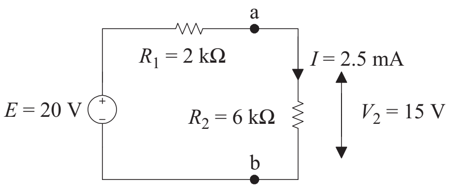

a
$R_1 = 2\ \text{k}\Omega$
$I = 2.5$ mA
$E = 20$ V
+
−
$R_2 = 6\ \text{k}\Omega$
$V_2 = 15$ V
b

Figure 5.15 Circuit 1 of the substitution theorem

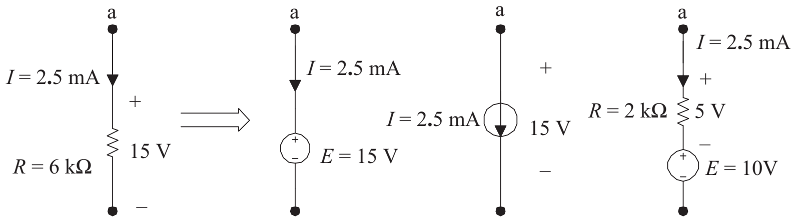

a
$I = 2.5$ mA
+
$R = 6\ \text{k}\Omega$
15 V
−
a
$I = 2.5$ mA
+
−
$E = 15$ V
a
$I = 2.5$ mA
+
15 V
−
a
$I = 2.5$ mA
+
$R = 2\ \text{k}\Omega$
5 V
−
+
−
$E = 10$V

Figure 5.16 Circuit 2 of the substitution theorem

5.4.4 Substitution theorem example

Example 5.9: Use a current source with a 30 Ω internal resistor to replace the a–b branch in the circuit of Figure 5.17(a).

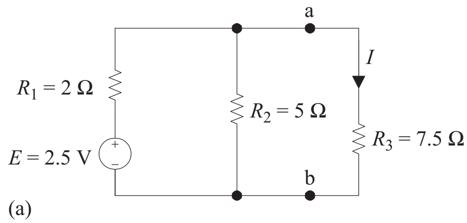

a
I
$R_1 = 2\ \Omega$
$R_2 = 5\ \Omega$
$R_3 = 7.5\ \Omega$
$E = 2.5$ V
+
–
b
(a)

Figure 5.17(a) Circuit for Example 5.9

Solution:

- Figure 5.17(b) shows the resultant circuit after the current source with a 30 Ω internal resistor replaced the a–b branch in the circuit of Figure 5.17(a).

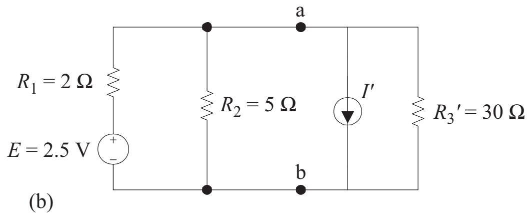

a
$R_1 = 2\ \Omega$
$R_2 = 5\ \Omega$
I'
$R_3' = 30\ \Omega$
$E = 2.5$ V
+
–
b
(b)

Figure 5.17(b)

- Determine the voltage and current in the a–b branch of the circuit in Figure 5.17(a).

$$V_{ab} = E\frac{R_2//R_3}{R_1 + R_2//R_3} = 2.5\text{ V}\frac{\dfrac{5\times 7.5}{5+7.5}\ \Omega}{\left(2+\dfrac{5\times 7.5}{5+7.5}\right)\Omega} = 1.5\text{ V} \qquad V_2 = V_T\frac{R_2}{R_1+R_2}$$

$$I = \frac{V_{ab}}{R_3} = \frac{1.5\text{ V}}{7.5\ \Omega} = 0.2\text{ A} = 200\text{ mA} \qquad I = \frac{V}{R}$$

- Determine the currents in the substituted branch and the current source branch using the circuit in Figure 5.17(c).

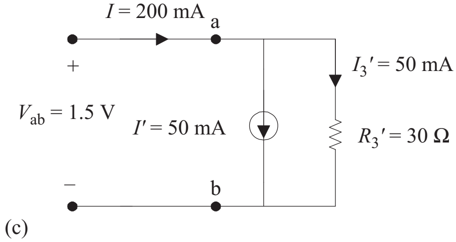


Figure 5.17(c)

$$I_3' = \frac{V_{ab}}{R_3'} = \frac{1.5\ \text{V}}{30\ \Omega} = 0.05\ \text{A} = 50\ \text{mA} \qquad I = \frac{V}{R}$$

- To maintain the terminal voltage $V_{ab} = 1.5$V in the original branch, the current I_3' in the R_3' branch should be 50 mA. Using KCL, we can get the current I' in the current source branch:

$$I' = I - I_3' = 200\ \text{mA} - 50\ \text{mA} = \boxed{150\ \text{mA}} \qquad \Sigma I = 0$$

Summary

Basic concepts

- Network: a complicated circuit.
- Linear circuit: a circuit that includes the linear components (such as resistors).
- The linear two-terminal network with the sources: It is a linear complex circuit that has power sources and two terminals.
- The pre-requirement of applying some of the network theorems is that the analyzed network must be a linear circuit.

Superposition theorem

- Theorem: The unknown voltages or currents in any linear network are the sum of the voltages or currents of the individual contributions from each single power supply, by setting the other inactive sources to zero.
- Steps to apply the superposition theorem
  1. Turn off all power sources except one, i.e., replace the voltage source with the short circuit, and replace the current source with an open circuit. Redraw the original circuit with a single source.

2. Analyze and calculate this circuit by using the single source series—parallel analysis method, and repeat steps 1 and 2 for the other power sources in the circuit.
3. Determine the total contribution by calculating the algebraic sum of all contributions due to single sources.

   The result should be positive when the reference polarity of the unknown in the single-source circuit is the same as the reference polarity of the unknown in the original circuit; otherwise it should be negative.

Thevenin's and Norton's theorems

- Theorems: Any linear two-terminal network with power supplies can be replaced by a simple equivalent circuit that has a single power source and a single resistor.
  - Thevenin's theorem: The equivalent circuit is a voltage source (with an equivalent resistance R_{TH} in series with an equivalent voltage source V_{TH}).
  - Norton's theorem: The equivalent circuit is a current source (with an equivalent resistance R_N in parallel with an equivalent current source I_N).
- Steps to apply Thevenin's and Norton's theorems

1. Open and remove the load branch (or any unknown current or voltage branch) in the network, and mark the letters a and b on the two terminals.
2. Determine the equivalent resistance R_{TH} or R_N: It equals the equivalent resistance, looking at it from the a and b terminals when all sources are turned off or equal to zero in the network. That is $\boxed{R_{TH} = R_N = R_{ab}}$
   - A voltage source should be replaced by a short circuit.
   - A current source should be replaced by an open circuit.
3.
   - Determine Thevenin's equivalent voltage V_{TH}. It equals the open-circuit voltage from the original linear two-terminal network of a and b, that is, $\boxed{V_{TH} = V_{ab}}$
   - Determine Norton's equivalent current I_N. It equals the short-circuit current from the original linear two-terminal network of a and b, that is, $\boxed{I_N = I_{sc}}$
4. Plot Thevenin's or Norton's equivalent circuit, and connect the load branch (or unknown current or voltage branch) to a and b terminals of the equivalent circuit. Then the load (or unknown) voltage or current can be determined.

Maximum power transfer theorem

When the load resistance is equal to the internal resistance of the source ($R_L = R_S$); or when the load resistance is equal to the Thevenin's/Norton's equivalent resistance of the network ($R_L = R_{TH} = R_N$), maximum power will be transferred to the load.

Millman's theorem
When several voltage sources or branches consisting of a resistor are in parallel, they can be replaced by a branch with a voltage source.

$$\boxed{V_m = R_m I_m = R_m\left(\frac{E_1}{R_1} + \frac{E_2}{R_2} + \ldots + \frac{E_n}{R_n}\right)}$$

$$\boxed{R_m = R_1 // R_2 // \ldots // R_n}$$

Substitution theorem
A branch in a network that consists of any component can be replaced by an equivalent branch that consists of any combination of components, as long as the currents and voltages on that branch do not change after the substitution.

Practice problems

5.1

1. Calculate the branch currents I_{R_1} and I_{R_3} in the circuit of Figure 5.18 using the superposition theorem.

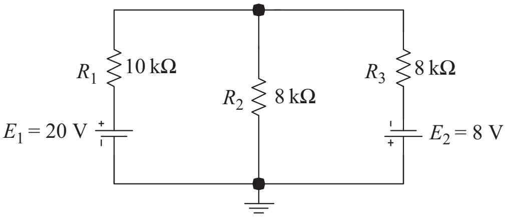


Figure 5.18

2. Calculate the branch currents I_{R_2} in the circuit of Figure 5.19 using the superposition theorem.

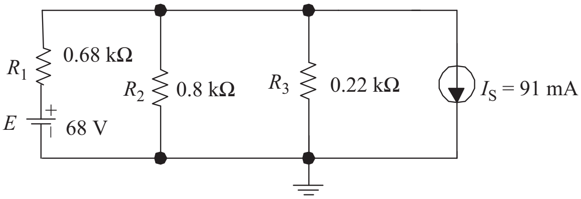
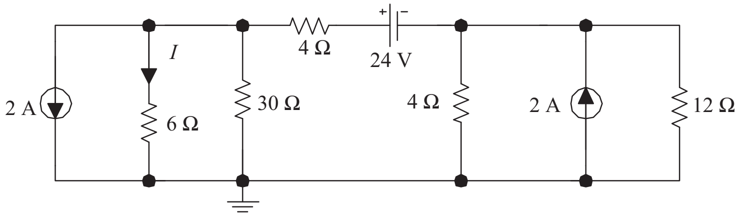


Figure 5.19

3. Calculate the branch current I flowing though the 6 Ω resistor in the circuit of Figure 5.20 using the superposition theorem.

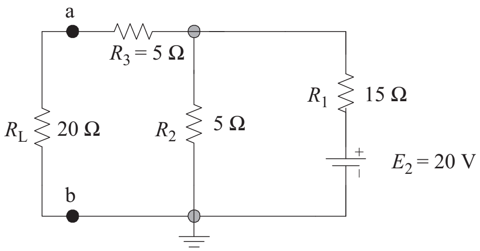


+
−
4 Ω
24 V
I
2 A
6 Ω
30 Ω
4 Ω
2 A
12 Ω

Figure 5.20

5.2

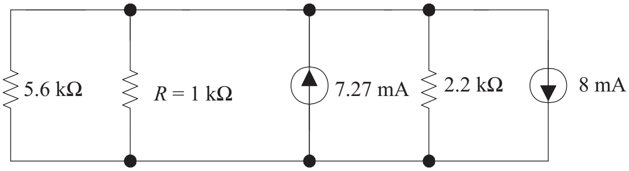
4. Determine Thevenin's and Norton's equivalent circuits for the terminals a and b of the circuit in Figure 5.21 (the viewpoints a-b from the terminals of the load R_L).



a
$R_3 = 5$ Ω
R_1
15 Ω
R_L
20 Ω
R_2
5 Ω
+
$E_2 = 20$ V
b

Figure 5.21

5. Calculate the current flowing through the 1 kΩ resistor in the circuit of Figure 5.22 using Thevenin's and Norton's theorems, respectively.



5.6 kΩ
R = 1 kΩ
7.27 mA
2.2 kΩ
8 mA

Figure 5.22

6. Calculate the current flowing through the 5 kΩ resistor in the circuit of Figure 5.23.

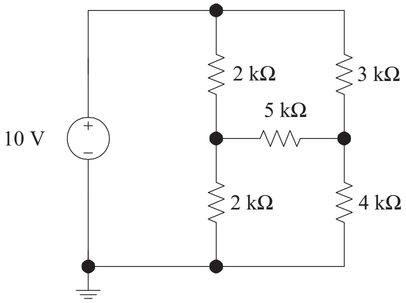

2 kΩ
3 kΩ
5 kΩ
10 V
+
−
2 kΩ
4 kΩ

Figure 5.23

5.3

7. Determine the load resistance R_L in the circuit of Figure 5.24(a) and (b) when the power dissipation on R_L is maximum.

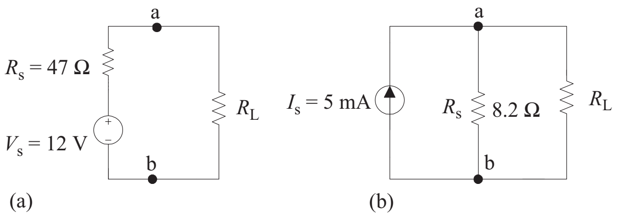

a
$R_s = 47\ \Omega$
R_L
$V_s = 12$ V
b
(a)
a
$I_s = 5$ mA
R_s
8.2 Ω
R_L
b
(b)

Figure 5.24

8. Determine the load resistance R_L in the circuit of Figure 5.25 when the power dissipation on R_L is at maximum. Then calculate the maximum power dissipated on R_L. (Hint: Determine Thevenin's equivalent circuit first; when $R_L = R_{TH}$, maximum power will be dissipated on R_L.)

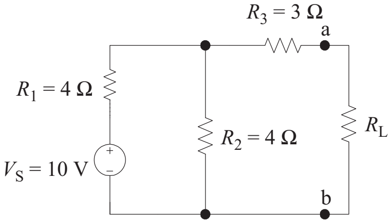

$R_3 = 3\ \Omega$
a
$R_1 = 4\ \Omega$
$R_2 = 4\ \Omega$
R_L
$V_S = 10$ V
b

Figure 5.25

5.4

9. Calculate the current flowing through the resistor R_L in the circuit of Figure 5.26 using Millman's theorem.

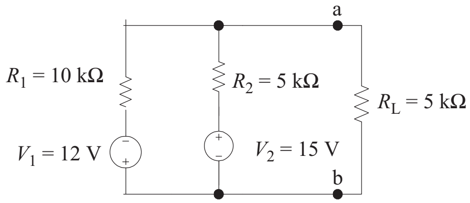

a
$R_1 = 10\ k\Omega$
$R_2 = 5\ k\Omega$
$R_L = 5\ k\Omega$
$V_1 = 12\ V$
$V_2 = 15\ V$
b

Figure 5.26

10. Calculate the current flowing through resistor R_L in the circuit of Figure 5.27 using Millman's theorem.

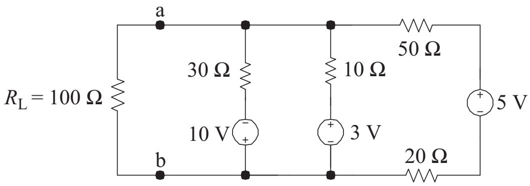

a
$R_L = 100\ \Omega$
30 Ω
10 Ω
50 Ω
5 V
10 V
3 V
20 Ω
b

Figure 5.27

11. Plot three different equivalent circuits to replace the a–b branch of the circuit in Figure 5.28 using the substitution theorem.

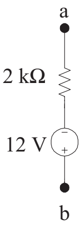


Figure 5.28

Chapter 6

Capacitors and inductors

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6.1 Capacitors

6.1.1 Three basic circuit components

Three basic circuit components

- There are three important fundamental circuit elements: the resistor, capacitor, and inductor.
- The resistor (R) has been discussed in circuit analysis in the previous chapters. The other two elements—the capacitor (C) and inductor (L) will be introduced in this chapter.
- The capacitor and inductor can store energy that has been absorbed from the power supply, and release it to the circuit.
  - A capacitor can store energy in the electric field.
  - An inductor can store energy in the magnetic field.
  - A resistor consumes or dissipates electric energy.
- A circuit containing only resistors has limited applications. Practical electric circuits usually combine the above three basic elements and possibility along with other devices.

Three basic circuit components	– Resistor (R) – Capacitor (C) – Inductor (L)

Introduction to capacitors

- A capacitor has applications in many areas of electrical and electronic circuits, and it extends from households to industry and the business world.
- For instance, it is used in flash lamps (for flash camera), power systems (power supply smoothing, surge protections), electronic engineering, communications, computers, etc.
- There are many different types of capacitors, but no matter how differently their shapes and sizes, they all have the same basic construction.

Capacitor (C)	An energy storage element that has two parallel conductive metal plates separated by an isolating material (the dielectric).

6.1.2 Capacitors

The construction of a capacitor

- A capacitor has two parallel conductive metal plates separated by an isolating material (the dielectric).
- The dielectric can be of insulating material such as paper, vacuum, air, glass, plastic film, oil, mica, and ceramics. The basic construction of a capacitor is shown in Figure 6.1.

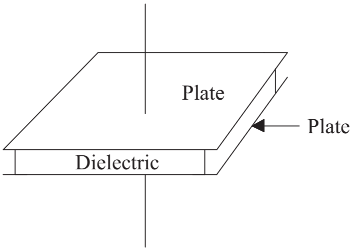


Figure 6.1 The basic construction of a capacitor

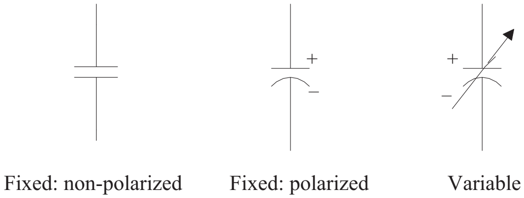


Figure 6.2 Symbols of capacitor

Capacitor schematic symbols

- A capacitor can be represented by a capacitor schematic symbol as its circuit model.
- Similar to resistors, there are two basic types of capacitors, variable and fixed, and their schematic symbols are shown in Figure 6.2.

Fixed and variable capacitors

- A variable capacitor is a capacitor that possesses a value that may be changed manually or automatically.
- A fixed capacitor is a capacitor that possesses a fixed value and cannot be adjusted.
  - For a fixed polarized capacitor, connect its positive lead (+) to the higher voltage point in the circuit, and negative lead (–) to the lower voltage point.
  - For a non-polarized capacitor, it does not matter which lead connects to where.

Electrolytic capacitors are usually polarized, and non-electrolytic capacitors are non-polarized. Electrolytic capacitors can have higher working voltages and store more charges than non-electrolytic capacitors.

6.1.3 Charging a capacitor

Initial condition ($V_C = 0$)

- A purely capacitive circuit with an uncharged capacitor ($V_C = 0$), a three-position switch, and a DC (direct current) voltage source (E) is shown in Figure 6.3(a).

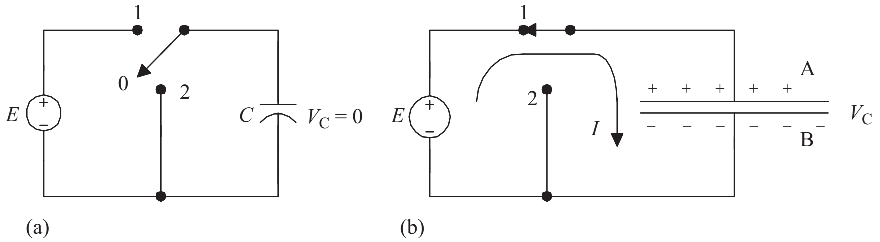


Figure 6.3 Charging a capacitor

- With the switch at position 0, the circuit is open, and the potential difference between the two metal plates of the capacitor is zero ($V_C = 0$).

Two plates of the capacitor have the same size and are made by the same conducting material, so they should have the same number of charges at the initial condition.

Charging a capacitor

- Once the three-position switch is turned on to position 1 as shown in Figure 6.3(b), the DC voltage source is connected to the capacitor, current I will flow in the circuit.

  From the rule "opposites attract and likes repel," we know that the positive pole of the voltage source will attract electrons from the positive plate of the capacitor, and the negative pole of the voltage source will attract positive charges from the negative plate of the capacitor; this causes current I to flow in the circuit.
- Plate A loses electrons and shows positive; plate B loses positive charges and thus shows negative. Thus, the electric field is built up between the two metal plates of the capacitor, and the potential difference (V_C) appears on the capacitor with positive (+) on plate A and negative (–) on plate B, as shown in Figure 6.3(b).
- Once voltage across the capacitor V_C has reached the source voltage E, i.e., $V_C = E$, there is no more potential difference between the source and capacitor, the charging current ceases to flow ($I = 0$), and the process of charging the capacitor is completed. This is the process of charging a capacitor.

Charging a capacitor	– Once the three-position switch is turned on to position 1, current I will flow in the circuit. – Plate A loses electrons and shows positive; plate B loses positive charges and thus shows negative. V_C appears on the capacitor. – When $V_C = E$, $I = 0$, the process of charging the capacitor is completed.

6.1.4 How does a capacitor store energy?

Energy storage element

- When the switch is turned off to position 0 in the circuit shown in Figure 6.3(a), the capacitor and power supply will be disconnected.
- If the voltage across the capacitor V_C is measured at this time using a multimeter (voltmeter function), V_C should still be the same with the source voltage ($V_C = E$) even without a power supply connected to it.
- This is why a capacitor is called an energy storage element, as it can store charges absorbed from the power supply and store electric energy obtained from charging.
- Once a capacitor has transferred some charges through charging, an electric field is built up between the two plates of the capacitor, and it can maintain the potential difference across it.

Capacitor will keep its charged voltage for a long time

- The isolating material (dielectric) between the two metal plates isolates the charges between the two plates. Charges will not be able to cross the insulating material from one plate to another.
- So, the energy storage element capacitor will keep its charged voltage V_C for a long time (duration will depend on the quality and type of the capacitor).
- Since the insulating material will not be perfect and a small leakage current may flow through the dielectric, this may eventually slowly dissipate the charges.

Capacitor stores energy	– When the switch is turned off, $V_C = E$ (after charging) even without a power supply connected to it. – Charges will not be able to cross the insulating material from one plate to another. – Capacitor will keep its charged voltage for a long time. Insulating material will not be perfect and a small leakage current may flow through the dielectric.

6.1.5 Discharging a capacitor

Discharging a capacitor

- When the switch is closed to position 2 as shown in the circuit of Figure 6.4, the capacitor and wires in the circuit form a closed path.
- At this time, the capacitor is equivalent to a voltage source, as voltage across the capacitor V_C will cause the current to flow in the circuit.
- Since there is no resistor in this circuit, it is a short circuit, and a high current causes the capacitor to release its charges or stored energy in a short time. This is known as discharging a capacitor.

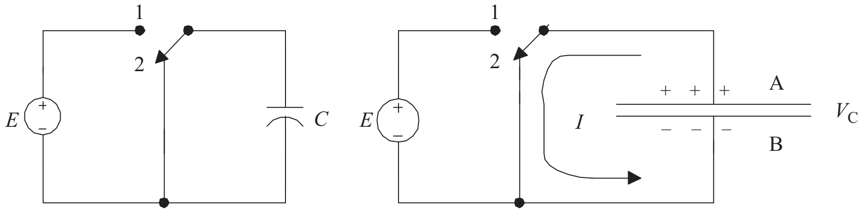


Figure 6.4 Discharging a capacitor

- After the capacitor has released all its stored energy, the voltage across the capacitor will be zero ($V_C = 0$), the current in the circuit ceases to flow ($I = 0$) and the discharge process is completed.

Release energy

- The capacitor cannot release energy that is more than it has absorbed and stored, therefore it is a passive component. A passive component is a component that absorbs (but does not produce) energy.
- The concept of a capacitor may be analogous to a small reservoir. It acts as a reservoir that stores and releases water. The process of charging a capacitor from the power supply is similar to a reservoir storing water. The process of discharging a capacitor is similar to a reservoir releasing water.
- There is an important characteristic that implies in the charge and discharge of a capacitor. That is, the voltage on the capacitor cannot change instantly; it will always take time, i.e., gradually increase (charging) or decrease (discharge).

Charging / discharging a capacitor	A capacitor is an electric element that can store and release charges that it absorbed from the power supply. – Charging: the process of storing energy. – Discharging: the process of releasing energy.

6.1.6 Capacitance

The relationship between *Q* and *V*

- Once the source voltage is applied to two leads of a capacitor, the capacitor starts to store energy or charges. The charges (Q) that are stored are proportional to the voltage (V) across it. This can be expressed by the following formula:

$$\boxed{Q = CV} \quad \text{or} \quad \boxed{C = \frac{Q}{V}}$$

- The higher the voltage, the more charges a capacitor can store.

This is analogous to a pump pumping water to a reservoir. The higher the pressure, the more water will be pumped into the reservoir.

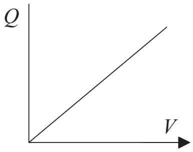


Figure 6.5 Q–V characteristic of a capacitor

Q–V **characteristic of a capacitor**

- The voltage and charge (*V–Q*) characteristic of a capacitor is shown in Figure 6.5, demonstrating that the capacitor voltage is proportional to the amount of charges a capacitor can store.

Capacitance

- *C* is the capacitance, which is the value of the capacitor and describes the amount of charges stored in the capacitor.
- Capacitor is a component and capacitance is the value of a capacitor.
  Just as a resistor is a component and resistance is the value of a resistor,
- Capacitor is symbolized by C while capacitance is *C*.
  Resistor is symbolized by R while resistance is *R*.

Capacitance (*C*)	*C*, the value of the capacitor, is directly proportional to its stored charges, and inversely proportional to the voltage (*V*) across it. $\text{Capacitance} = \dfrac{\text{Charge}}{\text{Voltage}} \qquad C = \dfrac{Q}{V}$

6.1.7 Calculating capacitance

Calculating capacitance

- $\boxed{\text{Capacitance} = \dfrac{\text{Charge}}{\text{Voltage}}}$ or $\boxed{C = \dfrac{Q}{V}}$
- Units:

$$\boxed{\underset{\text{Farad (F)}}{C} = \frac{\overset{\text{Coulomb (C)}}{Q}}{\underset{\text{Volt (V)}}{V}}}$$

- A capacitor can store 1 C charge when 1 V of voltage is applied to it.

That is, $1\ \text{F} = \dfrac{1\ \text{C}}{1\ \text{V}}$

Units of capacitance

- Microfarad (μF) or picofarad (pF) is a more commonly used unit for capacitors. Farad is a very large unit of measurement for most practical capacitors.
- Recall: $1\ \mu\text{F} = 10^{-6}\ \text{F}$ and
  $1\ \text{pF} = 10^{-12}\ \text{F}$

Note: μ is a Greek letter called "mu" (see "Appendix A" for a list of Greek letters).

Example 6.1: If a 50 μC charge is stored on the plates of a capacitor, determine the voltage across the capacitor if the capacitance of the capacitor is 1,000 pF.

Solution:

$$Q = 50\ \mu\text{C}, \qquad C = 1{,}000\ \text{pF}, \qquad V = ?$$

$$V = \frac{Q}{C} = \frac{50\ \mu\text{C}}{1{,}000\ \text{pF}} = \frac{50 \times 10^{-6}\ \text{C}}{1{,}000 \times 10^{-12}\ \text{F}} = 0.05 \times 10^{6}\ \text{V}$$

$$= 50 \times 10^{3}\ \text{V} = \boxed{50\ \text{kV}}$$

6.1.8 *Factors affecting capacitance*

Three factors affecting capacitance

There are three basic factors affecting the capacitance of a capacitor, and they are determined by the construction of a capacitor as shown below:

- The area of plates (A): A is directly proportional to the charge Q; the larger the plate area, the more electric charges that can be stored.
- The distance between the two plates (d): The shorter the distance between two plates, the stronger the produced electric field that will increase the ability to store charges. Therefore, the distance (d) between the two plates is inversely proportional to the capacitance (C).
- The dielectric constant (k): Different insulating materials (dielectrics) will have a different impact on the capacitance. The dielectric constant (k) is directly proportional to the capacitance (C).

The factors affecting the capacitance of a capacitor are illustrated in Figure 6.6. **Dielectric constants for some commonly used capacitor materials** are listed in Table 6.1.

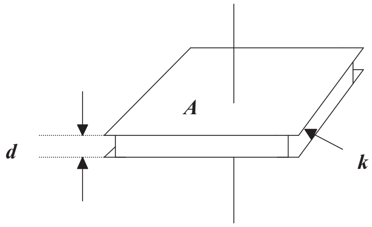


Figure 6.6 Factors affecting capacitance

Table 6.1 Dielectric constants of some insulating materials

Material	**Dielectric constant**
Vacuum	1
Air	1.0006
Paper (dry)	2.5
Glass (photographic)	7.5
Mica	5
Oil	4
Polystyrene	2.6
Teflon	2.1

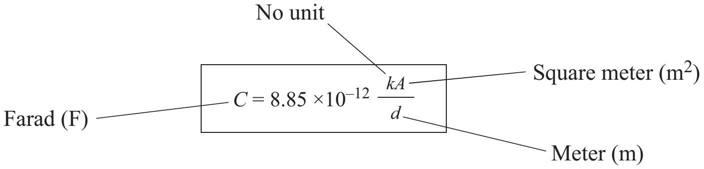
Factors affecting capacitance	– The area of plates (A) – The distance between the two plates (d) – The dielectric constant (k) $C = 8.85 \times 10^{-12} \dfrac{kA}{d}$

Calculating capacitance

- $$\boxed{\text{Capacitance} = 8.85 \times 10^{-12} \frac{(\text{Dielectric constant}) \times (\text{Plate area})}{\text{Distance between the plates}}}$$

  or $$\boxed{C = 8.85 \times 10^{-12} \frac{kA}{d}}$$

- Units:



Example 6.2: Determine the capacitance if the area of plates for a capacitor is 0.004 m^2, the distance between the plates is 0.006 m, and the dielectric for this capacitor is mica.

Solution:

$$A = 0.004\ \text{m}^2, \qquad d = 0.006\ \text{m}, \qquad \text{and} \quad k = 5 \qquad (\text{Table 6.1})$$

$$C = 8.85 \times 10^{-12} \frac{kA}{d}$$

$$= 8.85 \times 10^{-12} \frac{5 \times 0.004\ \text{m}^2}{0.006\ \text{m}}$$

$$= \boxed{29.5\ \text{pF}}$$

6.1.9 Leakage current and breakdown voltage

Leakage current

- The dielectric between two plates of the capacitor is insulating material, and practically no insulating material is perfect (i.e., 100% of the insulation).
- Once voltage is applied across the capacitor, there may be a very small current through the dielectric, and this is called the leakage current in the capacitor.
- Although the leakage current is very small, it is always there. That is why the charges or the energy stored on the capacitor plates will eventually leak off.
- The leakage current is so small that it can be ignored for the application. (Electrolytic capacitors have higher leakage current).

Leakage current	A very small current through the dielectric of a capacitor.

Breakdown voltage

- A capacitor charging acts as a pump pumping water into a reservoir, or a water tank.
  - The higher the pressure, the more water will be pumped into the tank.
  - If the tank is full and still continues to increase the pressure, the tank may break down or become damaged by such high pressure.
- This is similar to a capacitor. If the voltage across a capacitor is too high and exceeds the capacitor's working or breakdown voltage, the capacitor's dielectric will break down, causing current to flow through it.
- As a result, this may explode or permanently damage the capacitor.
- When using a capacitor, pay attention to the maximum working voltage, which is the maximum voltage a capacitor can have. The applied voltage of the capacitor can never exceed the capacitor's breakdown voltage.

Breakdown voltage	The voltage that causes a capacitor's dielectric to become electrically conductive. It may explode or permanently damage the capacitor.

6.1.10 Relationship between the v and i of a capacitor

Instantaneous quantity

- A quantity that varies with time is called instantaneous quantity (such as a capacitor that takes time to charge/discharge), which is the quantity at a specific time.
- Usually the lowercase letters symbolize instantaneous quantities, and the uppercase letters symbolize the constants or average quantities.
- The equation $Q = CV$ in terms of instantaneous quantity is $q = Cv$.

Note: if you have not learned calculus, just keep in mind that $i = C\frac{\Delta v}{\Delta t}$ or $i_c = C\frac{\mathrm{d}v_c}{\mathrm{d}t}$ is Ohm's law for a capacitor, and skip the following mathematic derivation process, where Δv and Δt or $\mathrm{d}v$ and $\mathrm{d}t$ are very small changes in voltage and time.

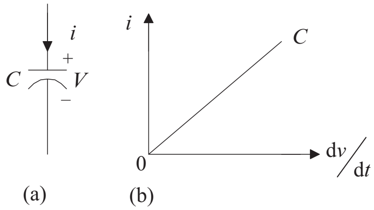


Figure 6.7 Relationship between v and i of a capacitor

- Differentiating the equation $q = Cv$ yields: $\frac{dq}{dt} = C\frac{dv}{dt}$
- Recall that current is the rate of movement of charges, and has the $i = \frac{dq}{dt}$ notation in calculus.
- Substitute $i = \frac{dq}{dt}$ into the equation of $\frac{dq}{dt} = C\frac{dv}{dt}$ yields:

$$\boxed{i = C\frac{dv}{dt}} \quad \text{or} \quad \boxed{i = C\frac{\Delta v}{\Delta t}}$$

Relationship between the current and voltage

- The relationship of voltage and current for a capacitor shows that when the applied voltage at two leads of the capacitor changes, the charges (q) stored on the plates of the capacitor will also change. This will cause current to flow in the capacitor circuit.
- Current and the rate of change of voltage are directly proportional to each other.
- The reference polarities of capacitor voltage and current should be mutually related. That is, the reference polarities of voltage and current of a capacitor should be consistent, as shown in Figure 6.7(a).
- The relationship between voltage and current of a capacitor can be expressed by Figure 6.7(b).

6.1.11 Ohm's law for a capacitor

Ohm's law for a capacitor

- The relationship between the current and voltage for a resistor is Ohm's law for a resistor.
- The relationship between the current and voltage for a capacitor is Ohm's law for a capacitor.
- $\boxed{i = C\frac{dv}{dt}}$ or $\boxed{i = C\frac{\Delta v}{\Delta t}}$ is Ohm's law for a capacitor.

Ohm's law for a capacitor	The current of a capacitor i_c is directly proportional to the ratio of capacitor voltage $\frac{dv_C}{dt}$ $\left(\text{or } \frac{\Delta v_C}{\Delta t}\right)$ and capacitance C. $i_c = C\frac{dv_C}{dt}$ or $i = C\frac{\Delta v_C}{\Delta t}$

where dv_c and dt or Δv_c, and Δt are very small changes in voltage and time.

DC blocking

- The relationship of voltage and current in a capacitive circuit shows that:
  - The faster the voltage changes with time, the greater the amount of capacitive current flows through the circuit.
  - The slower the voltage changes with time, the smaller the amount of current.
  - If voltage does not change with time, the current will be zero. Zero current means that the capacitor acts like an open circuit for DC voltage at this time.
- Voltage that does not change with time is DC (direct current) voltage, meaning that current is zero when DC voltage is applied to a capacitor. Therefore, the capacitor may play an important role for blocking the DC current. (This is a very important characteristic of a capacitor.)

DC blocking	Current through a capacitor is zero when DC voltage applied to it (open-circuit equivalent). A capacitor can block DC current.

Note:

- Although there is a DC voltage source applied to the capacitive circuit in Figures 6.3 and 6.4, the capacitor charging/discharging happened at the moment when the switch turned to different locations, i.e., when the voltage across the capacitor changes within a moment.
- When the capacitor charging/discharging has finished, the capacitor is equivalent to an open circuit for that circuit.

6.1.12 Energy stored by a capacitor

Energy stored by a capacitor

- A capacitor is an energy storage element. It can store energy that it absorbed from charging and maintain voltage across it.
- Energy stored by a capacitor in the electric field can be derived as follows.
  - The instantaneous electric power of a capacitor is given by $p = vi$. Substituting this into the capacitor's current $i = C\frac{dv}{dt}$ yields $p = Cv\frac{dv}{dt}$

– Since the relationship between power and work is $p = \frac{w}{t}$ (energy is the capacity to do work), and, instantaneous power for this expression is $p = \frac{\mathrm{d}w}{\mathrm{d}t}$, substituting it into $p = Cv\frac{\mathrm{d}v}{\mathrm{d}t}$ yields $\frac{\mathrm{d}w}{\mathrm{d}t} = Cv\frac{\mathrm{d}v}{\mathrm{d}t}$

– Integrating the above expression: $\int_0^t \frac{\mathrm{d}w}{\mathrm{d}t}\mathrm{d}t = C\int_0^v v\frac{\mathrm{d}v}{\mathrm{d}t}\mathrm{d}t$ gives

$$\boxed{W = \frac{1}{2}Cv^2}$$

Note: If you have not learned calculus, just keep in mind that $W = \frac{1}{2}Cv^2$, and skip the above mathematic derivation process.

Energy stored by a capacitor	Capacitor energy = ½(Capacitance)(Voltage)^2 $W = \frac{1}{2}Cv^2$

- The expression for energy stored by a capacitor shows that the capacitor's energy depends on the values of the capacitor (C) and voltage across the capacitor (v).

Calculating capacitor energy

- $\boxed{\text{Capacitor energy} = \frac{1}{2}(\text{Capacitance})(\text{Voltage})^2}$ or $\boxed{W = \frac{1}{2}Cv^2}$
- Units: $\boxed{\underset{\text{Joule (J)}}{W} = \frac{1}{2}\underset{\text{Farad (F)}}{C}\overset{\text{Volt (V)}}{v^2}}$

Example 6.3: A 15 V voltage is applied to a 2.2 μF capacitor. Determine the energy this capacitor has stored.

Solution: $W = \frac{1}{2}Cv^2 = \frac{1}{2}(2.2\ \mu\text{F})(15\ \text{V})^2 = \boxed{247.5\ \mu\text{J}}$

6.2 Capacitors in series and parallel

6.2.1 Total or equivalent capacitance

Total or equivalent capacitance C_{eq}

- Same as resistors, capacitors may also be connected in series or parallel to obtain a suitable resultant value that may be either higher or lower than a single capacitor value.
- The total or equivalent capacitance C_{eq} will decrease for a series capacitive circuit and it will increase for a parallel capacitive circuit.
- The total or equivalent capacitance has the opposite form with the total or equivalent resistance R_{eq}.

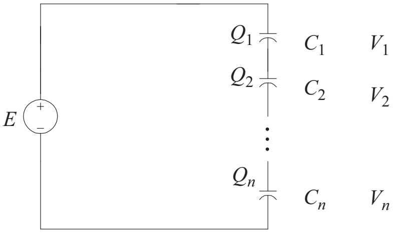


Figure 6.8 n capacitors in series

Derive the series equivalent (total) capacitance C_{eq}

- A circuit of n capacitors is connected in series as shown in Figure 6.8:
- Applying KVL to the above circuit gives $E = V_1 + V_2 + \cdots + V_n$ $\sum E = \sum V$
- Since $V = \frac{Q}{C}$, substituting it into the above equation yields:

  $$\frac{Q_{eq}}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \cdots + \frac{Q_n}{C_n}$$
- Where $E = \frac{Q_{eq}}{C_{eq}}$, Q_{eq} is the equivalent (or total) charges and C_{eq} is the equivalent (or total) capacitance for a series capacitive circuit, respectively.
- Since only one current flows in a series circuit, each capacitor will store the same amount of charges, i.e., $Q_{eq} = Q_1 = Q_2 = \cdots = Q_n = Q$
- Therefore $\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \cdots + \frac{Q}{C_n}$
- Dividing by Q on both sides of the above equation gives

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n} \qquad \text{or} \qquad \boxed{C_n = \frac{1}{\frac{1}{C_1} + \frac{1}{C_1} + \cdots \frac{1}{C_n}}}$$

- This is the equation for calculating the series equivalent (total) capacitance. This formula has the same form with the formula for calculating equivalent parallel resistance $\left(\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}\right)$.
- When there are two capacitors in series, it also has the same form with the formula for calculating two resistors in parallel, i.e., $\boxed{C_{eq} = \frac{C_1 C_2}{C_1 + C_2}}$ $\left(R_{eq} = \frac{R_1 R_2}{R_1 + R_2}\right)$.

6.2.2 Capacitors in series

Equivalent (total) series capacitance

Equivalent (total) series capacitance C_{eq}	– n capacitors in series: $C_{\text{eq}} = \dfrac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_1} + \cdots \dfrac{1}{C_n}}$ – Two capacitors in series: $C_{\text{eq}} = \dfrac{C_1 C_2}{C_1 + C_2}$

Example 6.4: Determine the charges Q stored by each capacitor in the circuit of Figure 6.9.

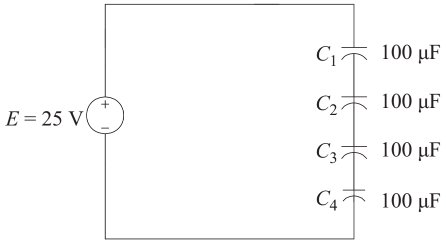


Figure 6.9 Figure for Example 6.4

Solution: Since $Q = CV$, or $Q = C_{\text{eq}} E$

– Solve for C_{eq} first,

$$C_{\text{eq}} = \frac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} + \dfrac{1}{C_4}} = \frac{1}{\dfrac{1}{100\,\mu\text{F}} + \dfrac{1}{100\,\mu\text{F}} + \dfrac{1}{100\,\mu\text{F}} + \dfrac{1}{100\,\mu\text{F}}} = 25\,\mu\text{F}$$

– Therefore $Q = C_{\text{eq}}E = (25\,\mu\text{F})(25\text{ V}) = \boxed{625\,\mu\text{C}}$

Characteristics of the series equivalent capacitance

- Example 6.4 shows that when capacitors are connected in series, the total or equivalent capacitance C_{eq} (25 μF) is less than any one of the individual capacitances (100 μF).
- The physical characteristic of the series equivalent capacitance is that the single series equivalent capacitance C_{eq} has the total dielectric (or total distance between the plates) of all the individual capacitors.

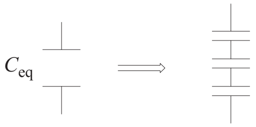


Figure 6.10 The physical characteristic of series C_{eq}

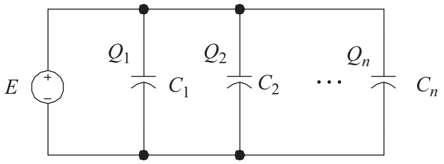


Figure 6.11 n capacitors in parallel

- The formula for factors affecting the capacitance $\left(C = 8.85 \times 10^{-12}\,\frac{kA}{d}\right)$ shows that if the distance between the plates of a capacitor (d) increases, the capacitance (C) will decrease. This is shown in Figure 6.10.

6.2.3 Capacitors in parallel

Capacitors in parallel

- A circuit of n capacitors connected in parallel is shown in Figure 6.11.
- Equivalent (total) parallel capacitance C_{eq}

Equivalent (total) parallel capacitance C_{eq}	$C_{eq} = C_1 + C_2 + \cdots + C_n$

Derive the equivalent (total) parallel capacitance C_{eq}

- The charge stored on the individual capacitor in this circuit is

$$Q_1 = C_1 V, \qquad Q_2 = C_2 V, \ldots, \qquad Q_n = C_n V \qquad (\text{where } V = E)$$

- The total charge Q_{eq} in this circuit should be the sum of all stored charges on the individual capacitor, i.e.,

$$Q_{eq} = Q_1 + Q_2 + \cdots + Q_n$$

therefore, $C_{eq}V = C_1 V + C_2 V + \cdots + C_n V$

- Dividing both sides by V on the above equation yields the equation for calculating the parallel equivalent (total) capacitance:

$$\boxed{C_{eq} = C_1 + C_2 + \cdots + C_n}$$

As you may have noticed, this equation has the same form with the equation for calculating series resistances ($R_{eq} = R_1 + R_2 + \cdots + R_n$).

Note: Equations for calculating capacitance are exactly opposite to the equations for calculating resistance.

- Capacitors in series result in parallel form as resistances. $R_{eq} = \dfrac{R_1 R_2}{R_1 + R_2}$
- Capacitors in parallel result in series form as resistances.
$R_{eq} = R_1 + R_2 + \cdots + R_n$

Example 6.5: Determine the total charge in all the capacitors in the circuit of Figure 6.12.

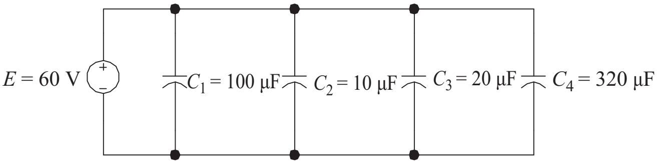


Figure 6.12 Figure for Example 6.5

Solution:

$$\begin{aligned} \text{Since } Q &= CV, \qquad \text{i.e.,} \qquad Q_{eq} = C_{eq}\,E \\ \text{and } C_{eq} &= C_1 + C_2 + \cdots + C_n \\ &= 100\,\mu\text{F} + 10\,\mu\text{F} + 20\,\mu\text{F} + 320\,\mu\text{F} = 450\,\mu\text{F} \\ \text{Therefore, } Q_{eq} &= C_{eq}E = (450\,\mu\text{F})(60\,\text{V}) = \boxed{27{,}000\,\mu\text{C}} \end{aligned}$$

From Example 6.5, we can see that when capacitors are connected in parallel, the total or equivalent capacitance C_{eq} (450 µF) is greater than any one of the individual capacitances ($C_1 = 100$ µF, $C_2 = 10$ µF, $C_3 = 20$ µF, and $C_4 = 320$ µF).

6.2.4 Physical properties of parallel C_{eq}

The physical characteristic of parallel C_{eq}

- The physical characteristic of the equation for calculating the parallel equivalent capacitance is that a single parallel equivalent capacitor C_{eq} has the total area of plates of the individual capacitors.
- If the area of plates (A) of a capacitor increases, the capacitance will increase $\left(C = 8.85 \times 10^{-12} \dfrac{kA}{d}\right)$. This is shown in Figure 6.13.

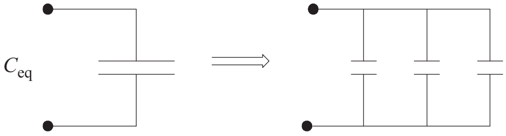


Figure 6.13 The physical characteristic of parallel C_{eq}

6.2.5 Capacitors in series–parallel

Series–parallel capacitor circuits

- Similar to resistors, capacitors may also be connected in various combinations.
- When serial and parallel capacitors are combined together, series—parallel capacitor circuits result and an example is shown in the following.

Example 6.6: Determine the equivalent capacitance through two terminals a and b in the circuit of Figure 6.14.

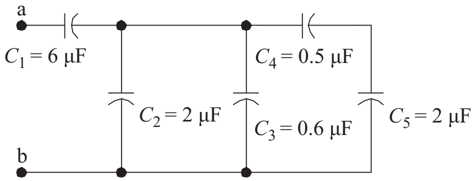


Figure 6.14 Figure for Example 6.6

Solution:

$$C_{4,5} = \frac{C_4 C_5}{C_4 + C_5} = \frac{(0.5\ \mu F)(2\ \mu F)}{0.5\ \mu F + 2\ \mu F} = 0.4\ \mu F \qquad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{2,3,4,5} = C_2 + C_3 + C_{4,5} = 2\ \mu F + 0.6\ \mu F + 0.4\ \mu F = 3\ \mu F \qquad C_{eq} = C_1 + C_2 + \cdots + C_n$$

$$C_{eq} = \frac{C_1 C_{2,3,4,5}}{C_1 + C_{2,3,4,5}} = \frac{(6\ \mu F)(3\ \mu F)}{6\ \mu F + 3\ \mu F} = \boxed{2\ \mu F} \qquad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

6.3 Inductors

6.3.1 Electromagnetic induction

Electromagnetic field

- All stationary electrical charges are surrounded by electric fields, and the movement of a charge will produce a magnetic field.
- When the charge changes its velocity of motion, an electromagnetic field is generated.
- Whenever a changing current flows through a conductor, the area surrounding the conductor will produce an electromagnetic field.
- The electromagnetic field can be visualized by inserting a current-carrying conductor (wire) through a hole in a cardboard and sprinkling some iron filings on it.
- As changing current flows through the conductor, the iron filings will align themselves with the circles surrounding the conductor; these are magnetic lines of force.
- The direction of these lines of force can be determined by the right-hand spiral rule, as shown in Figure 6.15.
- The area shows that the magnetic characteristics are called the magnetic field, as it is produced by the changing current-carrying conductor, and therefore, it is also called the electromagnetic field. This is the principle of electricity producing magnetism.

Right-hand spiral rule

- Thumb: the direction of current.
- Four fingers: The direction of magnetic lines of force or direction of the flux (the total magnetic lines of force).

Electromagnetic field	The surrounding area of a conductor with a changing current can generate an electromagnetic field.

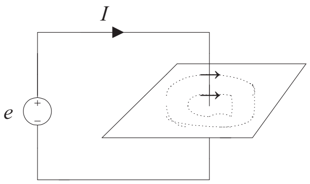


Figure 6.15 Electricity produces magnetism

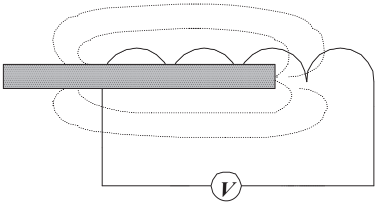


Figure 6.16 Magnet produces electricity

6.3.2 Faraday's law

Magnet produces electricity

- In 1831, the British physicist and chemist Michael Faraday discovered how an electromagnetic field can be induced by a changing magnetic flux.
- When there is a relative movement between a conductor and a magnetic field (or a changing current through the conductor), it will induce a changing magnetic flux Φ (the total number of magnetic lines of force) surrounding the conductor, hence an electromagnetic field is generated.

Induced voltage and current

- Electromagnetic field will produce an induced voltage (v_L) or electromotive force emf (e_L), and induced current (i_L).
- For example, in Figure 6.16, if a magnet bar is moved back and forth in a coil of wire (conductor), or if the coil is moved back and forth close to the magnet and through the magnetic field, the magnetic lines of flux will be cut and
  - a voltage v_L across the coil will be induced (v_L can be measured by using a voltmeter).
  - or, an electromotive force (e_L) that has an opposite polarity with v_L will be induced.
- This will result in an induced current in the coil. This is the principle of a magnet producing electricity.
- Faraday observed that the induced voltage (v_L) is directly proportional to the rate of change of flux $\left(\frac{d\phi}{dt}\right)$ and also the number of turns (N in the coil, and is expressed mathematically as $v_L = N\frac{d\phi}{dt}$.
- In other words, the faster the relative movement between the conductor and magnetic fields, or the more the turns the coil has, the higher the voltage will be produced.

Faraday's law	– When there is a relative movement between a conductor and magnetic field, the changing magnetic flux will induce an electromagnetic field and produce an induced voltage (v_L). – v_L is directly proportional to the rate of change of flux $\left(\frac{d\phi}{dt}\right)$ and the number of turns (N) in the coil, $v_L = N\frac{d\phi}{dt}$

6.3.3 Lenz's law

The polarity of induced effect

- In 1834, the Russian physicist Heinrich Lenz developed a companion result with the Faraday's law. Lenz defined the polarity of induced effect and stated that an induced effect is always opposed to the cause producing it.
- When there is a relative movement between a conductor and a magnetic field (or a changing current through the conductor), an induced voltage (v_L) or induced electromotive force emf (e_L) and also an induced current (i) will be produced.
- The polarity of the induced emf is always opposite to the change of the original current.

Lenz's law

- When the switch is turned on in the circuit of Figure 6.17, the current (cause) in the circuit will increase, but the induced emf (effect) will try to stop it from increasing.
- When the switch is turned off, the current i will decrease, but the polarity of induced emf (e_L) changes and will try to stop it from decreasing.
- An induced current in the circuit flows in a direction that can create a magnetic field that will counteract the change in the original magnetic flux.
- Mathematically, Lenz's law can be expressed as follows:

$$\text{If } i > 0,\ \frac{di}{dt} > 0, \qquad \text{then } e_L = -L\frac{di}{dt} \ \left(\text{or } \ v_L = L\frac{di}{dt}\right)$$

  - $\frac{di}{dt}$ is the rate of change of current.
  - The minus sign for e_L is to remind us that the induced emf always acts to oppose the change in magnetic flux that generates the emf and current.
- The induced voltage (v_L) and induced emf (e_L) have opposite polarities ($E = -V$); this emf is also called the counter emf.

However, the induced voltage (v_L) has the same polarity with the direction of induced current (i). This is similar to the concept of the mutually related reference polarity of voltage and current.

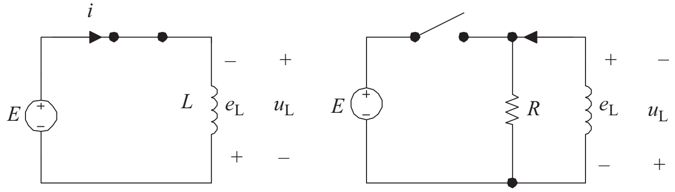


Figure 6.17 Lenz's Law

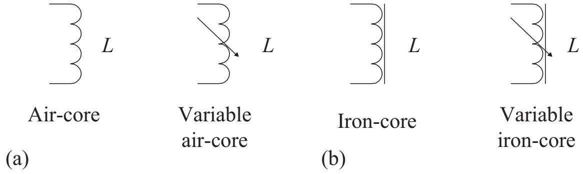


Figure 6.18 Schematic symbols for inductors

6.3.4 Inductors

Inductor (L)

- The resistor and the capacitor are two of the three important fundamental passive circuit elements (components that absorb but do not produce energy). The third element is the inductor (or coil).
- Inductors have many applications in electrical and electronic devices, including electrical generators, transformers, radios, TVs, radars, motors, etc.
- Both capacitors and inductors are energy storage elements.
- The difference between the two is that a capacitor stores transferred energy in the ***electric*** field, and an inductor stores transferred energy in the ***magnetic*** field.
- A basic inductor (L) is made by winding a given length of wire into a loop or coil around a core (center of the coil).

Air-core and iron-core inductors

- Inductors may be classified as air-core inductors or iron-core inductors.
  - An air-core inductor is simply a coil of wire. But this coil turns out to be a very important electric/electronic element because of its magnetic properties.
  - Iron-core provides a better path for the magnetic lines of force and a stronger magnetic field for the iron-core inductor as compared to the air-core inductor.
- The schematic symbol for an air-core inductor looks like a coil of wire as shown in Figure 6.18(a).
- The schematic symbol for an iron-core inductor is shown in Figure 6.18(b). Similar to resistors and capacitors, the inductor can also be classified as fixed and variable.

Inductor (L)	An inductor is an energy storage element that is made by winding a given length of wire into a loop or coil around a core.

6.3.5 Self-inductance

Lenz's law	– When there is a changing current through the conductor, an induced voltage (v_L) or induced emf (e_L) and also an induced current (i) will be produced. – The polarity of the induced emf (e_L) is always opposite to the change of the original current, $e_L = -L\frac{di}{dt}$ or $v_L = L\frac{di}{dt}$

The letter *L* in the equation $v_{\mathrm{L}} = L\frac{\mathrm{d}i}{\mathrm{d}t}$ or $e_{\mathrm{L}} = -L\frac{\mathrm{d}i}{\mathrm{d}t}$ is called inductance (or self-inductance).

Inductance (*L*)

- When current flows through an inductor (coil) that is the same as a current-carrying conductor, a magnetic field will be induced around the inductor.
- According to the principle of electromagnetic induction, Faraday's law and Lenz's law, when there is a relative movement between an inductor and magnetic field or when current changes in the inductor, the changing magnetic flux will induce an electromagnetic field resulting in an induced voltage (v_{L}), or induced emf (e_{L}), and also an induced current (i).
- The measurement of the changing current in an inductor that is able to generate induced voltage is called inductance (or self-inductance).

Inductance vs. inductor

- The resistor, capacitor, and inductor are circuit components.
- The resistance, capacitance, and inductance are the values or capacities of these components.
- Inductance is the capacity to store energy in the magnetic field of an inductor.
- The inductor is symbolized by L while inductance is symbolized by *L*, and the unit of inductance is henry (H).

Inductance *L* (or self-inductance)	– The measurement of the changing current in an inductor that can generate induced voltage is called inductance. – The unit of inductance is henry (H).

6.3.6 Ohm's law for an inductor

Ohm's law for an inductor (L)

- The equation $v_{\mathrm{L}} = L\frac{\mathrm{d}i}{\mathrm{d}t}$ shows the relationship between current and voltage for an inductor, and it is Ohm's law for an inductor.
- The inductance (L) and the current rate of change $\left(\frac{\mathrm{d}i}{\mathrm{d}t}\right)$ determine the induced voltage (v_{L}).
- The induced voltage v_{L} is directly proportional to the inductance L and the current rate of change $\frac{\mathrm{d}i}{\mathrm{d}t}$. This relationship can be illustrated as in Figure 6.19.

Ohm's law for an inductor	An inductor's voltage v_{L} is directly proportional to the inductance L and the rate of change of current $\frac{\mathrm{d}i}{\mathrm{d}t}$: $v_{\mathrm{L}} = L\frac{\mathrm{d}i}{\mathrm{d}t}$

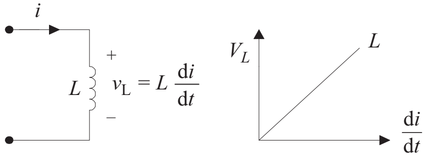


Figure 6.19 Characteristics of an inductor's voltage and current

- Ohm's law for an inductor $v_L = L\frac{di}{dt}$ has a similar form as Ohm's law for a capacitor $i_c = C\,\frac{dv_c}{dt}$. (These two are very important formulas that will be used in future circuits.)

Relationship between inductor voltage and current

- The larger the inductance (L), or the greater the change of current, the higher the induced voltage (v_L) in the coil.
- When the current does not change with time (DC current), i.e. $\frac{di}{dt} = 0$, the inductor voltage (v_L) is also zero.
- Zero voltage means that an inductor acts like a short circuit for DC current. Therefore, the inductor may play an important role for passing the DC current. This is a very important characteristic of an inductor and it is opposite to that of a capacitor.

Recall that a capacitor can block DC and acts like an open circuit for DC.

Passing DC	– Voltage across an inductor is zero when a DC current flows through it (short-circuit equivalent). – An inductor can pass DC.

6.3.7 Factors affecting inductance

Factors affecting inductance

- There are some basic factors affecting the inductance of an inductor (iron-core). These parameters are determined by the construction of an inductor as shown in the following (if all other factors are equal):
  - The number of turns (N) for the coil: More turns for a coil will produce a stronger magnetic field resulting in a higher induced voltage and inductance.
  - The length of the core (l): A longer core will make a loosely spaced coil and a longer distance between each turn, and therefore producing a weaker magnetic field, resulting in a smaller inductance.
  - The cross-sectional area of the core (A): A larger core area requires more wire to construct a coil, and therefore it can produce a stronger magnetic field resulting in a higher inductance.

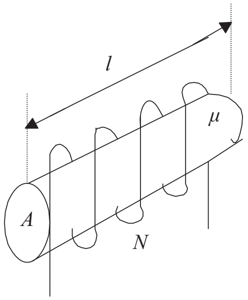


Figure 6.20 Factors affecting inductance

  - The permeability of the material of the core (μ): A core material with higher permeability will produce a stronger magnetic field resulting in a higher inductance.

    Permeability of the material of the core determines the ability of material to produce a magnetic field. Different materials have different degrees of permeability.

- Factors affecting the inductance of an inductor are illustrated in Figure 6.20.

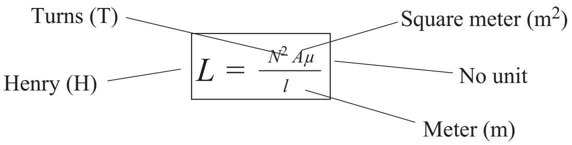
Factors effecting inductance	$L = \dfrac{N^2 A\mu}{l}$

- From the expression of the factor effecting inductance, we can see that when the number of turns of a coil (N) increases, or when the cross-sectional area of the core (A) increases, or when core material with higher permeability (μ) is chosen, or when the length of core (l) is reduced, the inductance of an inductor (L) will increase.

Calculating inductance

- $\boxed{\text{Inductance} = \dfrac{(\text{Number of turns})^2(\text{Area of the core})(\text{Permeability})}{\text{Length of the core}}}$ or $\boxed{L = \dfrac{(N^2 A\mu)}{l}}$

- Units:



6.3.8 Energy stored in an inductor

Inductor—an energy storage element

- Similar to a capacitor, an inductor is also an energy storage element.
- When voltage is applied to two leads of an inductor, the current flows through the inductor and will generate energy and this energy is then absorbed by the inductor and stored in the magnetic field as electromagnetic field builds up.

Derive the energy stored in an inductor

The energy stored by an inductor can be derived as follows:

- The instantaneous electric power of an inductor is given by: $p = iv_L$
- Since the relationship between power and work is $P = \frac{W}{t}$ (energy is the capacity to do work), and the instantaneous power for this expression is $= \frac{dW}{dt}$.
- Substituting $P = \frac{dW}{dt}$ and $v_L = L\frac{di}{dt}$ into the instantaneous power expression $p = iv_L$ gives $\frac{dW}{dt} = Li\frac{di}{dt}$
- Integrating both sides: $\int_0^t \frac{dw}{dt}dt = \int_0^t Li\frac{di}{dt}dt$
- Therefore, we have $w = L\int_0^t i\,di$

i.e., $\boxed{w = \frac{1}{2}Li^2}$

Note: If you have not learned calculus, just remember that $w_L = \frac{1}{2}Li^2$, and skip the above mathematic derivation process.

The energy stored in an inductor

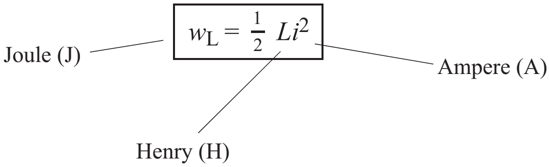
- The equation $w = \frac{1}{2}Li^2$ has a similar form with the energy equation of a capacitor $(w_c = \frac{1}{2}Cv^2)$.
- The equation for energy stored by an inductor shows that the inductor's energy depends on the inductance and the inductor's current.
  - When current increases, an inductor absorbs energy and stores it in the magnetic field of the inductor.
  - When current decreases, an inductor releases the stored energy to the circuit.
- Same as a capacitor, an inductor cannot release more energy than it has stored, so it is also called a passive element.

6.3.9 Calculating the energy stored in an inductor

Energy stored in an inductor	Energy = ½(Inductance)(Current)^2 $w_L = \frac{1}{2}Li^2$

Calculating the energy stored in an inductor

- $\boxed{\text{Energy} = \frac{1}{2}(\text{Inductance})(\text{Current})^2}$ or $\boxed{w_L = \frac{1}{2}Li^2}$
- Units:



Example 6.7: Current in a 0.01 H inductor is $i(t) = 5e^{-2t}$ A, determine the energy stored by the inductor and induced voltage v_L.

Solution:

$$
\begin{aligned}
- \quad w_L &= \frac{1}{2}Li^2 \\
&= \frac{1}{2}(0.01\ \text{H})(5e^{-2t}\ \text{A})^2 \\
&= \frac{1}{2}(0.01\ \text{H})(25e^{-4t}\ \text{A}) \\
&= \boxed{0.125\ e^{-4t}\ \text{J}} \\
- \quad v_L &= L\frac{di}{dt} \\
&= (0.01\text{H})\frac{d}{dt}(5e^{-2t})\text{A} \\
&= (0.01\ \text{H})(-2)(5)e^{-2t}\ \text{A} \\
&= \boxed{-0.1\ e^{-2t}\ \text{V}}
\end{aligned}
$$

Note: If you have not learned calculus, skip the v_L part.

6.3.10 Winding resistor of an inductor

Winding resistance of a coil (R_w)

- When winding a given length of wire into a loop or coil around a core, an inductor is formed. A coil or inductor always has resistance.
- There is always a certain internal resistance distributed in the wire, and the longer the wire, the more turns of coils there are, and thus the wire will have a significantly higher internal resistance.
- The internal resistance in the wire of an inductor is called the wounding resistance of a coil (R_w). An inductor circuit with winding resistance is shown in Figure 6.21.

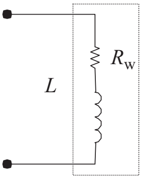


Figure 6.21 Winding resistance

Winding resistance (R_w)	The internal resistance in the wire of an inductor.

Example 6.8: The winding resistance for an inductor in the circuit of Figure 6.22 is 5 Ω. When the current approaches steady state (does not change any more), the energy stored by the inductor is 4 J. What is the inductance of the inductor?

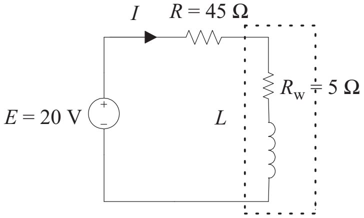


Figure 6.22 Circuit for Example 6.8

Solution: $E = 20\text{ V},\ R = 45\ \Omega,\ R_w = 5\ \Omega,$ and $W_L = 4$ J. $L = ?$

$$I = \frac{E}{R + R_w} = \frac{20\text{ V}}{(45+5)\Omega} = 0.4\text{ A}$$

From $w_L = \frac{1}{2}Li^2$

Solving for L: $L = \frac{2W_L}{I^2} = \frac{2(4\text{ J})}{(0.4\text{ A})^2} = \boxed{50\text{ H}}$

$i = I$, since the current approaches steady state.

6.4 Inductors in series and parallel

6.4.1 Series and parallel inductors

The equivalent inductance

- Similar to resistors and capacitors, inductors may also be connected in series or in parallel to obtain a suitable resultant value that may be either higher or lower than a single inductor value.

- The equivalent (total) series or parallel inductance has the same form as the equivalent (total) series or parallel resistance.
- The equivalent inductance (L_{eq}) will increase if inductors are in series, and the equivalent (total) inductance (L_{eq}) will decrease if inductors are in parallel.

Inductors in series

- A circuit of n inductors connected in series is shown in Figure 6.23.

Equivalent series inductance	$L_{eq} = L_1 + L_2 + \cdots + L_n$

As you may have noticed, this formula has the same form as the formula for calculating series resistances $(R_{eq} = R_1 + R_2 + \cdots + R_n)$.

Inductors in parallel

- A circuit of n inductors connected in parallel is shown in Figure 6.24.

Equivalent parallel inductance	– n inductors in parallel: $L_{eq} = \dfrac{1}{\dfrac{1}{L_1} + \dfrac{1}{L_1} + \cdots + \dfrac{1}{L_n}}$ – Two inductors in parallel: $L_{eq} = \dfrac{L_1 L_2}{L_1 + L_2}$

As you may have noticed, these equations have the same forms as the equations for calculating parallel resistance $\left(R_{eq} = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots + \dfrac{1}{R_n}},\ R_{eq} = \dfrac{R_1 R_2}{R_1 + R_2} \right)$.

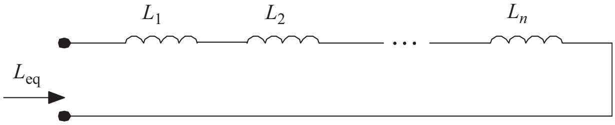


Figure 6.23 Inductors in series

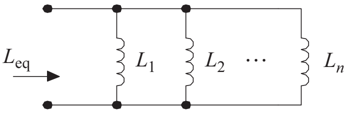


Figure 6.24 Inductors in parallel

6.4.2 Inductors in series–parallel

Series–parallel inductive circuit

- Similar to resistors and capacitors, inductors may also be connected in various combinations of series and parallel.
- An example of a series–parallel inductive circuit is shown in the following.

Example 6.9: Determine the equivalent inductance for the series—parallel inductive circuit shown in Figure 6.25.

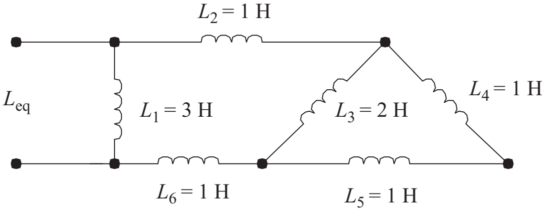


Figure 6.25 Circuit for Example 6.9

Solution: $L_{eq} = [(L_4 + L_5)//L_3 + (L_2 + L_6)]//L_1$

Recall: parallel can be expressed by a symbol of "//".

$$L_{eq} = \frac{\left[\dfrac{2(1+1)}{2+(1+1)} + 1 + 1\right] \times 3}{\left[\dfrac{2(1+1)}{2+(1+1)} + 1 + 1\right] + 3} \text{H} \qquad L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

$$= \boxed{1.5 \text{ H}}$$

Example 6.10: There are three inductors in a series—parallel inductive circuit: 40 H, 40 H, and 50 H. If $L_{eq} = 70$ H, how are these inductors connected?

Solution: $L_{eq} = 50\text{ H} + 40\text{ H}//40\text{ H} = 70\text{ H}$

$$\text{Or } L_{eq} = 50\text{ H} + \frac{40 \times 40}{40 + 40}\text{H} = 70\text{ H} \qquad L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

So, $\boxed{\text{two 40 H inductors are in parallel, and then in series with a 50 H inductor.}}$

Summary

Capacitor

- Capacitor (C): An energy storage element that has two conductive plates separated by an isolating material (the dielectric).
- Capacitor charging: Capacitor stores absorbed energy.
- Capacitor discharging: Capacitor releases energy to the circuit.
- Capacitance (C): the value of the capacitor, $C = \frac{Q}{V}$.
- Factors affecting capacitance: $C = 8.85 \times 10^{-12} \frac{kA}{d}$
- Leakage current: A very small current through the dielectric.
- Breakdown voltage: The voltage that causes a capacitor's dielectric to become electrically conductive; it can explode or permanently damage the capacitor.
- Ohm's law for a capacitor: $i_\mathrm{c} = \frac{\mathrm{d}v_c}{\mathrm{d}t}$ or $i_\mathrm{c} = \frac{\Delta v_c}{\Delta t}$
- Blocking DC: a capacitor can block DC current (open-circuit equivalent).
- Energy stored by a capacitor: $W_\mathrm{c} = \frac{1}{2}Cv^2$

Electromagnetic induction

- Electromagnetic field: The surrounding area of a conductor with a changing current can generate an electromagnetic field.
- Faraday's law:
  - When there is a relative movement between a conductor and magnetic field, the changing magnetic flux will induce an electromagnetic field and produce an induced voltage (v_L).
  - v_L is directly proportional to the rate of change of flux $\left(\frac{\mathrm{d}\phi}{\mathrm{d}t}\right)$ and the number of turns (N) in the coil, $v_\mathrm{L} = N\frac{\mathrm{d}\phi}{\mathrm{d}t}$
- Lenz's law:
  - When there is a changing current through the conductor, an induced voltage (v_L) or induced emf (e_L) and also an induced current (i) will be produced.
  - The polarity of the induced emf (e_L) is always opposite to the change of the original current $\left(e_\mathrm{L} = -L\frac{\mathrm{d}i}{\mathrm{d}t} \quad \text{or} \quad v_\mathrm{L} = L\frac{\mathrm{d}i}{\mathrm{d}t}\right)$.

Inductor

- Inductor (L): An energy storage element that is made by winding a given length of wire into a loop or coil around a core.

- Inductance (L): The measurement of the changing current in an inductor that produces the ability to generate induced voltage.
- Ohm's law for an inductor: $v_L = L\frac{di}{dt}$
- Passing DC: Voltage across an inductor is zero when a DC current flows through it (short-circuit equivalent). An inductor can pass DC.
- Factors affecting inductance: $L = \frac{N^2 A\mu}{l}$
- Energy stored in an inductor: $w_L = \frac{1}{2}Li^2$
- Winding resistance (R_w): The internal resistance in the wire of an inductor.

The characteristics of the resistor, capacitor, and inductor:

Characteristic	**Resistor**	**Capacitor**	**Inductor**
Ohm's law	$V = IR$	$i_c = \frac{dv_c}{dt}$	$v_L = L\frac{di}{dt}$
Energy	$W = pt$ or $dw = pdt$	$W_C = \frac{1}{2}Cv^2$	$W_L = \frac{1}{2}Li^2$
Series	$R_{eq} = R_1 + R_2 + \cdots + R_n$	$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}}$ Two capacitors: $C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2 + \cdots + L_n$
Parallel	$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$ Two resistors: $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2 + \cdots + C_n$	$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}}$ Two inductors: $L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$
Elements in DC		Open-circuit equivalent (blocking DC)	Short-circuit equivalent (passing DC)

Practice problems

6.1

1. Which of the following statements is right, (a) or (b)?
   (a) There is current flowing through the dielectric of a capacitor when it is charging.
   (b) When a DC voltage source is connected to a capacitor, this capacitor will charge to the same value as the voltage source.

2. Which of the following statements is right, (a) or (b)?
   (a) The plates of the capacitor are made from insulating material.
   (b) The plates of the capacitor are made from conducting material.
3. The capacitance of a capacitor is 0.05 μF. Determine the capacitance when a 3 kV source voltage is applied to this capacitor.
4. The plate area of a capacitor is 0.008 m^2, the distance between two plates is 0.00095 m, and the dielectric material is paper. Determine the capacitance of this capacitor.
5. Four capacitors (100 μF, 50 μF, 25 μF, and 10 μF) are connected in series, and a 25 V voltage source is applied to them. Determine the total (or equivalent) capacitance and the amount of charge that is stored.
6. Derive the formula to calculate the energy stored by a capacitor.

6.2

7. A 3 μF capacitor and an unknown capacitor are connected in series. A 10 V voltage source is applied to them and the voltage across the 3 μF capacitor is 3 V. Determine the unknown capacitance.

$$\left(\text{Hint: } C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2},\ C_2 = \frac{?C_{\text{eq}} C_1}{C_1 - C_{\text{eq}}},\ C_{\text{eq}} = \frac{Q?}{E},\ Q = Q_1 = C_1 V_1\right)$$

8. Three capacitors are connected in parallel. Their capacitances are 50 pF, 0.005 μF and 20 pF, respectively. Determine the parallel equivalent capacitance.
9. Determine the equivalent capacitance in the circuit of Figure 6.26.

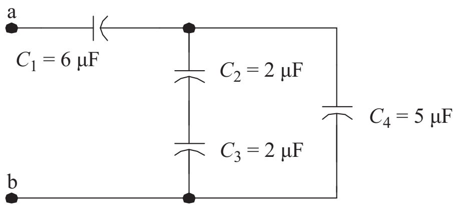


Figure 6.26

10. Determine the equivalent capacitance in the circuit of Figure 6.27.

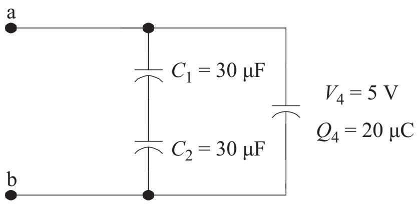


Figure 6.27

6.3

11. Whenever a changing current flows through a ( ), the area surrounding the conductor will produce an ( ).
12. The ( ) of a coil is proportional to the rate of the change of the magnetic flux and ( ).
13. The inductance of a coil is proportional to (, , and), and inversely proportional to ( ).
14. The magnetic flux of a 150-turn coil increased from 0 to 0.18 Wb in 0.5 s. Determine its induced voltage.
15. The current flowing through a 0.5 H inductor is 2A. Determine the energy stored in this inductor.
16. A 10 V voltage source is connected to the two terminals of an inductor with a 10 Ω internal resistor. Determine the current flowing through this inductor.

6.4

17. Calculate the series equivalent inductance for a series circuit that has three inductors 35 μH, 40 μH, and 30 mH, respectively.
18. Four inductors are connected in parallel. Their values are 200 mH, 15 mH, 230 μH and 3H, respectively. Calculate the parallel equivalent inductance in this circuit.
19. Calculate the parallel equivalent inductance in the circuit of Figure 6.28.

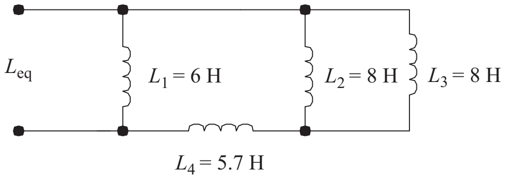


Figure 6.28

20. The equivalent inductance is 8 H in a circuit, and the values of three inductors in this circuit are 6 H, 6 H, and 5 H, respectively. How are these inductors connected?

Chapter 7

Transient analysis of circuits

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7.1 The first-order circuit and its transient response

7.1.1 First-order circuit

RL or RC circuits

- There are three basic elements in an electric circuit, the resistor R, capacitor C, and inductor L.
- The circuits in this chapter will combine the resistor(s) R with an energy storage element capacitor C or an inductor L to form an RL (resistor–inductor) or RC (resistor–capacitor) circuit.
- These circuits exhibit the important behaviors that are fundamental to much of analogue electronics, and they are used very often in electric and electronic circuits.
- Analysis RL or RC circuits still use KCL and KVL.

First-order circuit

- The main difference between RL or RC circuits and pure resistor circuits is that the pure resistor circuits can be analyzed by algebraic methods.
- The relationship of voltages and currents in the capacitor and inductor circuits is expressed by the derivative and differential equations (the equations with the derivative).
- RL or RC circuits that are described by the first-order differential equations, or the circuits that include resistor(s), and only one single energy storage element (inductor or capacitor), are called the first-order circuits.

First-order circuit	– The circuit that contains resistor(s), and a single energy storage element (L or C). – RL or RC circuits that are described by the first-order differential equations.

7.1.2 Transient and steady state

Charging/discharging and energy storing/releasing

- We have discussed the concept of charging/discharging behavior of the energy storage element capacitor C. Another energy storage element inductor L also has the similarly energy-storing/releasing behavior.
- The difference is that charging/discharging of a capacitor is in the electric field, and the energy-storing/releasing of an inductor is in the magnetic field.

Transient state and steady state

- There are two types of circuit states in RL or RC circuit: the transient state and steady state.

- The transient state is the dynamic state that occurs by a sudden change of voltage, current, etc. in a circuit. That means the dynamic state of the circuit has been changed, such as
  - the process of charging/discharging a capacitor or
  - energy-storing/releasing for an inductor. (As the result of the operation of a switch).
- The steady state is an equilibrium condition that occurs in a circuit when all transients have finished.
  - It is the stable circuit state when all the physical quantities in the circuit have stopped changing.
  - For the process of charging/discharging a capacitor or energy storing/ releasing for an inductor, it is the result of the operation of a switch in the circuit after a certain time interval.

Transient state	The dynamic state that occurs when the physical quantities have been changed suddenly.
Steady state	An equilibrium condition that occurs when all physical quantities have stopped changing and all transients have finished.

7.1.3 Step response

Circuit responses

- A response is the effect of an output resulting from an input.
- The first-order RL or RC circuit has two responses, one is called the step response, and the other is the source-free response.

Step response

- The step response for a general system states that the time behavior of the outputs when its inputs change from 0 to unity value (1) in a very short time.
- The step response for an RC or RL circuit is the circuit responses (outputs) when
  - the initial state of the energy store elements L or C is zero,
  - the input (DC power source) is not zero in a very short time.
- The step response is
  - when a DC source voltage is instantly applied to the circuit, the energy storage element L or C has not stored energy yet and the output current or voltage generated in this first-order circuit.
  - Or the charging process of the energy-storing process of the capacitor or inductor.
- The step response can be analogized as a process to fill up water in a reservoir or a water bottle.

Basic terms for a step response

- The initial state: the state when an energy storage element has not stored energy yet.
- Input (excitation): the power supply.
- Output (response): the resultant current and voltage.

Step response	The circuit response when the initial condition of the energy store elements (L or C) is zero, and the input (DC power source) is not zero in a very short time, i.e., the charging/storing process of the C or L.

7.1.4 Source-free and unit-step response

Source-free response

- The source-free response or natural response is opposite to the step response.
  - It is the circuit response when the input is zero, and the initial condition of the capacitor or inductor is not zero (the energy has been stored to the capacitor or inductor).
  - It is the discharging or energy-releasing process of the capacitor or inductor in an RC or RL circuit.
- The source-free response can also be analogized as the process to release water in a reservoir or a water bottle.

Source-free (or natural) response	The circuit response when the input (DC power source) is zero, and the initial condition of the energy storage elements (L or C) is not zero, i.e., the discharging/releasing process of the C or L.

Unit-step response

- When an RC or RL circuit that is initially at "rest" with zero initial condition and a DC voltage source is switched on to this circuit instantly, this DC voltage source can be analogized as an unit step function, since it "steps" from 0 to a unit constant value (1).
- The step response can be also called the unit-step response.
- The unit-step response is defined as follows:
  - All initial conditions of the circuit are zero at times less than zero ($t < 0$), i.e., at the moment of time before the power turns on.
  - The response $v(t)$ or output voltage for this condition is obviously also zero.
  - After the power turns on ($t > 0$), the response $v(t)$ will be a constant unit value 1, as shown in the following mathematic expression and also can be illustrated in Figure 7.1(b).

$$v(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

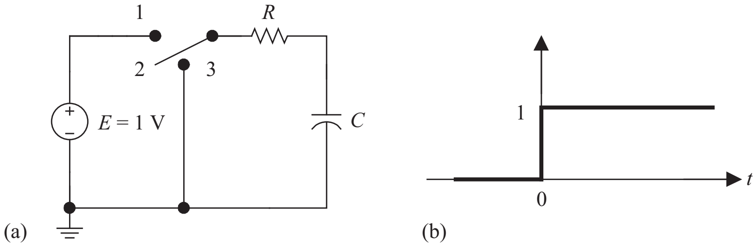


Figure 7.1 Step function

- The unit-step function can be expressed as the switch in the circuit of Figure 7.1(a), when $t = 0$, the switch turns to position 1, a DC power source is connected to the RC circuit, and produces an unit-step response to the circuit.

7.1.5 The initial condition of the dynamic circuit

Switching circuit

- The process of charging and discharging of a capacitor needs a switch to connect or disconnect to the DC source in the RC circuit, as shown in the circuit of Figure 7.1(a).
- The instantly turned on or turned off the switch, or the source input that is switched "on" or "off" in an RC or RL circuit is called the switching circuit.

$t = 0^-$ and $t = 0^+$

- At the moment when the circuit is suddenly switched, the capacitor voltage and inductor current will not change instantly, this concept can be described as $t = 0^-$ and $t = 0^+$.
  - $t = 0^-$ is the instant time interval before switching the circuit (turn off the switch).
  - $t = 0^+$ is the instant time interval after switching the circuit (turn on the switch).

Non-zero initial capacitor voltage and inductor current

- At this switching moment, the non-zero initial capacitor voltage and inductor current can be expressed as follows:

$$v_c(0^+) = v_c(0^-) \qquad i_L(0^+) = i_L(0^-)$$

- v_c (0^-) is the capacitor voltage at the instant time before the switch is closed.
- v_c (0^+) is the capacitor voltage at the instant time after the switch is closed.
- i_L (0^-) is the inductor current at the instant time before the switch is closed.
- i_L (0^+) is the inductor current at the instant time after the switch is closed.

Initial conditions	– Switching circuit: the instantly turned on or turned off switch in the circuit. – $t = 0^-$: the instant time interval before the switch is closed. – $t = 0^+$: the instant time interval after the switch is closed. – At the instant time before/after the switch is closed, v_c and i_L do not change instantly: $v_c(0^+) = v_c(0^-)$ and $i_L(0^+) = i_L(0^-)$

7.2 The step response of an RC circuit

7.2.1 The charging process of an RC circuit

An RC circuit

- In Chapter 6, we have discussed the charging and discharging process of a capacitor. When there are no resistors in the circuit, a pure capacitive circuit will fill with electric charges instantly, or release the stored electric charges instantly.
- But there is always some small amount of resistance in the practical capacitive circuits.
- Sometimes a resistor will be connected to a capacitive circuit that is used very often in the different applications of the electronic circuits.
- Figure 7.2 is a resistor—capacitor series circuit that has a switch connecting to the DC power supply. Such a circuit is generally referred to as an RC circuit.
- The most important concepts of step response (charging) or source-free response (discharging) and transient and steady state of an RC circuit can be analyzed by this simple circuit.

The charging process of an RC circuit

- Assuming the capacitor has not been charged yet in the circuit of Figure 7.2, the switch is in position 2 (middle).
- What will happen when the switch is turned to position 1, and the DC power source (E) is connected to the RC series circuit as shown in the circuit of Figure 7.3?

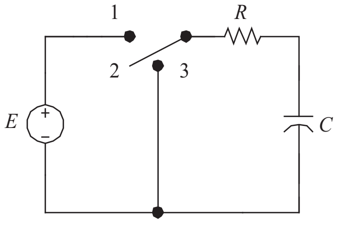


Figure 7.2 An RC circuit

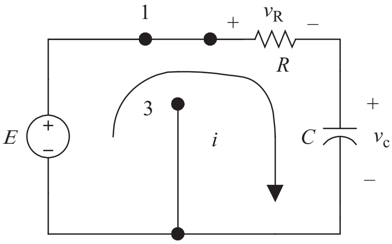


Figure 7.3 RC charging circuit

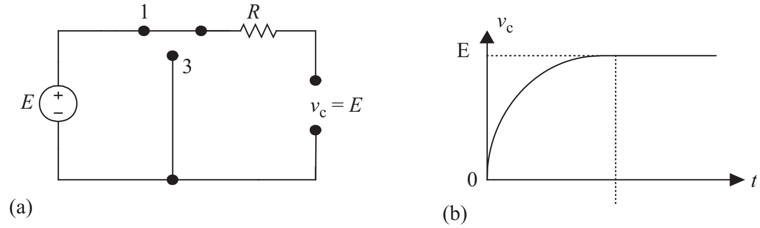


Figure 7.4 The charging process of an RC circuit

- The energy-storing element capacitor C will start charging. Since there is a resistor in this circuit, the process of the capacitor's charging will not finish instantly, the capacitor will gradually store the electric charges.
- The RC circuit is similar to a reservoir (or a water bottle) filling with water to capacity. If the door of the reservoir only opened to a certain width, the reservoir will need more time to fill up with the water (or the water bottle will need more time to fill up the water if the tab did not fully open).

Capacitor voltage v_c increases exponentially

- The voltage across the capacitor v_c is not instantaneously equal to the source voltage E when the switch is closed to 1.
- The capacitor voltage is zero at the beginning. It needs time to overcome the resistance R of the circuit to gradually charge to the source voltage E.
- After this charging time interval or the transient state of the RC circuit, the capacitor can be fully charged; this is shown in Figure 7.4.
- Figure 7.4(b) indicates that capacitor voltage v_c increases exponentially from zero to its final value (E).
- The voltage across the capacitor will be increased until it reaches the source voltage (E), at that time no more charges will flow onto the plates of the capacitor, i.e., the circuit current stops flowing. And the capacitor will reach a state of dynamic equilibrium (steady state).

Steady state

- The state of the circuit voltage or current after charging is called the steady state.
  - Once they reach the steady state, the current and voltage in the circuit will not change any more, and at this time, the capacitor voltage is equal to the source voltage, i.e.,

    $$v_c = E$$

  - The circuit current is zero, and the capacitor is equivalent to an open circuit as shown in the circuit of Figure 7.4(a). For this open circuit, the current stops to flow. Therefore, there is no voltage drop across the resistor.
- The phenomenon of the capacitor voltage v_c increases exponentially from zero to its final value E (or a charging process) in a RC circuit can be also analyzed by the quantity analysis method as follows.

7.2.2 Quantity analysis of the RC charging process

Applying KVL to Figure 7.3

- The polarities of the capacitor and resistor voltages of the RC circuit are shown in Figure 7.3. Applying KVL to this circuit will result in

$$v_R + v_c = E \tag{7.1}$$

- The voltage drop across the resistor is Ri (Ohm's law) while the current through this circuit is $i = C\frac{dv_c}{dt}$ (from Chapter 6),

i.e., $$v_R = Ri \qquad\qquad i = C\frac{dv_c}{dt}$$

Therefore, $$v_R = RC\frac{dv_c}{dt} \tag{7.2}$$

- Substituting (7.2) into (7.1) yields

$$RC\frac{dv_c}{dt} + v_c = E \tag{7.3}$$

Determine the capacitor voltage v_c

Note: If you have not learned calculus, then just remember that (7.4) is the equation for the capacitor voltage v_c during the charging process in an RC circuit, and skip the following mathematic derivation process.

- The first-order differential equation (7.3) can be rearranged as

$$v_c - E = -RC\frac{dv_c}{dt}$$

- Divide both sides by $-RC$ $\quad -\frac{1}{RC}(v_c - E) = \frac{dv_c}{dt}$

- Rearrange $\quad -\frac{dt}{RC} = \frac{dv_c}{v_c - E}$

- Integrating the above equation on both sides yields

$$-\frac{1}{RC}\int_0^t dt = \int_0^{v_c} \frac{dv_c}{v_c - E}$$

$$-\frac{t}{RC}\bigg|_0^t = \ln|v_c - E|\big|_0^{v_c}$$

- Rearrange $\quad -\frac{t}{RC} = \ln|v_c - E| - \ln|-E|$

$$\ln\left|\frac{v_c - E}{-E}\right| = -\frac{t}{RC}$$

- Taking the natural exponent (e) on both sides results in

$$e^{\ln\left|\frac{v_c - E}{-E}\right|} = e^{\frac{-t}{RC}}$$

$$\frac{v_c - E}{-E} = e^{\frac{-t}{RC}}$$

- Solve for v_c $\quad \boxed{v_c = E\left(1 - e^{\frac{-t}{RC}}\right)}$ (7.4)

- The above equation is the capacitor voltage during the charging process in an RC circuit.

7.2.3 Charging equations for an RC circuit

Determine the resistor voltage v_R

- Applying KVL in the circuit of Figure 7.3,

$$v_R + v_c = E$$

- Rearrange $\quad v_R = E - v_c$ (7.5)

- Substitute the capacitor voltage $v_c = E(1 - e^{\frac{-t}{RC}})$ into (7.5) yields

$$v_R = E - E(1 - e^{\frac{-t}{RC}})$$

- Therefore, the resistor voltage is $\boxed{v_R = E\, e^{\frac{-t}{RC}}.}$

Determine the charging current *i*

- Dividing both sides of the equation $v_R = E\,e^{\frac{-t}{RC}}$ by R yields

$$\frac{v_R}{R} = \frac{E}{R}\,e^{\frac{-t}{RC}}$$

- Applying Ohm's law to the left side of the above equation will result in the charging current i

$$\boxed{i = \frac{E}{R}\,e^{\frac{-t}{RC}}} \qquad\qquad I = \frac{V}{R}$$

Charging equations

Charging equations for an RC circuit	– Capacitor voltage: $v_c = E(1 - e^{\frac{-t}{RC}})$
	– Resistor voltage: $v_R = E\,e^{\frac{-t}{RC}}$
	– Charging current: $i = \frac{E}{R}\,e^{\frac{-t}{RC}}$

Mathematically, these three equations indicate that:

- Capacitor voltage increases exponentially from initial value zero to the final value E.
- The resistor voltage and the charging current decay exponentially from initial value E and E/R (or I_{max}) to zero, respectively.
- And t is the charging time in the equations.

7.2.4 Example with RC circuit

According to the above mathematic equations, the curves of v_c, v_R, and i versus time can be plotted as in Figure 7.5.

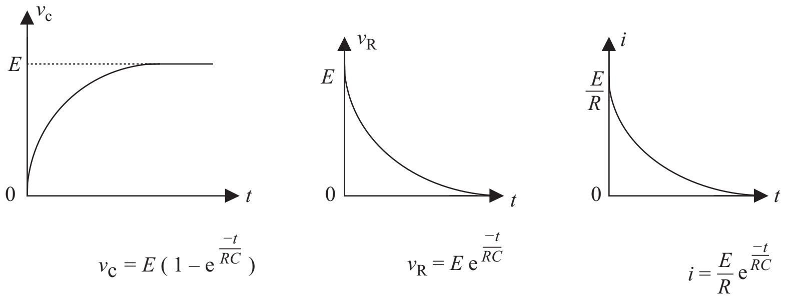


Figure 7.5 v_c, v_R, *and i versus t*

Example 7.1: For the circuit shown in Figure 7.3, if $E = 25$ V, $R = 2.5$ kΩ, and $C = 2.5$ μF, the charging time $t = 37.5$ ms. Determine the resistor voltage v_R and capacitor voltage v_c.

Solution: $RC = (2.5\ \text{k}\Omega)\ (2.5\ \mu\text{F}) = 6.25$ ms

- $$\begin{aligned} v_R &= E\,\mathrm{e}^{\frac{-t}{RC}} \\ &= (25\ \text{V})(\mathrm{e}^{\frac{-37.5\ \text{ms}}{6.25\ \text{ms}}}) \\ &= (25\ \text{V})(\mathrm{e}^{-6}) \\ &\approx \boxed{0.062\ \text{V}} \end{aligned}$$
- $$\begin{aligned} v_c &= E(1 - \ \mathrm{e}^{\frac{-t}{RC}}) \\ &= (25\ \text{V})(1 - \mathrm{e}^{\frac{-37.5\ \text{ms}}{6.25\ \text{ms}}}) \\ &= (25\ \text{V})(1 - \mathrm{e}^{-6}) \\ &\approx \boxed{24.938\ \text{V}} \end{aligned}$$
- These results can be checked up by using KVL: $v_R + v_c = E$
  Substituting the values into KVL yields

  $$0.062\ \text{V} + 24.938\ \text{V} = 25\ \text{V}, \qquad v_R + v_c = E \quad \text{(checked)}$$

  Thus. the sum of the capacitor voltage and resistor voltage must be equal to the source voltage in the RC circuit.

7.3 The source-free response of the RC circuit

7.3.1 The discharging process of the RC circuit

Initial condition: $v_c = E$

- Consider a capacitor C that has initially charged to a certain voltage value V_0 (such as the DC source voltage E) through the charging process of the last section in the circuit of Figure 7.4(a).
- The voltage across the capacitor is $v_c = E$, whose function will be the same as a voltage source in the right loop of the RC circuit in Figure 7.6.

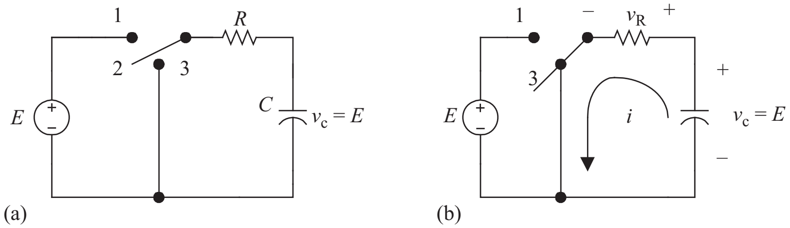


Figure 7.6 Discharging process of the RC circuit

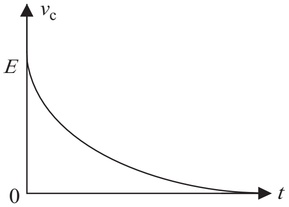


Figure 7.7 Discharge curve of the RC circuit

Discharging process

- Once the switch turns to position 3 as shown in the circuit of Figure 7.6(b), the capacitor will start discharging, but now it will be different than a pure capacitive circuit that can discharge instantly.
- The discharging time will increase since there is a resistor in the circuit. It needs some time to overcome the resistance and eventually release all the charges from the capacitor.
- Once the capacitor has finished the discharge, the capacitor voltage v_c will be 0, the discharging curve is shown in Figure 7.7.
- This is similar to a reservoir that has an open door to release the water (or the water bottle has opened the lid to pour water). But the releasing door of a reservoir is not open wide enough, so it will need some time to release all the water.

7.3.2 Quantity analysis of the RC discharging process

Quantity analysis

- The equations used to calculate the capacitor voltage v_c, resistor voltage v_R, and discharging current i of the capacitor discharging circuit can be determined by the following mathematic analysis method.
- Applying KVL to the circuit in Figure 7.6(b) will result in

$$v_R - v_c = 0, \qquad \text{or} \qquad v_R = v_c \tag{7.6}$$

- Since $v_R = iR$ and $i = -C\dfrac{dv_C}{dt}$
- Substitute i into the equation of v_R $\quad v_R = -RC\dfrac{dv_c}{dt} \quad (7.7)$
- The negative sign in the above equation is because the current i and voltage v_c in the circuit of Figure 7.6(b) have opposite polarities.
- Substitute (7.7) into the left-hand side of (7.6):

$$-RC\frac{dv_c}{dt} = v_c$$

- Divide both sides of the above equation by $-RC$

$$\frac{dv_c}{dt} = -\frac{1}{RC}v_c \tag{7.8}$$

Determine the capacitor voltage v_c

Note: If you have not learned calculus, then just remember that (7.10) is the equation for the capacitor voltage v_c during the discharging process in an RC circuit, and skip the following mathematic derivation process.

- Integrating (7.8) on both sides yields $\int \frac{dv_c}{u_c} = -\frac{1}{RC}\int dt$

  $\ln|v_c| = -\frac{1}{RC}t + \ln A$ $\quad$ $\ln A$ – the constant of the integration

  or $\quad \ln|v_c| - \ln A = -\frac{1}{RC}t$

- Rearrange $\quad \ln\left|\frac{v_c}{A}\right| = -\frac{t}{RC}$
- Taking the natural exponent (e) on both sides of the above equation: $e^{\ln\left|\frac{v_c}{A}\right|} = e^{-\frac{t}{RC}}$
- Therefore, $\quad \frac{v_c}{A} = e^{\frac{-t}{RC}} \quad$ or $\quad v_c = Ae^{\frac{-t}{RC}}$ (7.9)
- As the capacitor has been charged to an initial voltage value V_0 before being connected to the circuit in Figure 7.6(b), therefore the initial condition (initial value) of the capacitor voltage should be $v_c\,(0^-) = V_0$
- V_0 can be any initial voltage value for the capacitor such as the source voltage E.
- Immediately before/after the switch is closed to the position 3 in the circuit of Figure 7.6(b), v_c does not change instantly (from Section 7.1), therefore,

  $v_c(0+) = v_c(0-) \quad$ or $\quad v_c = V_0$

- When $t = 0$, substituting $v_c = V_0$ into (7.9) yields $\quad V_0 = Ae^{\frac{-0}{RC}}$
  That is $\quad V_0 = A$
- Substitute $V_0 = A$ into (7.9) $\quad \boxed{v_c = V_0\, e^{\frac{-t}{RC}}}$ (7.10)

  This is the equation of the capacitor voltage for the *RC* discharging circuit.

Determine the resistor voltage v_R

- According to (7.6), $\quad v_R = v_c$
- Substitute (7.10) into it yields: $\quad \boxed{v_R = V_0\, e^{\frac{-t}{RC}}}$ (7.11)

Determine the discharge current *i*

- Since $i = \frac{v_R}{R}$ $\quad$ Ohm's law
- Substitute (7.11) into the above Ohm's law will result in

$$\boxed{i = \frac{V_0}{R}\, e^{\frac{-t}{RC}}}$$

Discharging equations and curves

Discharging equations for an RC circuit	– Capacitor voltage: $v_c = V_0\, e^{\frac{-t}{RC}}$ – Resistor voltage: $v_R = V_0\, e^{\frac{-t}{RC}}$ – Discharging current: $i = \frac{V_0}{R}\, e^{\frac{-t}{RC}}$

- In the above equations, t is the discharging time. V_0 is the initial capacitor voltage.
- The three equations mathematically indicate that:
  - The capacitor voltage v_c and the resistor voltage v_R decay exponentially from the initial value V_0 to the final value zero.
  - The discharging current i decays exponentially from the initial value V_0/R (or I_{max}) to the final value zero.
- The curves of v_c, v_R, and i versus time t can be illustrated as shown in Figure 7.8.
- The capacitor gradually releases the stored energy, and eventually the energy stored in the capacitor will be released to the circuit completely, and it will be received by the resistor and convert to heat energy.

7.3.3 RC time constant

RC time constant τ

- In an RC circuit, the charging and discharging is a gradual process that needs some time.
- The time rate of this process depends on the values of circuit capacitance C and resistance R. The variation of the R and C will affect the rate of charging and discharging.
- The product of the R and C is called the RC time constant and it can be expressed as a Greek letter τ (tau), i.e., $\tau = RC$.
- In general, the time constant is the time interval required for a system or circuit to change from one state to another, i.e., the time required from the transient to the steady state or to charge or discharge in an RC circuit.

RC time constant τ	RC time constant = (Resistance) (Capacitance) $\tau = RC$

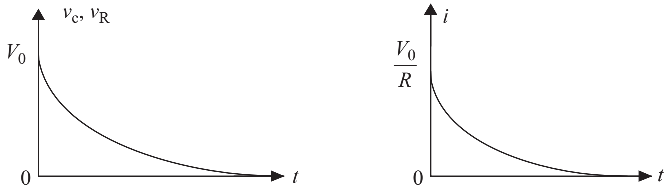
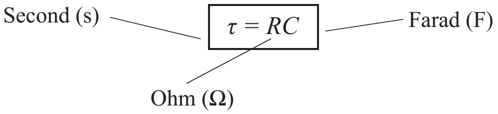


Figure 7.8 The curves of v_c, v_R, and i versus t

Calculating RC time constant

- $\boxed{\text{RC time constant} = \text{(Resistance) (Capacitance)}}$ or $\boxed{\tau = RC}$

- Units:



- The time constant τ represents
  - the time the capacitor voltage reaches (increases) to 63.2% of its final value (steady state),
  - or the time the capacitor voltage decays (decreases) below to 36.8% of its initial value.

The effect of the τ on v_c

The higher the R and C values (or when the time constant τ increases), the longer the charging or discharging time; lesser the v_c variation, longer the time to reach the final or initial values. This can be shown in Figure 7.9.

7.3.4 The RC time constant and charging/discharging

Capacitor charging/discharging voltages and τ

- The capacitor charging/discharging voltages v_c when the time is 1 τ and 2 τ can be determined from the equations of the capacitor voltage in the RC charging/discharging circuit.

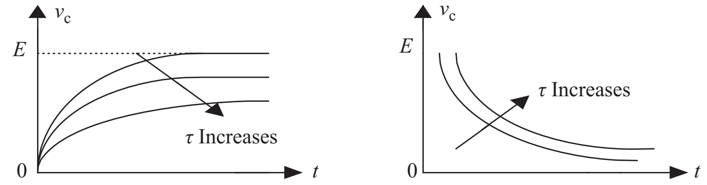
  i.e., $v_c = E(1 - e^{\frac{-t}{\tau}})$ and $v_c = V_0 e^{\frac{-t}{\tau}}$

- For example, when $V_0 = E = 100$ V,
  - At $t = 1\ \tau$:

$$\text{Capacitor charging voltage: } v_c = E(1 - e^{-\frac{t}{\tau}})$$

$$= 100\ \text{V}(1 - e^{-\frac{1\tau}{\tau}}) \approx 63.2\ \text{V}$$

$$\text{Capacitor discharging voltage: } v_c = V_o\, e^{-\frac{t}{\tau}} = 100\ \text{V}e^{-\frac{1\tau}{\tau}} \approx 36.8\ \text{V}$$



(a) Charging (b) Discharging

Figure 7.9 The effect of the time constant τ on v_c

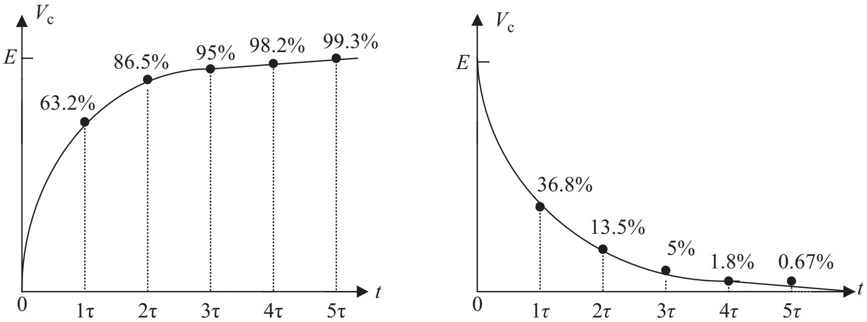


Figure 7.10 The charging/discharging curves of the capacitor voltage

Table 7.1 The capacitor charging/discharging voltages

Charging/discharging time	**Capacitor charging voltage** $v_c = E(1 - e^{\frac{-t}{\tau}})$	**Capacitor discharging voltage** $v_c = V_0 e^{\frac{-t}{\tau}}$
1τ	63.2% of E	36.8% of E
2τ	86.5% of E	13.5% of E
3τ	95.0% of E	5% of E
4τ	98.2% of E	1.8% of E
5τ	99.3% of E	0.67% of E

  - At $t = 2\,\tau$:

$$\text{Capacitor charging voltage: } v_c = E(1 - e^{-\frac{t}{\tau}})$$
$$= 100\text{ V}(1 - e^{-\frac{2\tau}{\tau}}) \approx 86.5\text{ V}$$
$$\text{Capacitor discharging voltage: } v_c = V_0 e^{-\frac{t}{\tau}} = 100\ V e^{-\frac{2\tau}{\tau}} \approx 13.5\text{ V}$$

- Using the same method as above, the capacitor charging/discharging voltages v_c can be determined when the time is 3τ, 4τ, and 5τ. These results are summarized in Table 7.1 and Figure 7.10.

7.3.5 Different time constants for charging/discharging

When the time constant is 1τ and 2τ

- When the time constant is 1τ (the data in Table 7.1 and graphs in Figure 7.10),
  - the capacitor will charge to 63.2% of the final value (source voltage),
  - and discharge to 36.8% of the initial value (the initial capacitor voltage).

    If the final and initial values are 100 V, it will charge to 63.2 V and discharge to 36.8 V.
- When the time constant is 2τ,
  - the capacitor will charge to 86.5% of the final value,
  - and discharge to 13.5% of the initial value.

When the time constant is 5τ

- When the time constant is 5τ,
  - the capacitor will charge to 99.3% of the final value,
  - and discharge to 0.67% of the initial value.
- When the time is 5τ, the circuit will reach the steady state, which means that
  - the capacitor will charge approaching to the source voltage E,
  - or discharge approaching to zero.
- When time has passed 4τ to 5τ, charging/discharging of the capacitor will be almost finished.
- After 5τ, the transient state of RC circuit will be finished and enter the steady state of the circuit.

Time constant τ and charging/discharging	– When $t = 1\tau$: the capacitor charges to 63.2% of the final value and discharges to 36.8% of the initial value. – When $t = 5\tau$: the capacitor charges to 99.3% of the final value and discharges to 0.67% of the initial value.

7.3.6 Discharging process examples

Example 7.2: In the circuit of Figure 7.6(a), the source voltage is 100 V, the resistance is 10 kΩ, and the capacitance is 0.005 μF. How long can the capacitor voltage be discharged to 5 V after the switch is turned to position 3?

Solution: $E = 100$ V, $R = 10$ kΩ, $C = 0.005$ μF, $t = ?$

- The time constant τ for discharging is:

$$\begin{aligned}\tau = RC &= (10\ \text{k}\Omega)(0.005\ \mu\text{F})\\ &= (10{,}000\ \Omega)(0.005\ \mu\text{F})\\ &= 50\ \mu\text{s}\end{aligned}$$

- The capacitor voltage discharging to 5 V is 5% of the initial value E (100 V). Table 7.1 and Figure 7.10 indicate that the time capacitor discharges to 5% of the initial value is 3τ.
- Therefore, the capacitor discharging time is

$$\begin{aligned}t &= 3\tau\\ &= 3(50\ \mu\text{s})\\ &= \boxed{150\ \mu\text{s}}\end{aligned}$$

Example 7.3: In an RC circuit, $R = 5\ \text{k}\Omega$, the transient state has last 1 s in this circuit. Determine the capacitance C.

Solution: The transient state in the RC circuit will last 5τ, therefore,

$$5\tau = 1\ \text{s}, \qquad \text{or} \qquad \tau = \frac{1}{5} = 0.2\ \text{s}$$

$$\because\ \tau = RC$$

$$\text{Therefore,}\quad C = \frac{\tau}{R} = \frac{0.2\ \text{s}}{5\ \text{k}\Omega} = \frac{0.2\ \text{s}}{5{,}000\ \Omega}$$

$$= 0.00004\ \text{F} = \boxed{40\ \mu\text{F}}$$

7.4 The step response of an RL circuit

7.4.1 RL circuit

An RL circuit

- Figure 7.11(a) is a resistor and inductor series circuit, it runs through a switch connecting to the DC power supply. Such a circuit is generally referred to as an RL circuit.
- An RC circuit stores the charges in the electric field, and an RL circuit stores the energy in the magnetic field.
- We will use the term "charging/discharging" in an RC circuit, and the term "energy-storing/releasing" in an RL circuit.

A circuit can be used to analyze RL transient and steady state

- The most important concepts of the magnetic storing/releasing or transient and steady state of RL circuit can be analyzed by a simple circuit as shown in Figure 7.11.
- The step response (storing) and source-free response (releasing) of an RL circuit is similar to the step response and source-free response of an RC circuit.

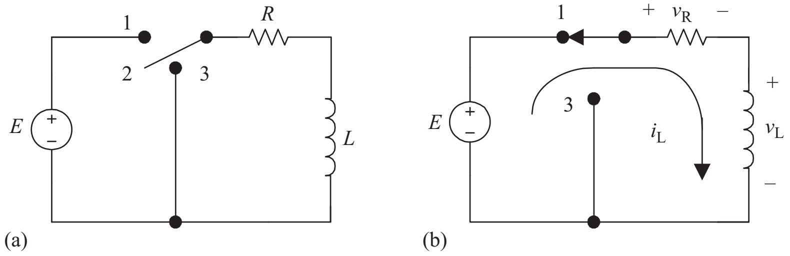


Figure 7.11 RL circuit

- After understanding the RC circuit, its method of analysis can be used to analyze the RL circuit in a similar fashion.
- Figure 7.11(a) is a circuit that can be used to analyze RL step response and source-free response.

7.4.2 Energy-storing process of the RL circuit

The energy-storing process of an RL circuit

- In the circuit of Figure 7.11(a), assuming the energy has not been stored in the inductor yet, the switch is in position 2.
- What will happen when the switch is turned to position 1, and the DC power source is connected to the RL series circuit as shown in the circuit of Figure 7.11(b)?
- As it has been mentioned in Chapter 6, when the switch in Figure 7.11(a) turns to position 1, the current will flow through this RL circuit, the electromagnetic field will be built up in the inductor L, and will produce the induced voltage v_L.
- The inductor L absorbs the electric energy from the DC source and converts it to magnetic energy. This energy-storing process of the inductor in an RL circuit is similar to the electron charging process of the capacitor in an RC circuit.

Inductor current i_L increases exponentially

- Since there is a resistor R in the circuit of Figure 7.11(b), it will be different as a pure inductor circuit that can store energy instantly.
- After the switch is turned to position 1, the current needs time to overcome the resistance in this RL circuit. Therefore, the process of the inductor's energy-storing will not finish instantly.
- The current i_L in the RL circuit will reach the final value (maximum value) after a time interval, as shown in Figure 7.12.
- The phenomenon of the inductor current i_L in an RL circuit increases exponentially from zero to its final value (I_{max}) or from the transient to the steady state can also be analyzed by the quantitative analysis method below.

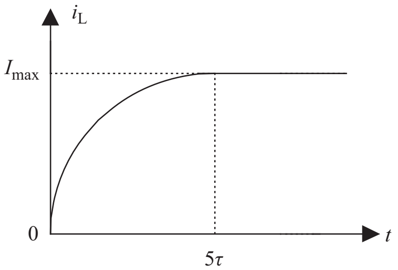


Figure 7.12 Current versus time curve in the RL Circuit

7.4.3 Quantity analysis of the RL energy-storing process

Quantitative analysis of the energy-storing process in an RL circuit

- The polarities of the inductor and resistor voltages of an RL circuit are shown in the circuit of Figure 7.11(b).
- Applying KVL to this circuit will result in

$$v_L + v_R = E \tag{7.12}$$

- Substituting $v_L = L\frac{di_L}{dt}$ Ohm's law for an inductor

  and $v_R = Ri \qquad (i = i_L)$

  into (7.12) yields $L\frac{di_L}{dt} + Ri_L = E$

- Applying a similar analysis method for the RC charging circuit in Section 7.2 will yield the equation of the current in RL circuit during the process of energy storing as given in the following sections.

Determine the current i_L

$$i_L = \frac{E}{R}\left(1 - e^{\frac{-t}{\frac{L}{R}}}\right) = \frac{E}{R}\left(1 - e^{\frac{-t}{\tau}}\right)$$

$$\boxed{i_L = \frac{E}{R}\left(1 - e^{\frac{-t}{\tau}}\right)} \tag{7.13}$$

- The time constant of RL circuit is $\boxed{\tau = \frac{L}{R}}$
- The final value for the current is $\boxed{I_{max} = \frac{E}{R}}$

Determine the resistor voltage v_R

- Applying Ohm's law $v_R = Ri$ (7.14)
- Keep in mind that $i = i_L$ and substituting i by the current i_L in (7.14) yields

$$v_R = R \cdot \frac{E}{R}\left(1 - e^{\frac{-t}{\tau}}\right)$$

$$\boxed{v_R = E\left(1 - e^{\frac{-t}{\tau}}\right)}$$

- The final value for the resistor voltage is $E = I_{max}R$

Determine the inductor voltage v_L

- According to (7.12), $v_L + v_R = E$
- Substitute v_R and solving for v_L $\quad v_L = E - v_R = E - E\left(1 - e^{\frac{-t}{\tau}}\right)$

$$\boxed{v_L = E\,e^{\frac{-t}{\tau}}}$$

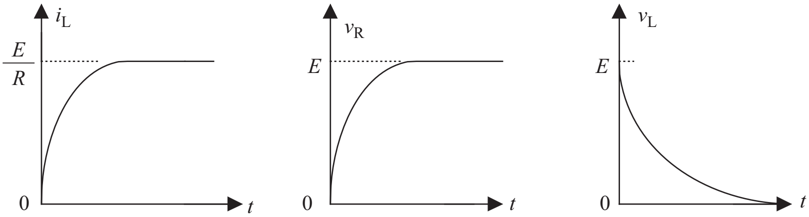


Figure 7.13 Curves of i_L, v_R, and v_L versus time

Energy-storing equations

Energy-storing equations for an RL circuit	– Circuit current: $i_L = \frac{E}{R}(1 - e^{\frac{-t}{\tau}})$ – Resistor voltage: $v_R = E(1 - e^{\frac{-t}{\tau}})$ – Inductor voltage: $v_L = Ee^{\frac{-t}{\tau}}$

- In the above equations,
  - t is the energy-storing time.
  - $\tau = \frac{L}{R}$ is the time constant of the RL circuit.
- These three equations mathematically indicate that
  - circuit current i_L and resistor voltage v_R increase exponentially from initial value zero to the final values E/R and E, respectively;
  - the inductor voltage v_c decays exponentially from the initial value E to zero.
- According to the above mathematical equations, the curves of i_L, v_R, and v_L versus time can be illustrated in Figure 7.13.

Example 7.4: The resistor voltage $v_R = 10(1 - e^{-2t})$ V and circuit current $i_L = 2(1 - e^{-2t})$ A in an RL circuit is shown in the circuit of Figure 7.11(b). Determine the time constant τ and inductance L in this circuit.

Solution:

- The given resistor voltage $\quad v_R = E(1 - e^{\frac{-t}{\tau}}) = 10(1 - e^{-2t})\text{V}$

  with $\quad E = 10\text{ V} \quad$ and $\quad -\frac{t}{\tau} = -2t$

  or $\quad \tau = \frac{1}{2}\text{ s} = 0.5\text{ s}$

- The given current $i_L = \frac{E}{R}(1 - e^{\frac{-t}{\tau}}) = 2(1 - e^{-2t})$ A

  with $\frac{E}{R} = 2$ A

  $E = 10$ V, $R = \frac{E}{I} = \frac{10\text{ V}}{2\text{ A}} = 5\ \Omega$
- The time constant $\tau = \frac{L}{R}$
- Solve for L $L = R\tau = (5\ \Omega)(0.5\text{ s}) = \boxed{2.5\text{ H}}$

7.5 Source-free response of an RL circuit

7.5.1 Energy-releasing process of an RL circuit

Initial condition: $v_L = E$

- Consider an inductor L that has initially stored energy and has the induced voltage v_L through the energy-storing process of the last section.
- If the switch turns to position 3 at this moment (Figure 7.14(b)), the inductor voltage v_L has a function just like a voltage source in the right loop of this RL circuit.

Energy-releasing process

- Without connecting the resistor R in this circuit, at the instant when the switch turns to position 3, the inductor will release the stored energy immediately. (This might produce a spark on the switch and damage the circuit components.)
- If there is a resistor R in the circuit, the resistance in the circuit will increase the time required for releasing energy, the current in the circuit will take time to decay from the stored initial value to zero.
- This means the inductor L releases the energy gradually, and the resistor absorbs the energy and converts it to heat energy.
- The current i_L curve of the energy release process in the RL circuit is illustrated in Figure 7.15.

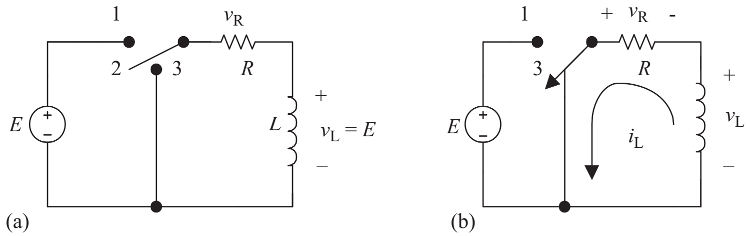


Figure 7.14 RL circuit

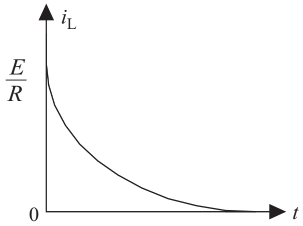


Figure 7.15 Energy release curve of the RL circuit

7.5.2 Quantity analysis of the RL energy-releasing process

Quantitative analysis of the energy-releasing process in an RL circuit

- The equations to calculate the inductor voltage v_L, resistor voltage v_R, and circuit current i_L of the RL energy-releasing circuit can be determined by the following mathematical analysis method.
- Applying KVL to the circuit in Figure 7.14(b) will result in

$$v_L + v_R = 0 \qquad \text{or} \qquad v_L = -v_R \tag{7.15}$$

- Substituting $v_L = L\frac{di_L}{dt}$ and $v_R = Ri_L$ into (7.15)

  yields $L\frac{di_L}{dt} = -Ri_L$ (7.16)

Determine the circuit current i_L

Note: If you have not learned calculus, then just remember that (7.18) is the equation for the current in the RL circuit during the energy releasing, and skip the following mathematic derivation process.

- Dividing both sides of (7.16) by L $\qquad \frac{di_L}{dt} = -\frac{R}{L}i_L$
- Integrating the above equation on both sides yields

$$\int \frac{di_L}{i_L} = -\int \frac{R}{L}dt, \qquad \ln|i_L| = -\frac{R}{L}t + \ln A$$

- Rearrange: $\ln|i_L| - \ln A = -\frac{R}{L}t$
- Taking the natural exponent (e) on both sides results in

$$e^{ln\left|\frac{i_L}{A}\right|} = e^{\frac{-R}{L}t} \qquad \text{or} \qquad \frac{i_L}{A} = e^{\frac{-R}{L}t}$$

- Solve for i_L $\qquad i_L = Ae^{\frac{-R}{L}t}$ (7.17)

- Since energy has been stored in the inductor before it is been connected to the circuit in Figure 7.14(b), its initial condition or value should be $i_L(0^-) = I_0$
  (I_0 can be any initial current, such as $I_0 = \frac{E}{R}$)
- Since immediately before/after the switch is closed to position 3, i_L does not change, therefore:

$$i_L(0^+) = i_L(0^-) \qquad \text{or} \qquad i_L = I_0$$

- When $t = 0$, substitute $i_L = I_0$ into (7.17) yields

$$I_0 = Ae^{\frac{-R}{L}\times 0}, \qquad \text{i.e.,} \qquad I_0 = \mathrm{A}$$

- Therefore, inductor current $\boxed{i_L = I_0 e^{\frac{-t}{\tau}}}$ (7.18)
- In the above equation, $\tau = \dfrac{L}{R}$ is the time constant for the RL circuit.

Determine the resistor voltage v_R

- Keep in mind that $i = i_L$
- Apply Ohm's law to (7.18)

$$v_R = R\,i = R(I_0 e^{\frac{-t}{\tau}})$$

- Resistor voltage

$$\boxed{v_R = I_0 R e^{\frac{-t}{\tau}}}$$

Determine the inductor voltage v_L

- Substituting (7.18) into $v_L + v_R = 0$ Equation (7.15)
- Results in $v_L = -v_R = -RI_0 e^{\frac{-t}{\tau}}$
- Inductor voltage $\boxed{v_L = -I_0 R e^{\frac{-t}{\tau}}}$

Energy-releasing equations and curves

Energy-releasing equations for an RL circuit	– Circuit current:	$i_L = I_0 e^{\frac{-t}{\tau}}$
	– Resistor voltage:	$v_R = I_0 R e^{\frac{-t}{\tau}}$
	– Inductor voltage:	$v_L = -I_0 R e^{\frac{-t}{\tau}}$

- In the above equations,
  - t is the energy-releasing time.
  - $I_0 = \dfrac{E}{R}$ is the initial current for the inductor.
  - $\tau = \dfrac{L}{R}$ is the time constant for the RL circuit.

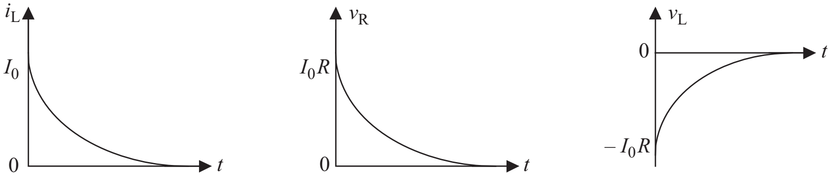


Figure 7.16 Curves of i_L, v_R, and v_L versus time

- These three equations mathematically indicate that inductor current i_L, resistor voltage v_R, and inductor voltage v_c decay exponentially from the initial values I_0, I_0 R, and $-I_0$ R, respectively, to the final value zero.
- The curves of i_L, v_R, and v_L versus time can be illustrated in Figure 7.16.

7.5.3 RL time constant

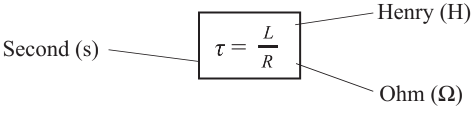
RL time constant τ

- In an RL circuit, the storing and releasing of energy is a gradual process that needs time. The time rate of this process depends on the values of the circuit inductance L and resistance R.
- The variation of R and L will affect the rate of the energy storing and releasing. The quotient of L and R is called the RL time constant $(\tau = \frac{L}{R})$.
- The RL time constant τ is the time interval required from the transient to the steady-state or the energy-storing/releasing time in an RL circuit.

RL time constant	RL time constant = Inductance/Resistance $\tau = \dfrac{L}{R}$

Calculating RL time constant

- $\boxed{\text{RL time constant} = (\text{Inductance})/(\text{Resistance})}$ or $\boxed{\tau = \dfrac{L}{R}}$
- Units:



The effect of the τ on i_L

- The time constant τ represents
  - the time the inductor current i_L reaches (increases) to 63.2% of its final value (steady state);
  - the time of the inductor current i_L decays (decreases) below to 36.8% of the its initial value.

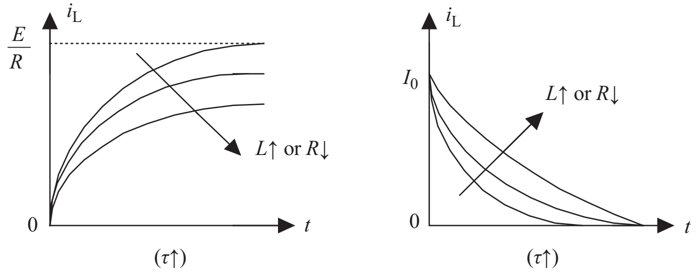


Figure 7.17 Effect of the time constant τ on i_L ($L\uparrow$ or $R\downarrow$)

Table 7.2 Relationship between the time constant and inductor current

Storing/releasing time	**Increasing inductor current (storing)** $i_L = \frac{E}{R}(1 - e^{\frac{-t}{\tau}})$	**Decreasing inductor current (releasing)** $i_L = I_0 e^{\frac{-t}{\tau}}$
1τ	63.2% of E/R	36.8% of I_0
2τ	86.5% of E/R	13.5% of I_0
3τ	95.0% of E/R	5% of I_0
4τ	98.2% of E/R	1.8% of I_0
5τ	99.3% of E/R	0.67% of I_0

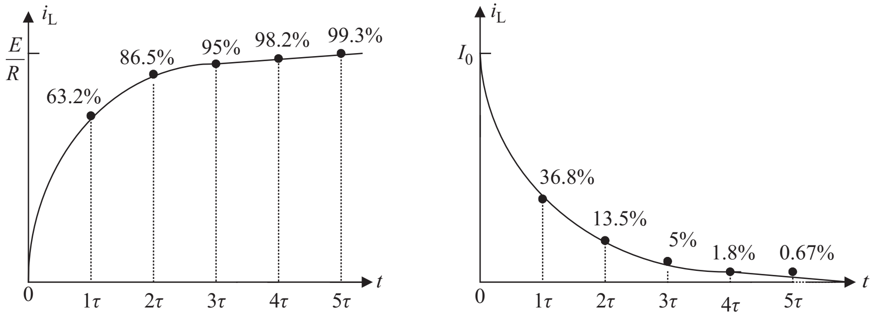


Figure 7.18 Relationship of inductor current and time constant

- The higher the value of L, the lower the R (or when the time constant τ increases), the longer the storing or releasing time, the lesser the i_L variation and the longer the time to reach the final or initial values. This can be shown in Figure 7.17.

7.5.4 RL time constant and energy storing/releasing

Inductor storing/releasing current and τ

- Similar to an RC circuit, the circuit current for an RL circuit can be determined when the time constant is 1τ, 2τ, … 5τ, according to the equations of $i_L = \frac{E}{R}(1 - e^{\frac{-t}{\tau}})$ and $i_L = I_0 e^{\frac{-t}{\tau}}$, respectively.
- These results are summarized in Table 7.2 and Figure 7.18.

Example 7.5: In the RL circuit of Figure 7.14, the resistance R is 100 Ω and the transient state has lasted 25 μs. Determine the inductance L.

Solution:

- The time of transient state usually lasts 5τ, and this transient state is

$$5\tau = 25\ \mu\text{s}, \qquad \therefore \tau = \frac{25\ \mu\text{s}}{5} = 5\ \mu\text{s}$$

- The time constant $\tau = \dfrac{L}{R}$, $\quad \therefore L = R\tau = (100\ \Omega)(5\ \mu\text{s}) = \boxed{500\ \mu\text{H}}$

Example 7.6: In the circuit of Figure 7.14(b), $R = 2\ \text{k}\Omega$, $L = 40\ \text{H}$, $E = 1\ \text{V}$, and $t = 0.2\ \text{ms}$. Determine the circuit current i_L in this energy-releasing circuit.

Solution:

$$\tau = \frac{L}{R} = \frac{40\ \text{H}}{2\ \text{k}\Omega} = 20\ \text{ms}$$

$$i_L = I_0 e^{\frac{-t}{\tau}} = \frac{E}{R} e^{\frac{-t}{\tau}} = \frac{1\ \text{V}}{2\ \text{k}\Omega} e^{\frac{-0.2\ \text{ms}}{20\ \text{ms}}} \approx \boxed{0.5\ \text{mA}}$$

Summary

First-order circuit and types of responses

- First-order circuit:
  - It is the circuit that contains resistor(s), and a single energy storage element (L or C).
  - RL or RC circuits that are described by the first-order differential equations.
- Transient state: The dynamic state that occurs when the physical quantities have been changed suddenly.
- Steady state: An equilibrium condition that occurs when all physical quantities have stopped changing and all transients have finished.
- Step response: The circuit response when the initial condition of the L or C is zero, and the input (DC power source) is not zero in a very short time, i.e., the charging/storing process of the C or L.
- Source-free response: The circuit response when the input is zero, and the initial condition of L or C is not zero, i.e., the discharging/releasing process of the C or L.
- The initial condition:
  - Switching circuit: the instantly turned on or turned off switch in the circuit.
  - $t = 0^-$: the instant time interval before the switch is closed.
  - $t = 0^+$: the instant time interval after the switch is closed.

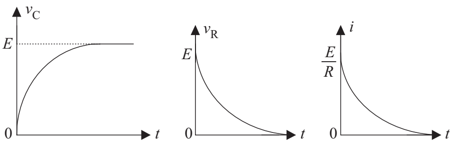
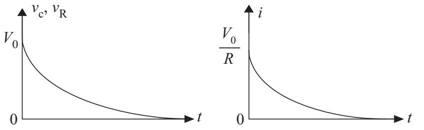
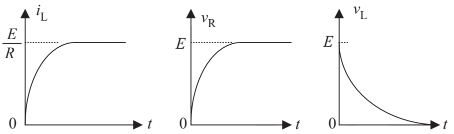
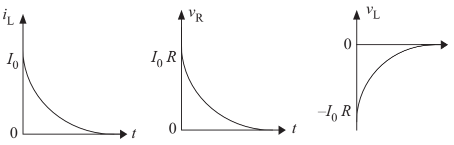
- At the instant time before/after the switch is closed, v_c and i_L do not change instantly:

$$v_c(0^+) = v_c(0^-), \quad i_L(0^+) = i_L(0^-)$$

The relationship between the time constants of RC/RL circuits

Time	**v_c and i_L increasing (charging/storing): $v_c = E(1 - e^{\frac{-t}{\tau}}), i_L = \frac{E}{R}(1 - e^{\frac{-t}{\tau}})$**	**v_c and i_L decaying (discharging/releasing): $v_c = V_0 e^{\frac{-t}{RC}}, i_L = I_0 e^{\frac{-t}{\tau}}$**
1τ	63.2%	36.8%
2τ	86.5%	13.5%
3τ	95.0%	5%
4τ	98.2%	1.8%
5τ	99.3%	0.67%

Summary of the first-order circuits

Circuit	**Equations**	**Waveforms**	**Time constant**
RC charging (step response)	$v_c = E(1 - e^{\frac{-t}{\tau}})$ $v_R = E\, e^{\frac{-t}{\tau}}$ $i = \frac{E}{R} e^{\frac{-t}{\tau}}$		$\tau = RC$
RC discharging (source-free response)	$v_c = V_0\, e^{\frac{-t}{\tau}}$ $v_R = V_0\, e^{\frac{-t}{\tau}}$ $i = \frac{V_0}{R} e^{\frac{-t}{\tau}}$		$\tau = RC$
RL storing (step response)	$i_L = \frac{E}{R}(1 - e^{\frac{-t}{\tau}})$ $v_R = E(1 - e^{\frac{-t}{\tau}})$ $v_L = E\, e^{\frac{-t}{\tau}}$		$\tau = \frac{L}{R}$
RL-releasing (source-free response)	$i_L = I_0\, e^{\frac{-t}{\tau}}$ $v_R = I_0\, R\, e^{\frac{-t}{\tau}}$ $v_L = -I_0\, R\, e^{\frac{-t}{\tau}}$		$\tau = \frac{L}{R}$

Practice problems

7.1

1. (   ) or (   ) circuits can be described by the first-order (   ) equations.
2. (   ) occurs when all physical quantities have stopped changing and all transients have finished.
3. (a) $t = 0^+$ is the instant time (   ) switching.
   (b) $t = 0^-$ is the instant time (   ) switching.
4. (a) v_c (   ) is the (   ) voltage at the instant time before the switch is closed.
   (b) v_R (   ) is the (   ) voltage at the instant time after the switch is closed.
5. (a) i_L (   ) is the (   ) current at the instant time before the switch is closed.
   (b) i_c (   ) is the (   ) current at the instant time after the switch is closed.

7.2

6. Determine the expression of the capacitor charging voltage v_c and current i_c in the circuit of Figure 7.19, and plot the waveform of v_c.

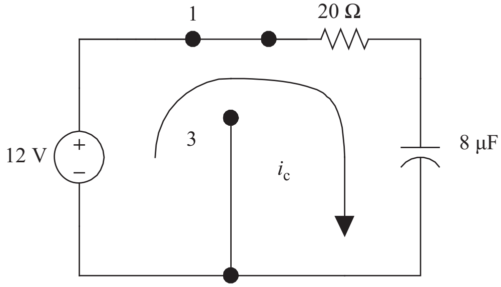


Figure 7.19

7 Determine the capacitor charging voltage v_c and circuit current i_c expressions in the circuit of Figure 7.20.

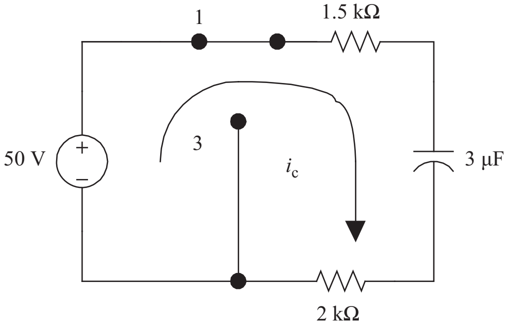


Figure 7.20

8. In an RC series circuit, the capacitance is 0.1 μF and the source voltage is 30 V. After charging for 65 μs, the voltage across the capacitor is 22 V. Determine the resistance of this circuit.

7.3

9. The resistance is 5 kΩ and the capacitance is 3 μF in an RC charging circuit. Determine the time required for the capacitor voltage to reach 63.2% of the steady-state voltage.
10. The resistance R is 8 kΩ and the capacitance C is 0.003 μF in an RC circuit. If the voltage across the terminals of this RC circuit is 100 V, calculate the capacitor voltage after a time period of 1 τ, and the time required for the capacitor to charge to the value of the source voltage.
11. Determine the values or expressions in the statements of (a), (b), and (c) for the circuit of Figure 7.21:

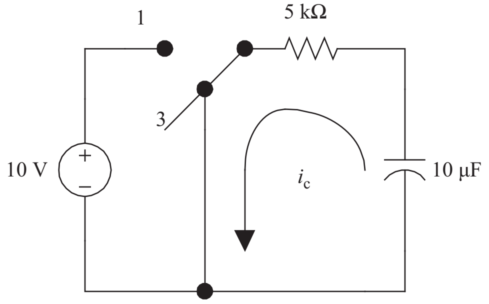


Figure 7.21

(a) The time constant of the circuit.
(b) The capacitor voltage and current expressions when the switch is turned to position 3 (assume the capacitor voltage is 10 V before the switch is turned to position 3).
(c) Calculate the capacitor voltages when the time is 1τ, 2τ, 3τ, 4τ, and 5τ.

7.4

12. The resistance is 4.7 kΩ, and the inductance is 10 mH in an RL circuit. Determine the time constant for this circuit.
13. Determine the time constant of the circuit in Figure 7.22. Also write the expressions for the inductor voltage, resistor voltage, and circuit current in this circuit.

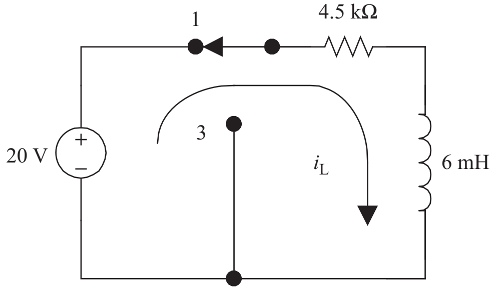


Figure 7.22

7.5

14. Determine the time constant of the circuit in Figure 7.23 when the inductor is releasing energy (determine the equivalent resistance first).

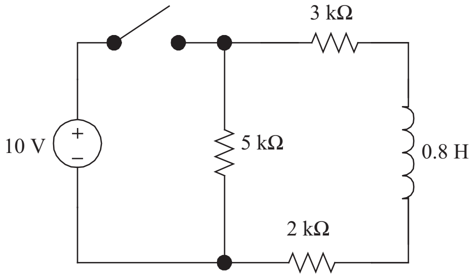


Figure 7.23

15. Determine the expressions of the inductor voltage and inductor current in the circuit of Figure 7.23.

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Chapter 8

Magnetism and electromagnetism

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8.1 The magnetism field

8.1.1 Magnetism

Magnet

It is a piece of iron (or steel, alloy . . .) that has the ability to attract another metal object.

Permanent magnet

It is a magnet that retains its magnetism over a long period of time after it is removed from a magnetic field.

Magnetism

It is the attraction or repulsion properties of a magnet.

Magnetic poles

A magnet has two areas of strongest force, called poles.

- Every magnet has a North and South Pole (N and S).
- The basic law of magnet: Like poles repel, unlike poles attract (Figure 8.1).

Magnetic field

It is a place near a magnet or a moving electric charge where magnetic properties are produced (an invisible area of magnetism produced by moving the electric charge).

Electromagnetic force

It is a force between charged objects around their electric and magnetic fields.

Magnetic field lines (the lines of force)

The imaginary lines around a magnet that describe the directions of the magnetic field (they can be plotted with iron filings and paper) (Figure 8.2).

- Outside: the magnetic field lines travel from the North Pole (N) to the South Pole (S).
- Inside: from the South Pole to the North Pole.

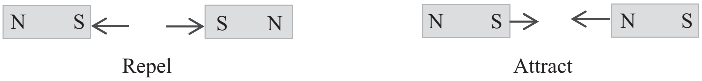


Figure 8.1 The basic law of magnet

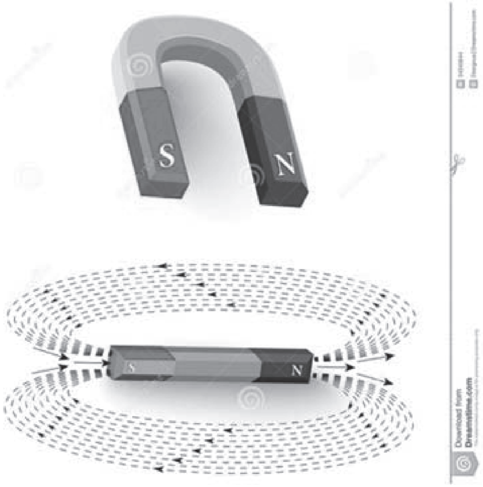


Figure 8.2 Magnetic field lines

8.1.2 Magnetic flux and magnetic flux density

Magnetic flux (ϕ)

It is a measure of the amount of magnetic field lines passing through a surface area in a magnetic field.

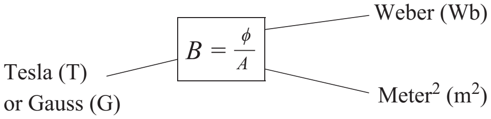
Magnetic flux density (B)

It is a measure of the amount of magnetic flux in an area.

- It is the measure of the amount of flux per unit area taken perpendicular to the flux's direction.
- It is the strength of a magnetic field at a given point.

Calculating magnetic flux density

- $\boxed{\text{Magnetic flux density} = \frac{\text{Magnetic flux}}{\text{Area}}}$ $\boxed{B = \frac{\phi}{A}}$
- Units:



- 1 Wb = 10^8 lines (magnetic field lines)
- 1 Tesla = 10^4 gauss
  - Tesla—in SI unit
  - Gauss—in CGS system (centimeter-gram-second system of units)

Example 8.1: The amount of flux present in a round magnetic bar was measured at 0.018 Wb. If the material has a diameter of 16 cm, what is the flux density?

Solution: Diameter: $d = 16\,\text{cm} = 0.16\,\text{m}$

Radius: $r = \frac{d}{2} = \frac{0.16\,\text{m}}{2} = 0.08\,\text{m}$

Area: $A = \pi r^2 = \pi\,(0.08\,\text{m})^2 \approx 0.02\,\text{m}^2$

Magnetic flux: $\phi = 0.018\,\text{Wb}$

Magnetic flux density: $B = \frac{\phi}{A} = \frac{0.018\,\text{Wb}}{0.02\,\text{m}^2} = \boxed{0.9\,\text{T}}$

8.1.3 Domain theory of magnetism

Magnetized material

Any material when placed in a magnetic field can be magnetized itself (can attract or repel metals).

Ferromagnetism

The physical phenomenon in which certain materials (like iron) can become permanent magnets when subjected to a magnetic field. (The ability of materials to be attracted to a magnet is called ferromagnetism.)

Ferromagnetic materials

These refer to materials that can maintain their magnetic properties when the magnetic field is removed.

- They are materials that can be magnetized.
- Examples of ferromagnetic materials: iron, cobalt, nickel, etc.

Domain theory of magnetism

It states that inside a magnet all the atoms are aligned in the same directions.

- The domain theory can explain why ferromagnetic materials get magnetized.
- The atoms of ferromagnetic materials may be thought of as little atomic magnets (with its own North and South Pole).
- These groups of atomic magnets join together so that their magnetic fields are all pointing in the same direction to form a magnetic domain.

Magnetic domain

It is a small region in which the magnetic fields of atoms are grouped together and aligned (Figure 8.3).

- The magnetic domains are indicated by the arrows in the metal material.
- Each magnetic domain acts as a miniature magnet within a material.
- In ferromagnetic materials the domains align themselves in the same direction.

8.2 Electromagnetism

8.2.1 Charging and electric field

Electric charge (or charge)

Electric charges are the basic properties of particles (electrons, protons, etc.) in matter.

- Protons are positively (+) charged.
- Electrons are negatively (−) charged.

Like charges repel and unlike charges attract (Figure 8.4).

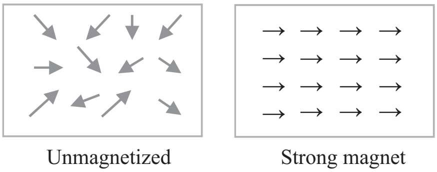


Figure 8.3 Magnetic domain

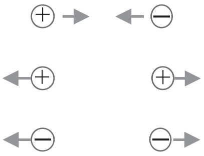


Figure 8.4 Electric charge

Charging

Charging is transfer of electrons between the two objects.

- The object that loses electrons has an excess (extra) of positive charge.
- The object that gains electrons has an excess of negative charge.
- $\begin{cases} \text{The total charge of an object} = 0\text{: the number of electrons} \\ \quad = \text{the number of protons} \\ \text{The total charge of an object} = +\text{: the object loses electrons} \\ \text{The total charge of an object} = -\text{: the object gains electrons} \end{cases}$

Charging by conduction and induction

- Conduction: transfer of charge from one object to another by direct ***contact.***
- Induction: transfer of charge from one object to another ***without*** direct ***contact***.
  Changing/discharging caused by conduction or induction.

Static electricity

It is an accumulation (build up) of an electric charge on the surface of an object (electric charge at rest rather than moving).

Static discharge

It is the release of static electric charge.

Law of conservation of electric charge

It states that the electric charge cannot be created or destroyed, but it can be transferred from one form to another (the total electric charge remains constant).

Electric field

It is the area near a charged object experiences electric forces that fill the area.

Electric field lines

These are the imaginary lines around a charged object that describe the electric field in an area. They begin as positive charges and end as negative charges (+ → −).

8.2.2 Electromagnetism

Electric current (or a moving electric charge) can produce a magnetic field

- Each electric charge has its own electric field, a moving charge creates a magnetic field.
- When an electric current passes through a wire (conductor), a magnetic field is formed around the wire.

Electromagnetism

It is the interaction of electric and magnetic fields.

- Moving charges (or current) produce a magnetic field.
- Spinning magnets cause an electric current to flow.

An electromagnetism is a relationship between electricity and magnetism.

Electromagnetic induction

Moving a loop of wire through a magnetic field, or moving a magnetic field relative to a coil will produce an electric current.

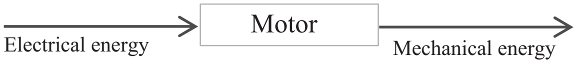
Right-hand rule

It is a memory aid used to remember the directions of current and the magnetic field around a wire (Figure 8.5).

- The thumb: the direction of the positive current.
- The fingers: the direction of the magnetic field. (The fingers of your hand circle the wire.)

Electric motor

It is a device that converts electrical energy into mechanical energy.



A motor uses a magnet to act a force on a wire coil, this force makes the motor rotate (turn).

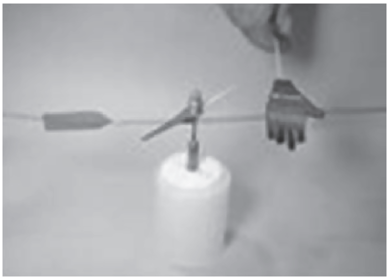


Figure 8.5 Right-hand rule

Electric generator

It is a device that converts mechanical energy into electrical energy.

Mechanical energy
Generator
Electrical energy

8.3 Electromagnetic characteristics of materials

8.3.1 Permeability and reluctance

Permeability (μ)

It is the measure of the ability of a magnetic material to respond magnetic field development.

- It is the ability of a magnetic material to respond to how much magnetic flux it can support to pass through the material.
- It is the degree of magnetization capability.
- Higher permeability means the more easily a magnetic field can be established and the more easily a material can be magnetized (the domains aligned).

Calculating relative permeability

- $\boxed{\text{Relative permeability} = \frac{\text{Absolute permeability}}{\text{Permeability of free space}}}$ $\boxed{\mu_r = \frac{\mu}{\mu_0}}$
- Permeability of free space μ_0: $\mu_0 = 4\,\pi \times 10^{-7}$ Weber/Ampere · meter (Wb/A · m) or Henry/Meter (H/m)

  (permeability of a vacuum)
- Units: $\boxed{\mu_r = \frac{\mu}{\mu_0}}$

  Weber/Ampere · meter (Wb/A · m)

  No unit

  Weber/Ampere · meter (Wb/A · m)

Example 8.2: The absolute permeability of a piece of steel is 2. 8 × 10^{-9} Wb/A · m. Calculate the relative permeability of this steel material.

Solution: The absolute permeability: $\mu = 2.8 \times 10^{-9}\ \text{Wb/A} \cdot \text{m}$

The permeability of free space μ_0: $\mu_0 = 4\,\pi \times 10^{-7}\ \text{Wb/A} \cdot \text{m}$

The relative permeability μ_r:

$$\mu_r = \frac{\mu}{\mu_0} = \frac{2.8 \times 10^{-9}\ \text{Wb/A·m}}{4\pi \times 10^{-7}\ \text{Wb/A} \cdot \text{m}}$$

$$\approx \boxed{0.223 \times 10^{-2}}$$

Reluctance ($\Re$)

Reluctance ($\Re$) is the opposition offered by a magnetic circuit to the magnetic flux.

- $\Re$—a curly capital *R*.
- Reluctance is analogous to the resistance in an electrical circuit. It is the magnetic resistance in a magnetic circuit.
- Reluctance is a measure of the ability of a material to pass magnetic flux.

Calculating reluctance

- $$\boxed{\text{Reluctance} = \frac{\text{Length of the coil}}{(\text{Permeability})(\text{The cross-sectional area of the material})}}$$

$$\boxed{\Re = \frac{\ell}{\mu A}}$$

- Units: Ampere · turn/Weber (A · t/Wb) $\boxed{\Re = \frac{l}{\mu A}}$

Meter (m)

Meter^2 (m^2)

Weber/Ampere · meter (Wb/A · m)

Example 8.3: Determine the reluctance of a material with a length of 0.34 m and a cross-sectional area of 0.06 m^2, if the absolute permeability is 130×10^{-6} Wb/A · m.

Solution: The absolute permeability: $\mu = 130 \times 10^{-6}\ \text{Wb/A}\cdot\text{m}$

The cross-sectional area: $A = 0.06\ \text{m}^2$

The length of the coil: $\ell = 0.34\ \text{m}$

The reluctance:
$$\Re = \frac{\ell}{\mu A} = \frac{0.34\ \text{m}}{(130 \times 10^{-6}\ \text{Wb/A}\cdot\text{m})(0.06\ \text{m}^2)}$$
$$\approx \boxed{0.044 \times 10^6\ \text{A}\cdot\text{t/Wb}}$$

Magnetomotive force (F_m)

It is a force that is the cause of a magnetic flux in a magnetic circuit.

- The magnetomotive force is the force produced by current through a coil of wire.
- The magnetomotive force is analogous to the electromotive force in an electric circuit.

Calculating magnetomotive force

- $\boxed{\text{Magnetomotive force} = (\text{Number of turns of wire})\ (\text{Current})}$ $\boxed{F_m = NI}$

- Units: $\boxed{F_m = NI}$

Turn (t)

Ampere-turn (A · t)

Ampere (A)

Example 8.4: The magnetomotive force for a one-turn coil carrying a current of 1 A is:

$$F_\mathrm{m} = N\,I = (1\ \mathrm{turn})(1\ \mathrm{A}) = \boxed{1\ \mathrm{A}\cdot\mathrm{t}}$$

Example 8.5: What is the magnetomotive force in an 80 turns coil of wire when there is 4.4 A of current through it?

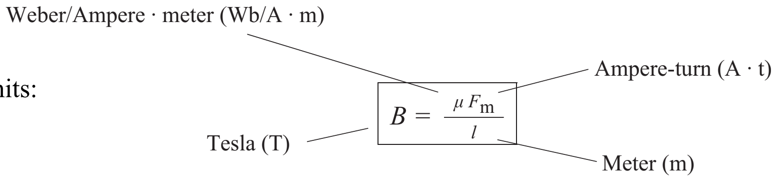
$$F_\mathrm{m} = N\,I = (80\ \mathrm{turns})(4.4\ \mathrm{A}) = \boxed{352\ \mathrm{A}\cdot\mathrm{t}}$$

Factors affecting the flux density produced by a coil

- The permeability of the core
- Magnetomotive force
- The length of the coil.

Calculating flux density

- $\boxed{\text{Flux density} = \dfrac{(\text{Absolute permeability})(\text{Magnetomotive force})}{\text{Length of the coil}}}$ $\boxed{B = \dfrac{\mu\,F_\mathrm{m}}{\ell}}$
- Units:



Example 8.6: There is 0.3 A of current through an air-core coil of wire with 150 turns and a length of 250 cm. Determine the flux density for the coil.

Solution: The number of turns of wire: $N = 150\ \mathrm{t}$

The current: $I = 0.3\ \mathrm{A}$

The magnetomotive force: $F_\mathrm{m} = N\,I = (150\ \mathrm{t})(0.3\ \mathrm{A}) = \boxed{45\ \mathrm{A}\cdot\mathrm{t}}$

The permeability of free space μ_0: $\mu_0 = 4\,\pi \times 10^{-7}\ \mathrm{Wb/A}\cdot\mathrm{m}$ (Air core coil)

The length of the coil: $\ell = 250\ \mathrm{cm} = 2.5\ \mathrm{m}$

The flux density:

$$B = \frac{\mu\,F_\mathrm{m}}{\ell} = \frac{(4\,\pi \times 10^{-7}\ \mathrm{Wb/A}\cdot\mathrm{m})(45\mathrm{A}\cdot\mathrm{t})}{(2.5\ \mathrm{m})} \approx \boxed{226.2 \times 10^{-7}\ \mathrm{T}}$$

8.3.2 Ohm's law for magnetic circuits

Ohm's law for magnetic circuits

- Ohm's law: $\boxed{\text{Flux density} = \dfrac{\text{Magnetomotive force}}{\text{Reluctance}}}$ $\boxed{\phi = \dfrac{F_m}{\Re}}$
- Units:

Ampere-turn (A · t)

Weber (Wb) $\boxed{\phi = \dfrac{F_m}{\Re}}$ Ampere · turn/Weber (A · t/Wb)

Example 8.7: There is 0.22 A of current through a coil of wire with 380 turns. Determine the reluctance of the circuit if the magnetic flux is 0.25 μWb.

Solution: The number of turns of wire: $N = 380\,\text{t}$

The current: $I = 0.22\,\text{A}$

The flux: $\phi = 0.25\,\mu\text{Wb}$ μWb = microweber

The magnetomotive force: $F_m = N\,I = (380\,\text{t})(0.22\,\text{A})$

$= \boxed{83.6\,\text{A}\cdot\text{t}}$

The reluctance: $\Re = \dfrac{F_m}{\phi}$ $\phi = \dfrac{F_m}{\Re}$

$= \dfrac{83.6\,\text{A}\cdot\text{t}}{0.25\,\mu\text{Wb}}$

$= 334.4\,\text{A}\cdot\text{t}/\mu\text{Wb}$ $\mu - 10^{-6}$

$= \boxed{334.4 \times 10^{6}\,\text{A}\cdot\text{t/Wb}}$

Basic electric and magnetic quantities and Ohm's law

Electric circuit	**Magnetic circuit**
Electromotive force (emf)	Magnetomotive force (F_m)
Current (I)	Flux (ϕ)
Resistance (R)	Reluctance ($\Re$)
Ohm's law: $\boxed{I = \dfrac{E}{R}}$	Ohm's law: $\boxed{\phi = \dfrac{F_m}{\Re}}$

8.4 Magnetic hysteresis

8.4.1 Magnetic field intensity

Magnetic field intensity or magnetizing force (*H*)

It is a measure of the actual magnetic field within a material.

- It is the strength of an magnetic field at any point.
- It is the amount of magnetomotive force (F_m) available per unit length (l).

Calculating magnetic field intensity

- $\boxed{\text{Magnetic field intensity}(H) = \frac{\text{Magnetomotive force } (F_m)}{\text{Length of the coil}(\ell)}}$ $\boxed{H = \frac{F_m}{\ell}}$
- Units: $\boxed{H = \frac{F_m}{l}}$

  Ampere-turn (A · t)/Meter (m)

  Ampere-turn (A · t)

  Meter (m)

Example 8.8: There is 0.4 A of current through a coil of wire with 150 turns. If the length of the magnetic circuit is 15 cm, determine the magnetic field intensity.

Solution: The number of turns of wire: $N = 150\,\text{t}$

The current: $I = 0.4\,\text{A}$

The magnetomotive force: $F_m = N\,I = (150\,\text{t})(0.4\,\text{A}) = \boxed{60\,\text{A} \cdot \text{t}}$

The length of the coil: $\ell = 15\,\text{cm} = 0.15\,\text{m}$

The magnetic field intensity:

$$H = \frac{F_m}{\ell} \qquad \phi = \frac{F_m}{\Re}$$

$$= \frac{60\,\text{A} \cdot \text{t}}{0.15\,\text{m}}$$

$$= \boxed{400\,\text{A} \cdot \text{t/m}}$$

Factors affecting the magnetic field intensity (*H*) produced by a coil

- The number of turns of wire (N). $N \uparrow \to F_m \uparrow \to H \uparrow$ $F_m = NI,\ H = \frac{F_m}{\ell}$
- The current (I) through the coil. $I \uparrow \to F_m \uparrow \to H \uparrow$ $F_m = NI,\ H = \frac{F_m}{\ell}$
- The magnetic flux (ϕ). $\phi \uparrow \to F_m \uparrow \to H \uparrow$ $\phi = \frac{F_m}{\Re},\ H = \frac{F_m}{\ell}$
- The length of the coil (ℓ). $\ell \uparrow \to H \downarrow$ $H = \frac{F_m}{\ell}$

8.4.2 Magnetic hysteresis

Hysteresis

It is a lag between cause and effect (or a lag between an input and an output).

“Hysteresis” originates from the Greek word “hysterein” meaning “to lag behind.”

Magnetic hysteresis

It is the phenomenon of changes of flux density *B* lagging behind the magnetizing force *H* in a ferromagnetic material, such as iron.

- It is the lagging of the magnetization of a ferromagnetic substance when the magnetizing force acting on it is changed.
- It is the lag in the response of magnetic induction to change of magnetic intensity.

Hysteresis curve (or hysteresis loop)

- A hysteresis curve (*B*–*H* curve) shows the relationship between the induced magnetic flux density (*B*) and the magnetizing force (*H*).
- When a ferromagnetic material is magnetized in one direction, it will not return back to zero magnetization when the applied magnetizing field is removed.
- If an alternating magnetic field is applied to the material, its magnetization will trace out a loop called a hysteresis curve (or loop).

Hysteresis loop and retentivity

- Positive cycle
  - $H = 0$, $B = 0$ (A magnetic core is unmagnetized.)
  - $H\uparrow \rightarrow B\uparrow$ (The current *I* through the coil $\uparrow \rightarrow H\uparrow \rightarrow B\uparrow$)
  - $H\uparrow \rightarrow H_{\max}$, $B\uparrow \rightarrow B_{\max}$
    - ○ *B* reaches its maximum value ($B_{\max}$) when *H* reaches its maximum value ($H_{\max}$) (Figure 8.6).
    - ○ Saturation: when the magnetic domains are all fully aligned, $H\uparrow \rightarrow B$ does not change → saturation
  - $H\downarrow \rightarrow B\downarrow \rightarrow B_{\text{res}} \rightarrow H_c$ B_{res} – residual value, H_c – coercive value
    - ○ Retentivity: the ability of a material to maintain a certain amount of magnetism after the magnetic field is removed (Figure 8.7).
    - ○ Coercive force H_C: the magnetomotive force (*H*) required to return the value of the flux density (*B*) to zero.

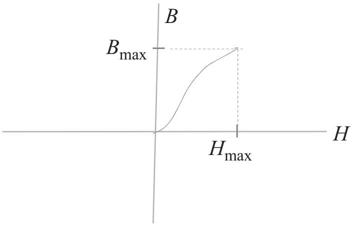


Figure 8.6 Development of a hysteresis loop

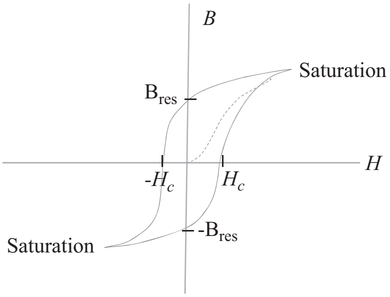


Figure 8.7 Hysteresis loop

- Negative cycle: When the current through the coil is in the reverse direction, the negative cycle repeats (Figure 8.7).
  - $-H\uparrow\ \rightarrow -H_{\max}$
  - $-B\uparrow\ \rightarrow -B_{\max}$
  - $-H\downarrow\ \rightarrow -B\downarrow\ \rightarrow -B_{\mathrm{res}} \rightarrow -H_{\mathrm{c}}$

Summary

Magnetism

- Magnet: a piece of iron (or steel, alloy ...) that has the ability to attract another metal object.
- Permanent magnet: a magnet that retains its magnetism over a long period time after it is removed from a magnetic field.
- Magnetism: the attraction or repulsion properties of a magnet.
- Magnetic poles: a magnet has two areas of strongest force, called poles.
  - Every magnet has a North and South Pole (N and S).
  - The basic law of magnet: like poles repel, unlike poles attract.
- Magnetic field: a place near a magnet or a moving electric charge where magnetic properties are produced.
- Electromagnetic force: a force between charged objects around their electric and magnetic fields.
- Magnetic field lines (the lines of force): the imaginary lines around a magnet that describe the directions of the magnetic field.
  - Outside: the magnetic field lines travel from the North Pole (N) to the South Pole (S).
  - Inside: the magnetic field lines travel from the South Pole to the North Pole.

Magnetic flux and magnetic flux density

- Magnetic flux (ϕ): a measure of the amount of magnetic field lines passing through a surface area in a magnetic field.
- Magnetic flux density (B): a measure of the amount of magnetic flux in an area.
- Calculating magnetic flux density:
  - $\boxed{\text{Magnetic flux density} = \dfrac{\text{Magnetic flux}}{\text{Area}}}$ $\boxed{B = \dfrac{\phi}{A}}$
  - Units: $$\boxed{\underset{\text{Tesla (T) or Gauss (G)}}{B} = \frac{\overset{\text{Weber (Wb)}}{\phi}}{\underset{\text{Meter}^2\ (\text{m}^2)}{A}}}$$

Domain theory of magnetism

- Magnetized material: any material when placed in a magnetic field can get magnetized itself.
- Ferromagnetism*:* the physical phenomenon in which certain materials (like iron) can become permanent magnets when subjected to a magnetic field.
- Ferromagnetic materials: materials that can maintain their magnetic properties when the magnetic field is removed.
- Domain theory of magnetism: it states that inside a magnet all the atoms are aligned in the same directions.
- Magnetic domain: a small region in which the magnetic fields of atoms are grouped together and aligned.

Charging and electric field

- Electric charge (or charge): the basic properties of particles (electrons, protons, etc.) in matter.
  - Protons are positively (+) charged.
  - Electrons are negatively (−) charged.
- Like charges repel and unlike charges attract.
- Charging: a transfer of electrons between the two objects.
  - The object that loses electrons has an excess (extra) of positive charge.
  - The object that gains electrons has an excess of negative charge.
  - $$\begin{cases} \text{The total charge of an object} = 0\text{: the number of electrons} \\ \quad = \text{the number of protons} \\ \text{The total charge of an object} = +\text{: the object loses electrons} \\ \text{The total charge of an object} = -\text{: the object gains electrons} \end{cases}$$
- Charging by conduction and induction:
  - Conduction: transfer of charge from one object to another by direct *contact*.
  - Induction: transfer of charge from one object to another *without* direct *contact*.

Electromagnetism

- Static electricity: an accumulation of an electric charge on the surface of an object.
- Static discharge: the release of static electric charge.
- Law of conservation of electric charge: the electric charge cannot be created or destroyed, but it can be transferred from one form to another.
- Electric field: the area near a charged object experiences electric forces that fill the area.
- Electric field lines: the imaginary lines around a charged object that describe the electric field in an area. They begin as positive charges and end as negative charges.
- Electric current (or a moving electric charge) can produce a magnetic field.
- Electromagnetism: a magnetic field that is created by an electric current.
- Electromagnetic induction: moving a loop of wire through a magnetic field, or moving a magnetic field relative to a coil will produce an electric current.
- Right-hand rule: a memory aid used to remember the directions of current and the magnetic field around a wire.
  - The thumb: the direction of the positive current.
  - The fingers: the direction of the magnetic field. (The fingers of your hand circle the wire.)

Permeability

- Permeability (μ): the measure of the ability of a magnetic material to respond magnetic field development.
- Calculating relative permeability:
  - $\boxed{\text{Relative permeability} = \frac{\text{Absolute permeability}}{\text{Permeability of free space}}}$ $\boxed{\mu_r = \frac{\mu}{\mu_0}}$
  - Permeability of $\underline{\text{free space}}$ μ_0: $\mu_0 = 4\,\pi \times 10^{-7}$ Weber/Area · meter (Wb/A · m) or Henry/Meter (H/m)

    (permeability of a vacuum)
  - Units:

    Weber/Ampere · meter (Wb/A · m)

    No unit $\boxed{\mu_r = \frac{\mu}{\mu_0}}$

    Weber/Ampere · meter (Wb/A · m)
- Reluctance ($\Re$): the opposition offered by a magnetic circuit to the magnetic flux.

Ohm's law for magnetic circuits

- Magnetomotive force (F_m): a force that is the cause of a magnetic flux in a magnetic circuit.
- Calculating magnetomotive force:
  - $\boxed{\text{Magnetomotive force} = (\text{Number of turns of wire})(\text{Current})}$

    $\boxed{F_m = NI}$

- Units:

Turn (t)

$$\boxed{F_{\text{m}} = NI}$$

Ampere-turn (A · t) Ampere (A)

- Factors affecting the flux density produced by a coil:
  - The permeability of the core
  - Magnetomotive force
  - The length of the coil.
- Calculating flux density:
  - $$\boxed{\text{Flux density} = \frac{(\text{Absolute permeability})\ (\text{Magnetomotive force})}{\text{Length of the coil}}}$$

$$\boxed{B = \frac{\mu F_{\text{m}}}{\ell}}$$

Weber/Ampere · meter (Wb/A · *m*)

  - Units:

Ampere-turn (A · t)

$$\boxed{B = \frac{\mu F_{\text{m}}}{l}}$$

Tesla (T) Meter (m)

- Ohm's law for magnetic circuits:
  - Ohm's law: $\boxed{\text{Flux} = \frac{\text{Magnetomotive force}}{\text{Reluctance}}}$ $\boxed{\phi = \frac{F_{\text{m}}}{\Re}}$
  - Units:

Ampere-turn (A · t)

$$\boxed{\phi = \frac{F_{\text{m}}}{\Re}}$$

Weber (Wb) Ampere · turn/Weber (A · t/Wb)

- Basic electric and magnetic quantities and Ohm's law:

Electric circuit	**Magnetic circuit**
Electromotive force (emf)	Magnetomotive force (F_{m})
Current (I)	Flux (ϕ)
Resistance (R)	Reluctance ($\Re$)
Ohm's law: $I = \frac{E}{R}$	Ohm's law: $\phi = \frac{F_{\text{m}}}{\Re}$

Magnetic hysteresis

- Magnetic field intensity or magnetizing force (H): a measure of the actual magnetic field within a material.

- Calculating magnetic field intensity:
  - $\boxed{\text{Magnetic field intensity } (H) = \frac{\text{Magnetomotive force } (F_{\text{m}})}{\text{length of the coil } (\ell)}}$ $\boxed{H = \frac{F_{\text{m}}}{\ell}}$
  - Units:

Ampere-turn (A · t)

Ampere-turn (A · t)/Meter (m) $\boxed{H = \frac{F_{\text{m}}}{l}}$

Meter (m)

- Hysteresis: a lag between cause and effect (or a lag between an input and an output).
- Magnetic hysteresis: the phenomenon of changes of flux density *B* lagging behind the magnetizing force *H* in a ferromagnetic material, such as iron.
- Hysteresis curve (or hysteresis loop) shows the relationship between the induced magnetic flux density (*B*) and the magnetizing force (*H*).

Practice problems

8.1

1. ( ) force is a force between charged objects around their electric and magnetic fields.
2. ( ) is a place near a magnet or a moving electric charge where magnetic properties are produced.
3. The magnetic field lines travel from the South Pole to the North Pole ( ) the magnet.
4. Like poles ( ), unlike poles attract.
5. Magnetic flux is a measure of the amount of magnetic field ( ) passing through a surface area in a magnetic field.
6. Magnetic flux ( ) is a measure of the amount of magnetic flux in an area.
7. The unit of magnetic flux density is the ( ).
8. ( ) is the physical phenomenon in which certain materials can become permanent magnets when subjected to a magnetic field.
9. ( ) theory of magnetism states that inside a magnet all the atoms are aligned in the same directions.
10. A magnetic ( ) is a small region in which the magnetic fields of atoms are grouped together and aligned.
11. The amount of flux present in a round magnetic bar was measured at 0.016 Wb. If the material has a diameter of 20 cm, what is the flux density?

8.2

12. Charging is a transfer of ( ) between the two objects.
13. Charging by induction is transfer of charge from one object to another ( ) direct contact.

14. ( ) electricity is an accumulation of an electric charge on the surface of an object.
15. An electric ( ) is the area near a charged object experiences electric forces that fill the area.
16. Electric current (or a moving electric charge) can produce a ( ) field.
17. ( ) is a magnetic field that is created by an electric current.
18. Moving a loop of wire through a magnetic field, or moving a magnetic field relative to a coil will produce an electric ( ).
19. Electric motor is a device that converts ( ) energy into mechanical energy.

8.3

20. ( ) is analogous to the resistance in an electrical circuit.
21. Ohm's law for magnetic circuits gives the relationship between flux, magnetomotive force, and ( ).
22. The unit of magnetomotive force is the ( ).
23. The absolute permeability of a piece of steel is 3.2×10^{-9} Wb/A·m. Calculate the relative permeability of this steel material.
24. What is the magnetomotive force in a 100 turn coil of wire when there are 2.4 A of current through it?
25. There is 0.2 A of current through an air core coil of wire with 120 turns and a length of 220 cm. Determine the flux density for the coil.
26. There is 0.28 A of current through a coil of wire with 400 turns. Determine the reluctance of the circuit if the magnetic flux is 0.34 μWb.

8.4

27. Magnetic field ( ) is a measure of the actual magnetic field within a material.
28. Magnetic hysteresis is the phenomenon of changes of ( ) B lagging behind the magnetizing force H in a ferromagnetic material.
29. Hysteresis ( ) shows the relationship between the induced magnetic flux density (B) and the magnetizing force (H).
30. There is 0.5 A of current through a coil of wire with 180 turns. If the length of the magnetic circuit is 15 cm, determine the magnetic field intensity.

Chapter 9

Fundamentals of AC circuits

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9.1 Introduction to alternating current (AC)

9.1.1 The difference between DC and AC

DC (direct current)

- The DC power supply provides a constant voltage and current, hence all resulting voltages and currents in DC circuit are constant and do not change with time.
- The polarity of DC voltage and direction of DC current do not change, only their magnitude changes.
- Before the nineteenth century, the DC power supply was the main form of electrical energy to provide electricity.

AC (alternating current)

- An alternating voltage is called AC voltage and alternating current is called AC current.
- The AC voltage alternates its polarity and the AC current alternates its direction periodically.
- Since the AC power supply provides an alternating voltage and current, the resulting currents and voltages in AC circuit also periodically switch their polarities and directions.

Advantages of AC

- In the nineteenth century, DC and AC have had constant competition, AC gradually showed its advantages and rapidly developed in the latter of the nineteenth century, and is still commonly used in current industries, businesses, and homes throughout the world.
- This is because the AC power can be more cost-effective for long-distance transmission from power plants to industrial, commercial, or residential areas.
- This is why power transmission for electricity today is nearly all AC. It is also easy to convert from AC to DC, allowing for a wide range of applications.

9.1.2 DC waveforms

DC voltage and current

- The DC voltage and current do not change their polarity or direction over time and only their magnitude changes.
- A DC waveform (a graph of voltage and current versus time) is shown in Figure 9.1.

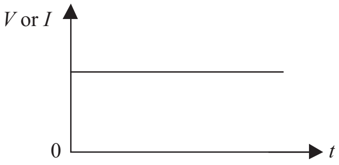


Figure 9.1 DC waveform

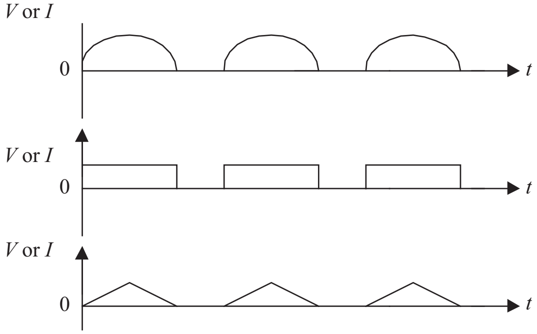


Figure 9.2 Pulsing DC waveforms

Pulsing DC

There is also a type of DC waveform known as the pulsing DC, in which

- the amplitude of DC pulse changes periodically from zero to the maximum with time.
- its polarity or direction do not change with time (always above zero), so it still belongs to the DC category.
- Figure 9.2 shows some pulsating DC waveforms.

Directcurrent (DC)	– The polarity of DC voltage and direction of DC current do not change. – The pulsing DC changes pulse amplitude periodically, but the polarity does not change.

9.1.3 AC waveforms

AC waveforms

- AC voltage and current periodically change polarity or direction with time. A few examples of AC waveforms are shown in Figure 9.3.
- The sinusoidal or sine AC wave is the most basic and widely used AC waveform, and is often referred to as AC, although other waveforms such as square wave, triangle wave, etc. also belong to AC.

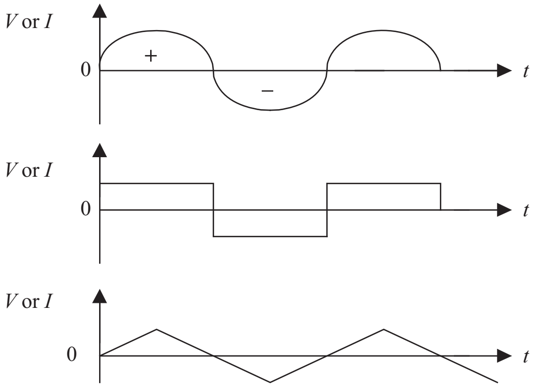


Figure 9.3 AC waveforms

Sine AC wave

- The sine AC wave energy is the type of power that is generated by the utility power industries around the world.
- AC voltage and current vary with sine (or we could use cosine by adding 90° to the sine wave) function, the symbol of AC source is —(∿)—.
- AC quantities are represented by lowercase letters (*e, v, i*, etc.) and DC quantities use uppercase letters (*E*, *V*, *I*, etc.).

Alternating current (AC)	– The polarity of voltage and direction of AC current periodically change with time (such as sine wave, square wave, sawtooth wave, etc.). – Sine AC (or AC) varies over time according to sine (or cosine) function, and is the most widely used AC.

- A sine function can be described as a mathematical expression of $f(t) = F_m \sin(\omega t + \psi)$. This is the expression of sine function in the time domain (the quantity versus time).
- Applying the expression of sine function to electrical quantities will obtain general expressions of AC voltage and current as follows.
  - Sinusoidal voltage: $v(t) = V_m \sin(\omega t + \psi)$
  - Sinusoidal current: $i(t) = I_m \sin(\omega t + \psi)$

9.1.4 Period and frequency

Period and frequency

- The waveform of a sinusoidal function is shown in Figure 9.4.
- Period *T*: the time to complete one full cycle of the waveform, or the positive and negative alternations of one revolution.

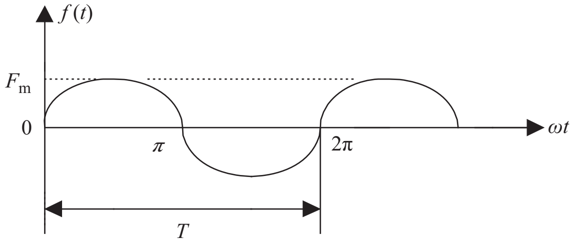


Figure 9.4 Sinusoidal waveform

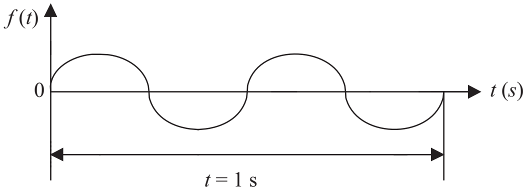


Figure 9.5 Frequency of sine waveform

- Frequency f: the number of cycles of waveforms within 1 s. The frequency is measured in Hertz (Hz).

For instance, in Figure 9.5, the number of complete cycles in 1 s is 2, so it has a frequency of 2 Hz.

Calculating period and frequency

- Relationship of T and f: The frequency f of the waveform is inversely proportional to the period T of the waveform, that is, $f = \frac{1}{T}$.

Period and frequency	– Period T: the time to complete one full cycle – Frequency f: number of cycles per second – $f = \frac{1}{T}$

Calculating frequency

- $\boxed{\text{Frequency} = \frac{1}{\text{Period}}}$ or $\boxed{f = \frac{1}{T}}$
- Units: Hertz (Hz) $\boxed{f = \frac{1}{T}}$ Second (s)

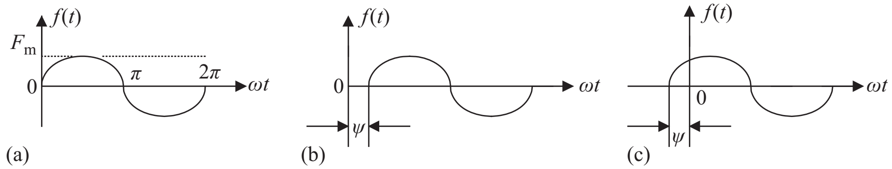


Figure 9.6 The peak value and phase of the sine wave

9.1.5 The peak value and angular velocity of a sine function

The peak value of a sine wave

- There are three important components in the expression of the sine function $f(t) = F_m \sin(\omega t + \psi)$: peak value F_m, angular velocity ω, and phase shift ψ.
- Peak value F_m:
  - F_m is the peak value or amplitude of the sine wave (I_m for current or V_m for voltage).
  - F_m is the distance from zero of the horizontal axis to the maximum point (positive or negative) that a waveform can reach during its entire cycle (Figure 9.6(a)).

The angular velocity of a sine wave

- Angular velocity ω (the Greek letter omega):
  - Angular velocity or angular frequency of a sine wave reflects the rate of change of the rotation of the wave.
  - Angular velocity = rotating distance/time

    (same with the linear motion: velocity = distance/time)
- Since the time required for a sine wave to complete one cycle is period T, the distance of one cycle is 2π as shown in Figure 9.4, so the angular velocity can be determined by: $\omega = \frac{2\pi}{T}$
- The relationship between the angular velocity and frequency is:

$$\boxed{\omega = \frac{2\pi}{T} = 2\pi f} \qquad \left(f = \frac{1}{T}\right)$$

So, the angular velocity is directly proportional to the frequency, this is also called the angular frequency.

9.1.6 The phase of a sine function

The phase shift ψ of a sine wave

- Phase shift or phase ψ (the Greek letter phi): an angle that represents the position of the wave shifted from a reference point at the vertical axis (0°).
- A sine wave may shift to the left or right of 0°. The range of phase shift is between $-\pi$ and $+\pi$.

$\psi = 0$, $\psi < 0$, and $\psi > 0$

- If phase shift $\psi = 0$, the waveform of sine function $f(t) = F_m \sin(\omega t + 0)$ or $f(t) = F_m \sin\omega t$ starts from $t = 0$ as shown in Figure 9.6(a).
- If phase shift ψ has a negative value ($\psi < 0$), the waveform of sine function $f(t) = F_m \sin(\omega t - \psi)$ will shift to the right side of 0° as shown in Figure 9.6(b).
- If phase shift ψ has a positive value ($\psi > 0$), the waveform of sine function $f(t) = F_m \sin(\omega t + \psi)$ will shift to the left side of 0° as shown in Figure 9.6(c).

Three important components of a sine function	$f(t) = F_m \sin(\omega t + \psi)$ – F_m: Peak value (amplitude) – ω: Angular velocity or angular frequency $\omega = \frac{2\pi}{T} = 2\pi f \quad (\pi = 180^\circ)$ – ψ: Phase or phase shift $\psi > 0$: waveform shifted to the left side of 0° $\psi < 0$: waveform shifted to the right side of 0°

Units:

Radian/second

- $\boxed{f(t) = F_m \sin(\omega t + \psi)}$

Radian or degree (rad or °)

- $\boxed{\omega = \frac{2\pi}{T} = 2\pi f}$

Hertz (Hz)

Second (s)

9.1.7 *An example of a sine voltage*

Example 9.1: Given a sinusoidal voltage $v(t) = 6 \sin(25t - 30^\circ)$ V, determine its peak voltage, phase angle, and frequency, and plot its waveform.

Solution:

- Peak value: $V_m = \boxed{6\text{ V}}$
- Phase: $\psi = \boxed{-30^\circ}$ $\quad (\because \psi < 0$, waveform shifted to the right side of $0^\circ)$

- Frequency: $f = \dfrac{1}{T}$

$$\because \omega = \frac{2\pi}{T}, \qquad \text{and} \qquad \omega = 25 \text{ rad/s}$$

$$\because T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{25 \text{ rad/s}} \approx 0.25 \text{ s}$$

$$\text{and} \quad f = \frac{1}{T} = \frac{1}{0.25 \text{ s}} = \boxed{4 \text{ Hz}}$$

- The waveform is shown below:

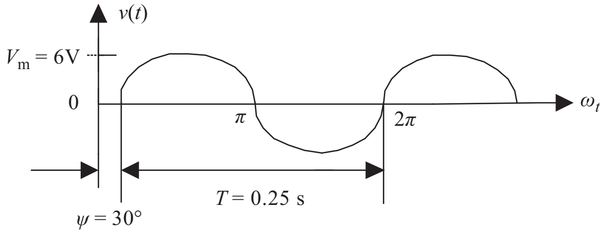


Figure 9.7 Waveform for Example 9.1

9.1.8 Phase difference of the sine function

Phase difference φ

- For two different sine waves with the same frequency, the angular displacement of their phases is called phase difference and denoted by φ (lowercase Greek letter phi).
- Phase difference is a phase angle by which one wave leads or lags another.

For instance, given the general expressions of sinusoidal voltage and current as

$$v(t) = V_m \sin(\omega t + \psi_v) \qquad \text{and} \qquad i(t) = V_m \sin(\omega t + \psi_i)$$

The phase difference between voltage and current is

$$\varphi = (\omega t + \psi_v) - (\omega t + \psi_i) = \psi_v - \psi_i$$

$\varphi = 0$, $\varphi > 0$, and $\varphi < 0$

- If $\varphi = \psi_v - \psi_i = 0$, the two waveforms are in phase as shown in Figure 9.8(a).
- If $\varphi = \psi_v - \psi_i > 0$, voltage leads current, or current lags voltage as shown in Figure 9.8(b).
- If $\varphi = \psi_v - \psi_i < 0$, current leads voltage, or voltage lags current, as shown in Figure 9.8(c).

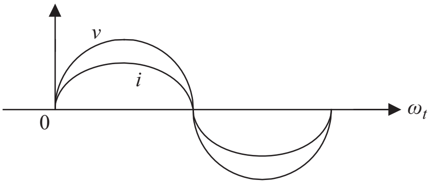


Figure 9.8(a) Two waveforms are in phase

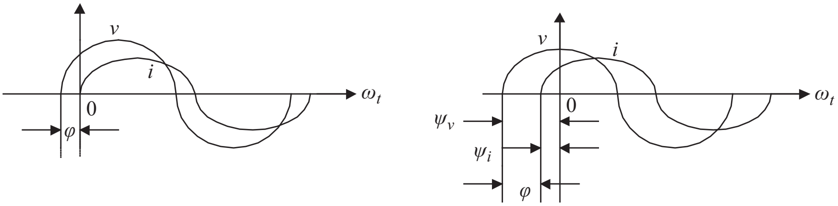

v
i
0
φ
ω_t
v
i
0
ψ_v
ψ_i
φ
ω_t

Figure 9.8(b) Current lags voltage

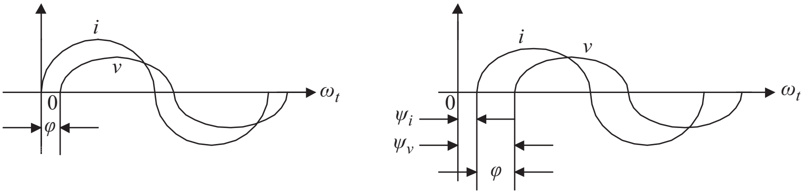

i
v
0
φ
ω_t
i
v
0
ψ_i
ψ_v
φ
ω_t

Figure 9.8(c) Current leads voltage

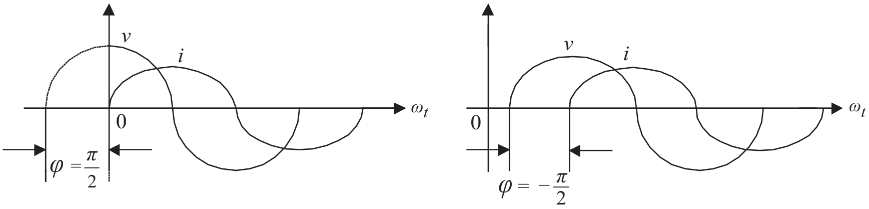

v
i
0
$\varphi = \frac{\pi}{2}$
ω_t
v
i
0
$\varphi = -\frac{\pi}{2}$
ω_t

Figure 9.8(d) Voltage and current are orthogonal

$\boldsymbol{\varphi = \pm\frac{\pi}{2}, \varphi > 0}$, and $\boldsymbol{\varphi < 0}$

- If $\varphi = \psi_\mathrm{v} - \psi_\mathrm{i} = \pm\frac{\pi}{2}$ (or ±90°), then voltage and current are orthogonal, or is a right angle. It is shown in Figure 9.8(d).
  (The Greek *orthos* means "straight", and *gonia* means "angle".)
- If $\varphi = \psi_\mathrm{v} - \psi_\mathrm{i} = \pm\pi$ (or ±180°), voltage and current are 180 degrees out of phase as shown in Figure 9.8(e).

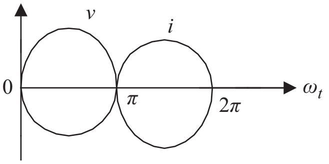


Figure 9.8(e) Voltage and current are out of phase

Phase difference φ $\varphi = \psi_v - \psi_i$	For two waves with the same frequency such as $v(t) = V_m \sin(\omega t + \psi_v), \quad i(t) = I_m \sin(\omega t + \psi_i)$ – If $\varphi = 0$: v and i in phase – If $\varphi > 0$: v leads i – If $\varphi < 0$: v lags i – If $\varphi = \pm\frac{\pi}{2}$: v and i are orthogonal – If $\varphi = \pm\pi$: v and i are 180 degrees out of phase

9.1.9 An example of phase difference

Example 9.2: Determine the phase difference of the following functions and plot their waveforms.

(a) $v(t) = 20 \sin(\omega t + 30^{\circ})$ V, $i(t) = 12 \sin(\omega t + 60^{\circ})$ A

(b) $v(t) = 5 \sin(\omega t + 60^{\circ})$ V, $i(t) = 2.5 \sin(\omega t + 20^{\circ})$ A

Solution:

(a) $\varphi = \psi_v - \psi_i = 30^{\circ} - 60^{\circ} = 30^{\circ} < 0$

So, voltage lags current by 30° as shown in Figure 9.9(a).

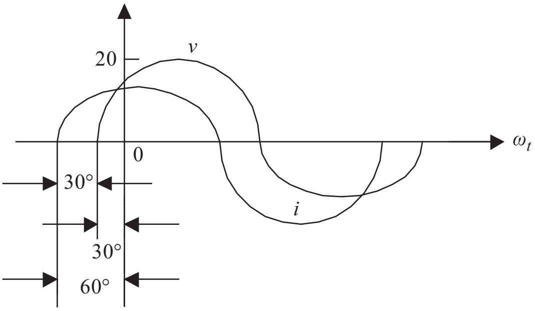


Figure 9.9(a) Figure for Example 9.2(a)

(b) $\varphi = \psi_v - \psi_i = 60° - 20° = 40° > 0$

So voltage leads current by 40° as shown in Figure 9.9(b).

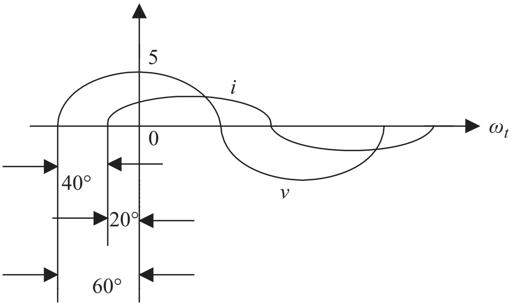


Figure 9.9(b) Figure for Example 9.2(b)

9.2 Sinusoidal AC quantity

9.2.1 Peak value and peak–peak value

AC voltage or current can be described in a number of ways

- A sinusoidal AC quantity such as AC voltage or current can be described in a number of ways. They can be described by their peak value, peak–peak value, instantaneous value, average value or rms (root-mean-square) value.
- The different expressions will provide different ways to analyze the sinusoidal AC quantity, and it is also because a sinusoidal wave always varies periodically and there is no one single value that can truly describe it.

Peak value F_{pk}

- The peak value is the amplitude or maximum value F_m in sine function $f(t) = F_m \sin(\omega t + \psi)$.
- The peak value is denoted by F_{pk} as shown in Figure 9.10.

Peak–peak value $F_{p\text{-}p}$

- The peak–peak value $F_{p\text{-}p}$ represents the distance from negative to positive peak, or minimum to maximum peak, or between peak and trough of the waveform.
- The peak–peak value $F_{p\text{-}p} = 2F_{pk}$ as shown in Figure 9.10.

(To determine the maximum values that electrical equipment can withstand, the peak values or peak–peak values of the AC quantities should be considered.)

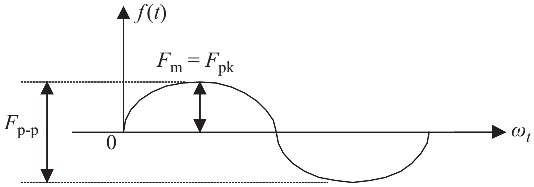


Figure 9.10 Peak and peak–peak value

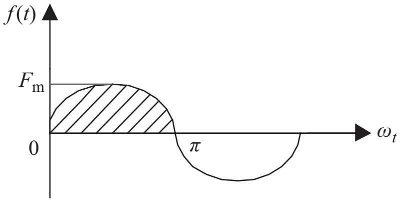


Figure 9.11 Average value

9.2.2 Average value

Average value is defined as the average of f(t)'s half-cycle

- Because of the symmetry of the sinusoidal waveform, its average value in a complete full cycle is always zero.
- For a sinusoidal function $f(t) = F_m \sin(\omega t + \psi)$, its average value is defined as the average of its half-cycle (0 to π), as shown in Figures 9.11 and 9.12.

Derive average value

- The average value of a half-cycle sinusoidal wave with a zero phase shift can be derived by using integration as follows:
- Note: If you have not learned calculus, then just keep in mind that $F_{Avg} = 0.637F_m$ is the equation for the average value of a half-cycle sinusoidal wave, and skip the following mathematic derivation process.

$$\begin{aligned} F_{Avg} &= \frac{\text{Area}}{\pi} = \frac{1}{\pi}\int_0^\pi f(t)\mathrm{d}t = \frac{1}{\pi}\int_0^\pi F_m \sin \omega t \mathrm{d}\omega t \\ &= \frac{F_m}{\pi}[-\cos \omega t]_0^\pi = -\frac{F_m}{\pi}[\cos \pi - \cos 0] \\ &= -\frac{F_m}{\pi}(-1-1) = \frac{2F_m}{\pi} \approx 0.637F_m \end{aligned}$$

i.e., $\boxed{F_{Avg} = 0.637F_m}$

- Therefore, the average value of a half-cycle sinusoidal wave is 0.637 times the peak value, as shown in Figure 9.12.

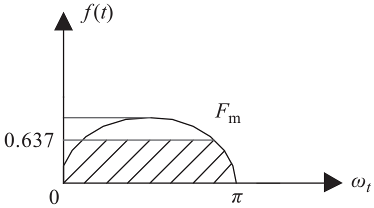


Figure 9.12 Average value

9.2.3 Instantaneous value

Instantaneous value of the sinusoidal waveform

- The instantaneous value of the sinusoidal waveform $f(t)$ varies with time, and it is the value at any instant time t (or ωt) in any particular point of a waveform.
- Instantaneous values of the variables are denoted by lowercase letters, such as voltage v, current i, etc.

Example 9.3: Given a sinusoidal AC voltage $v(t) = V_m \sin\omega t$ as shown in Figure 9.13, determine the instantaneous voltage v_1 (voltage at 30°) and v_2 (voltage at 135°) when $V_m = 5$ V.

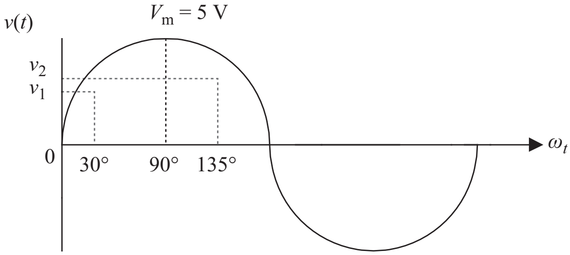


Figure 9.13 Figure for Example 9.3

Solution:

$$v_1 = V_m \sin\omega t = 5\sin 30^\circ = 2.5\ \text{V}$$
$$v_2 = V_m \sin\omega t = 5\sin 135^\circ \approx 3.54\ \text{V}$$

Peak value, peak–peak value, instantaneous value, and average value	For a sinusoidal waveform: – Peak value $F_{pk} = F_m$: the amplitude or maximum value – Peak–peak value $F_{p\text{-}p}$: $F_{p\text{-}p} = 2F_{pk}$ – Instantaneous value *f(t)*: the value at any time in any particular point of the waveform – Average value F_{Avg}: $F_{Avg} = 0.637F_m$

9.2.4 RMS (root-mean-square) value

Applications of RMS value

- RMS value (also referred to as the effective value) of the sinusoidal waveform is widely used in practice.
- For example, the values measured and displayed on instruments and the nominal ratings of the electrical equipment are rms values.
- In North America, the single-phase AC voltage 110 V from the wall outlet is a RMS value.

The physical meaning of RMS value

- For a sinusoidal waveform, the physical meaning of the AC RMS value is the heating effect of the sine wave.
- An AC source RMS value will deliver the equivalent amount of average power to a load as a DC source.
- For instance, whether turning on the switch 1 (connect to DC) or switch 2 (connect to AC) in Figure 9.14, 20 V DC or 20 V AC RMS value will deliver the same amount of power (40 W) to the resistor (lamp).
- If the lamp is replaced by an electric heater, then the heating effect delivered by 20 V DC and 20 V AC RMS will be the same.

9.2.5 Quantitative analysis of RMS value

The average power of AC

- The average power generated by an AC power supply is:

$$p_{\mathrm{ac}} = {i_{\mathrm{ac}}}^2 R = (I_{\mathrm{m}} \sin\omega t)^2 R = ({I_{\mathrm{m}}}^2 \sin^2\omega t) R$$

$$p_{\mathrm{ac}} = {I_{\mathrm{m}}}^2 \left[\frac{1}{2}(1 - \cos 2\omega t)\right] R = \frac{{I_{\mathrm{m}}}^2 \mathrm{R}}{2} - \frac{{I_{\mathrm{m}}}^2 \mathrm{R}}{2} \cos 2\omega t \qquad \sin^2\omega t = \frac{1}{2}(1 - \cos 2\omega t)$$

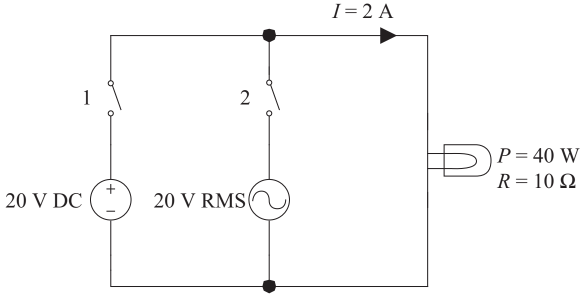


Figure 9.14 RMS value

- Only the first part in the above power expression represents the average power of AC, since the average value of the second part in the power expression (a cosine function) is zero, i.e., $\boxed{P_{\text{ac}} = \frac{{I_{\text{m}}}^2 R}{2}}$
- The average power generated by DC voltage is $\boxed{P_{\text{avg}} = I^2 R}$

RMS value of AC current

- According to the physical meaning of RMS, the average AC power is equivalent to the average DC power when the AC source is a RMS value, so

$$\frac{{I_{\text{m}}}^2 R}{2} = I^2 R \quad \text{or} \quad I^2 = \frac{{I_{\text{m}}}^2}{2}$$

- Taking square root on both sides of equation gives,

$$I = \sqrt{\frac{{I_{\text{m}}}^2}{2}} = \frac{I_{\text{m}}}{\sqrt{2}} \approx 0.707 I_{\text{m}} \quad \text{or} \quad \boxed{I_{\text{m}} = \sqrt{2} I \approx 1.414 I} \tag{9.1}$$

- The current I in the above equation is the RMS value of the AC current, and I_{m} is the peak value or amplitude of the AC current.

RMS value of AC voltage

- RMS value of AC voltage can be obtained in the same approach by obtaining the RMS value of the AC current, i.e.,

$$V = \frac{V_{\text{m}}}{\sqrt{2}} = 0.707 V_{\text{m}} \quad \text{or} \quad \boxed{V_{\text{m}} = \sqrt{2}\,\text{V} = 1.414\,\text{V}} \tag{9.2}$$

- The voltage V in (9.2) is the RMS value of AC voltage, and V_{m} is the peak value or amplitude of the AC voltage.

9.2.6 RMS value of a periodical function

The RMS value of a non-sine wave function

- Equations (9.1) and (9.2) indicate the relationship between the RMS value and the peak value of a sine wave, which is related by $\sqrt{2}$.
- That relation only applies to the sine wave. For a non-sine wave function $f(t)$, the following general equation can be used to determine its RMS value:

$$\boxed{F = \sqrt{\frac{1}{T}\int_0^{\text{T}} f^2(t)\,\mathrm{d}t}} \qquad (T \text{ is the period of the function})$$

Root-mean-square

The name root-mean-square (RMS) is obtained from the above equation, in which the term of

- $\sqrt{\ }$ denotes square root (**r**oot).
- $1/T$ denotes the average (**m**ean).
- $f^2(t)$ denotes square (**s**quare).

RMS value of AC function	– RMS value or effective value of AC: an AC source with RMS value will deliver the equivalent amount of power to a load as a DC source. – $V = 0.707V_m, \quad I = 0.707I_m$ or $V_m = \sqrt{2}\,V, \ I_m = \sqrt{2}I$ – The general equation to calculate RMS value: $F = \sqrt{\frac{1}{T}\int_0^T f^2(t)\mathrm{d}t}$

9.3 Phasors

9.3.1 Introduction to phasor notation

A phasor

- A phasor is a vector that contains both magnitude and direction or amplitude and phase information.
- A phasor can be used to represent AC quantities. Since phasors have magnitudes and directions, they can be represented as complex numbers.

A phasor notation

- A phasor notation or phasor-domain is a method that uses complex numbers to represent the sinusoidal quantities for analyzing AC circuits.

  Charles Proteus Steinmetz, a German-American mathematician and electrical engineer, developed the phasor notation in 1893.
- A phasor notation can represent sine waves in terms of their peak value (magnitude) and phase angle (direction). The peak value can be easily converted to the RMS value.
- The phasor notation can simplify the calculations for AC sinusoidal circuits, therefore, it is widely used in circuit analysis and calculations.

The same frequency

- The phasor notation can be used for sinusoidal quantities only when all waveforms have the same frequency.
- In an AC circuit, AC source voltage and the current are the sinusoidal values with the same frequency, so the resulting voltages and currents in the circuit

should also be sinusoidal values with the same frequency or angular frequency.

- Voltages and currents in an AC circuit can be analyzed by using the phasor notation, i.e., they can be determined by the peak value or RMS value and the phase shift of the phasor notation.

Phasor	– A phasor is a vector that contains both amplitude and angle information, and it can be represented as complex number. – Phasor notation is a method that uses complex numbers to represent the sinusoidal quantities for analyzing AC circuits when all quantities have the same frequency.

9.3.2 Complex numbers review

Complex numbers

- The key for understanding the phasor notation is to know how to use complex numbers. Therefore, we will review some important formulas of complex numbers that you may have learned in previous mathematics courses.
- The complex number has two main forms, the rectangular form and the polar form.

Rectangular form

- Rectangular form: $\boxed{A = x + jy}$ $(j = \sqrt{-1})$

  where x is the real part and y is the imaginary part. j is called the imaginary unit.
- The symbol i is used to represent imaginary unit in mathematics. Since i has been used to represent AC current in the circuit analysis, j is used to denote the imaginary unit rather than i to avoid confusion.

Polar form

- $\boxed{A = a\angle\psi}$ This is the abbreviated form of the exponential form $A = ae^{j\psi}$, in which a is called modulus of the complex number, and the angle ψ is called argument of the complex number.

Convert rectangular form to polar form

- Let $\boldsymbol{A} = x + jy = a\angle\psi$
- Apply $a = \sqrt{x^2 + y^2}$ Pythagorean theory

  gives $\boxed{A = x + jy = \sqrt{x^2 + Y^2}\tan^{-1}\frac{y}{x} = a\angle\,\psi}$ Refer to Figure 9.15.

Convert polar form to rectangular or triangular form

- $x = a\cos\psi$ and $y = a\sin\psi$ can be obtained from Figure 9.15.
- so, $\boxed{A = \mathrm{a}\angle\psi = x + jy = a(\cos\psi + j\sin\psi)}$

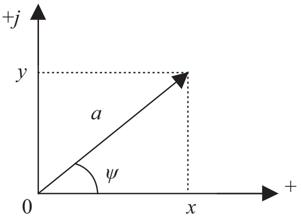


Figure 9.15 Complex number

Convert triangular form to exponential form

- Euler's formula can be used for the conversion from triangular form to exponential form:
- $e^{j\psi} = \cos\psi + j\sin\psi$ or $\boxed{ae^{j\psi} = a(\cos\ \psi + j\sin\ \psi)}$

Operations on complex numbers

- Given two complex numbers,

  $\boldsymbol{A}_1 = x_1 + jy_1 = a_1 < \psi_1$ and $\boldsymbol{A}_2 = x_2 + jy_2 = a_2 > \psi_2$
- Addition: $\boldsymbol{A}_1 + \boldsymbol{A}_2 = (x_1 + x_2) + j\ (y_1 + y_2)$
- Subtraction: $\boldsymbol{A}_1 - \boldsymbol{A}_2 = (x_1 - x_2) + j\ (y_1 - y_2)$
- Multiplication:
  - Polar form: $\boldsymbol{A}_1 \cdot \boldsymbol{A}_2 = a_1\ a_2 < (\psi_1 + \psi_2)$
  - Rectangular form:

$$\boldsymbol{A}_1 \cdot \boldsymbol{A}_2 = (x_1 + jy_1)(x_2 + jy_2) = (x_1x_2 - y_1y_2) + j(x_2y_1 + x_1y_2)$$

$$j^2 = \sqrt{-1}\sqrt{-1} = (\sqrt{-1})^2 = -1$$

- Division:
  - Polar form: $\dfrac{A_1}{A_2} = \dfrac{a_1}{a_2} \angle(\psi_1 - \psi_2)$
  - Rectangular form:

$$\frac{A_1}{A_2} = \frac{x_1 + jy_1}{x_2 + jy_2} = \frac{(x_1 + jy_1)(x_2 - jy_2)}{(x_2 + jy_2)(x_2 - jy_2)} = \frac{x_1x_2 + y_1y_2}{{x_2}^2 + {y_2}^2} + j\frac{x_2y_1 - x_1y_2}{{x_2}^2 + {y_2}^2}$$

(It will be much simpler to use the polar form on operations of multiplication and division.)

Complex numbers	– Rectangular form: $\boldsymbol{A} = x + j\,y$ – Polar form: $\boldsymbol{A} = a\angle\psi$ – Conversion between rectangular and polar forms: $\boldsymbol{A} = x + jy = \sqrt{x^2 + y^2}\tan^{-1}\frac{y}{x} = a\angle\psi$ $\boldsymbol{A} = a\angle\psi = x + jy = a(\cos\psi + j\sin\psi)$ – Addition and subtraction: $\boldsymbol{A}_1 \pm \boldsymbol{A}_2 = (x_1 \pm x_2) + j\,(y_1 \pm y_2)$ – Multiplication: $\boldsymbol{A}_1 \cdot \boldsymbol{A}_2 = a_1\,a_2 < (\psi_1 + \psi_2) = (x_1 + jy_1)\,(x_2 + jy_2)$ – Division: $\frac{A_1}{A_2} = \frac{a_1}{a_2}\angle(\psi_1 - \psi_2) = \frac{x_1 + jy_1}{x_2 + jy_2}$

9.3.3 Phasor domain

Real part and imaginary part of complex numbers

- Using the phase notation to represent the sinusoidal function is based on Euler's formula $e^{j\varphi} = \cos\varphi + j\sin\varphi$.
- For a sinusoidal function $f(t) = F_m \sin(\omega t + \psi)$, replacing φ with $(\omega t + \psi)$ in Euler's formula gives,

$$e^{j(\omega t+\psi)} = \cos(\omega t + \psi) + j\sin(\omega t + \psi)$$

where $\cos(\omega t + \psi) = \mathrm{R_e}\left[e^{j(\omega t+\psi)}\right]$

and $\sin(\omega t + \psi) = \mathrm{J_m}\left[e^{j(\omega t+\psi)}\right]$

- "$\mathrm{R_e}$ [ ]" and "$\mathrm{J_m}$ [ ]" stand for "real part" and "imaginary part" of the complex numbers, respectively.

The rotating factor and the phasor

- Sine function $f(t) = F_m \sin(\omega t + \psi) = \mathrm{J_m}\left[F_m e^{j(\omega t+\psi)}\right] = \mathrm{J_m}[F_m e^{j\psi} \cdot e^{j\omega t}]$

  That is, a sinusoidal function is actually taking the imaginary part of the complex number

$$\boxed{f(t) = \mathrm{J_m}[F_m e^{j\psi} \cdot e^{j\omega t}]} \tag{9.3}$$

- There are two terms in (9.3), $F_m e^{j\psi}$ and $e^{j\omega t}$.
  - The second term $e^{j\omega t}$ is called the rotating factor that varies with time t.
  - The first term is the phasor of the sinusoidal function

$$F_m e^{j\psi} = F_m\angle\psi = \boldsymbol{F}$$

- So, (9.3) of sine function can be written as:

$$f(t) = F_m \sin(\omega t + \psi) = \mathrm{J_m}[\boldsymbol{F} e^{j\omega t}]$$

- The first term in (9.3) is $\boldsymbol{F} = F_m \angle\psi$, where bold-face letter $\boldsymbol{F}$ represents a phasor (vector) quantity. (Similar to the boldface that indicates a vector quantity in mathematics and physics.)

- A phasor quantity can also be represented by a little dot on the top of the letter, such as $\dot{F} = F_{\mathrm{m}}\angle\psi$.
- There is no difference between operations on phasors and complex numbers, since both of them are vectors.

Sinusoidal currents and voltages in the phasor domain

- If the sinusoidal currents and voltages in an AC circuit are represented by vectors with the complex numbers, this is known as phasors.
- The sinusoidal voltage $v(t) = V_{\mathrm{m}}\sin(\omega t + \psi)$ and current $i(t) = I_{\mathrm{m}}\sin(\omega t + \psi)$ in an AC circuit can be expressed in the phasor domain as:
  - Peak value:

$$\dot{V} = V_{\mathrm{m}}\angle\psi_{\mathrm{v}} \quad \text{or} \quad \boldsymbol{V} = V_{\mathrm{m}}\angle\psi_{\mathrm{v}}$$

$$\dot{I} = I_{\mathrm{m}}\angle\psi_{\mathrm{i}} \quad \text{or} \quad \boldsymbol{I} = I_{\mathrm{m}}\angle\psi_{\mathrm{i}}$$

  - RMS value:

$$\dot{V} = V\angle\psi_{\mathrm{v}} \quad \text{or} \quad \boldsymbol{V} = V\angle\psi_{\mathrm{v}}$$

$$\dot{I} = I\angle\psi_{\mathrm{i}} \quad \text{or} \quad \boldsymbol{I} = I\angle\psi_{\mathrm{i}}$$

	Time domain	**Phasor domain**
Phasor	– $f(t) = F_{\mathrm{m}}\sin(\omega t + \psi)$	Peak value: $\boldsymbol{F}_{\mathbf{m}} = F_{\mathrm{m}}\angle\psi$ (or $\dot{F}_m = F_{\mathrm{m}}\angle\psi$)
		RMS value: $\boldsymbol{F} = F\angle\psi$ (or $\dot{F} = F\angle\psi$)
	– $v(t) = V_{\mathrm{m}}\sin(\omega t + \psi)$	Peak value: $\dot{V}_m = V_{\mathrm{m}}\angle\psi_v$
		RMS value: $\dot{V} = V\angle\psi_v$
	– $i(t) = I_{\mathrm{m}}\sin(\omega t + \psi)$	Peak value: $\dot{I}_m = I_{\mathrm{m}}\angle\psi_i$
		RMS value: $\dot{I} = I\angle\psi_i$

9.3.4 Phasor diagram

A phasor diagram

- Since a phasor is a vector that can be represented by a complex number, it can be presented with a rotating line in the complex plane as shown in Figure 9.16.
- The length of the phasor is the peak value F_{m} (or RMS value F). The angle between the rotating line and the positive horizontal axis is the phase angle ψ of the sinusoidal function. This diagram is called the phasor diagram.

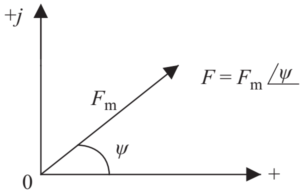


Figure 9.16 Phasor diagram

Example 9.4: Use the phasor notation to express the following voltage and current in which 12 and -10 are the peak values.

(a) $v = -10\sin(60t + 25^\circ)$ V

(b) $i = 12\sin(25t - 20^\circ)$ A

Solution:

(a) $\boxed{\dot{V}_\mathrm{m} = -10\angle 25^\circ \mathrm{V}}$

(b) $\boxed{\dot{I}_\mathrm{m} = 12\angle{-20^\circ}\ \mathrm{A}}$

Example 9.5: Use the instantaneous value to express the following voltage and current in which 120 and 12 are the RMS values.

(a) $\dot{V} = 120\angle 30^\circ$ V

(b) $\dot{I} = 12\angle 0^\circ$ A

Solution:

(a) $\boxed{v = 120\sqrt{2}\sin{(\omega t + 30)^\circ}\ \mathrm{V}}$

(b) $\boxed{i = 12\sqrt{2}\sin{\omega t}\ \mathrm{A}}$

9.3.5 Rotating factor

Rotating factor $\mathbf{e^{j\omega t}}$

- In the sinusoidal expression of $f(t) = F_\mathrm{m}\sin(\omega t + \psi) = \mathrm{J}_{\text{-m}}[F_\mathrm{m}e^{j\psi} \cdot e^{j\omega t}]$, the term "$e^{j\omega t}$" varies with time t, known as the rotating factor or time factor.
- As time changes, the rotating factor rotates counterclockwise at angular frequency ω in a radius F_m of the circle, as shown in Figure 9.17.

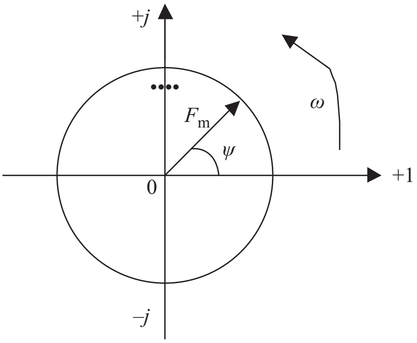


Figure 9.17 Rotating factor

- The rotating factor $e^{j\omega t}$ can be represented by Euler's formula

$$e^{j\omega t} = \cos\ \omega t + j \sin\ \omega t$$

$$\text{when}\ \ \omega t = \pm 90°:\ \ e^{\pm j90°} = \cos(\pm 90°) + j\sin(\pm 90°) = \pm j$$

$$\cos(\pm 90°) = 0,\ \ \sin(\pm 90°) = 1$$

Therefore, $\pm j$ is also the rotating factors ($\pm j = \pm 90°$).

Rotating factor	$e^{j\omega t}$ or $\pm j = \pm 90°$

Sine wave and phasors

- A sinusoidal function can be represented by a rotating phasor that rotates in 360° in a complex plane as shown in Figure 9.18.
- The instantaneous value of the sinusoidal wave at any time is equal to the projection of its relative rotating phasor on the vertical axis (*j*) at that time.

The geometric meaning of the sinusoidal function $f(t) = F_m \sin(\omega t + \psi) = J_m [\boldsymbol{F}_m e^{j\psi} \cdot e^{j\omega t}]$ represented by the rotational phasor motion can be seen from the following example.

Example 9.6:

In Figure 9.18,

- when $t = t_0 = 0$, the phasor is $\boldsymbol{F} = F_m \angle\psi$
- when $t = t_1$, the phasor is $\boldsymbol{F} = F_m \angle 90°$
- It goes from ψ to 360°.

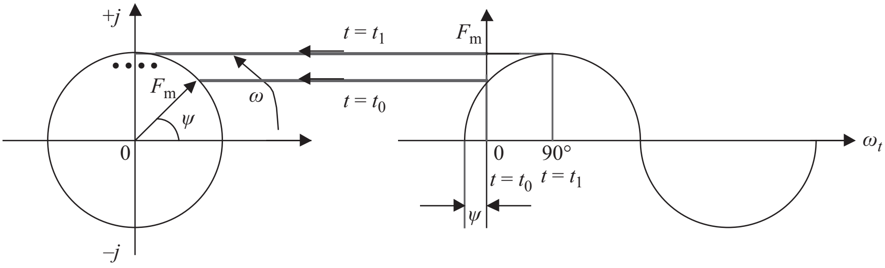


Figure 9.18 Sine wave and phasor motion

9.3.6 Differentiation and integration of the phasor

Note: Skip the following part and start from Example 9.8 if you have not learned calculus.

Differentiation of the phasor

- For a sinusoidal function $f(t) = F_m \sin(\omega t + \psi)$, the derivative of the sinusoidal function with respect to time can be obtained by its phasor $\boldsymbol{F}$ multiplying with $j\omega$,

  i.e., $\frac{df(t)}{dt} \Leftrightarrow j\omega F$

- This is equivalent to a phasor that rotates counterclockwise by 90° on the complex plane (since $+j = +90°$).

  (Appendix B provides the details for how to derive the above differentiation of the sinusoidal function in the phasor notation.)

Integration of the phasor

- The integral of the sinusoidal function with respect to time can be obtained from its phasor divided by $j\omega$, i.e.,

$$\int f(t)dt = \frac{\dot{F}}{j\omega}$$

- This is equivalent to a phasor that rotates clockwise on the complex plane by 90°.

  (since $\frac{1}{j} = -j = -90^\circ$.)

Differentiation and integration of the sinusoidal function in phasor notation	– Differentiation: $\frac{df(t)}{dt} \Leftrightarrow j\omega F$ or $j\omega\dot{F}$ $(+j = +90^\circ)$ – Integration: $\int f(t)dt \Leftrightarrow \frac{F}{j\omega}$ or $\frac{1}{j\omega}\dot{F}$ $\left(\frac{1}{j} = -j = -90^\circ\right)$

9.3.7 Examples of phasor domain

Example 9.7: Convert the following sinusoidal time domain expression to its equivalent phasor domain, and determine voltage $\dot{V}$ (or $\boldsymbol{V}$).

$$2v - 6\frac{dv}{dt} + 4\int v dt = 20\sin(4t + 30^\circ)$$

Solution:

$$\boxed{2\dot{V} - 6j\omega\dot{V} + 4\frac{\dot{V}}{j\omega} = 20\angle 30^\circ}$$

Since $\omega = 4$ in the original expression, so

$$2\dot{V} - 6j\,4\dot{V} + 4\frac{\dot{V}}{j4} = 20\angle 30^\circ$$

$$\dot{V}(2 - 24j - j) = 20\angle 30^\circ$$

$$\dot{V} = \frac{20\angle 30^\circ}{2 - j25} \approx \frac{20\angle 30^\circ}{25\angle -85.43^\circ} = \boxed{0.8\angle 115.43^\circ}$$

Note:

- The complex number of the denominator is $Z = x + jy = 2 - j25 = \sqrt{x^2 + y^2}\tan^{-1}\frac{y}{x}$
- Since x is positive and y is negative in ($2-j25$), the angle should be in the fourth quadrant, i.e., $-85.43°$.

Example 9.8: Convert the phasor domain voltage and current to their equivalent sinusoidal forms (time domain).

(a) $\dot{I} = j\,5e^{-j30^\circ}$ mA
(b) $\dot{V} = -6 + j8$ V

Solution: (a) $\dot{I} = j\,5\angle -30^\circ \text{ mA} = 5\angle 90^\circ \angle -30^\circ \text{ mA}$ $\qquad j = 90^\circ$

$$= 5\angle(90^\circ - 30^\circ)\text{ mA} = 5\angle 60^\circ \text{ mA}$$

$$\boxed{\therefore \quad i(t) = 5\sin(\omega t + 60^\circ)\text{ mA}}$$

(b) $\dot{V} = -6 + j8\text{ V} = \sqrt{(-6)^2 + 8^2}\ \tan^{-1}\angle\frac{8}{-6}\text{ V} \approx 10\angle 126.87^\circ\text{ V}$

(Since y is positive and x is negative, it should be in the second quadrant.)

$$\boxed{\therefore \ v(t) = 10\sin(\omega t + 126.87^\circ)\text{ V}}$$

If the phasors are used to express sinusoidal functions, the algebraic operations of sinusoidal functions of the same frequency can be replaced by algebraic operations of the equivalent phasors, which is shown in Example 9.9.

Example 9.9: Calculate the sum of the following two voltages.

$v_1(t) = 2\sin(\omega t + 60°)$ V and $v_2(t) = 10\sin(\omega t - 40°)$ V

Solution: Convert the sinusoidal time domain voltages to their equivalent phasor forms:

$$\dot{V}_1 = 2\angle 60^{\circ}\ \text{V} \quad \text{and} \quad \dot{V}_2 = 10\angle -40^{\circ}\ \text{V}$$

So
$$\begin{aligned}
\dot{V}_1 + \dot{V}_2 &= 2\angle 60^{\circ} + 10\angle{-40^{\circ}} \qquad a\angle\psi = a(\cos\psi + j\sin) \\
&= 2\cos 60^{\circ} + j\,2\sin 60^{\circ} + 10\cos(-40^{\circ}) + j\,10\sin(-40^{\circ}) \\
&\approx 1 + j1.732 + 7.66 - j6.43 \\
&= 8.66 - j4.698 \\
&= \sqrt{8.66^2 + (-4.698)^2}\ \tan^{-1}\left(\frac{-4.698}{8.66}\right) \\
&\approx 9.85\angle -28.48^{\circ}\ \text{V}
\end{aligned}$$

(Since y is negative and x is positive, it should be in the fourth quadrant.)

$$\boxed{\because\ v(t) = 9.85\sin(\omega t - 28.48^{\circ})\ \text{V}}$$

9.4 Resistors, capacitors, and inductors in sinusoidal AC circuits

9.4.1 Resistor's AC response

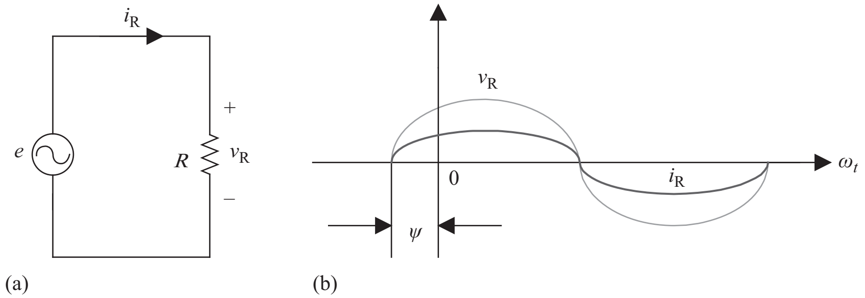
R, L, and C's AC response

- Any AC circuit may contain a combination of three basic circuit elements, resistor (R), inductor (L), and capacitor (C).
- When R, L, and C are connected to a sinusoidal AC voltage source, all resulting voltages and currents in the circuit are also sinusoidal and have the same frequency as AC voltage source.
- All voltages and currents in the AC circuit can be converted from the sinusoidal time domain form $f(t) = F_m \sin(\omega t + \psi)$ to the phasor domain $\boldsymbol{F} = F_m \angle\psi$.

Resistor's AC response

- A resistor is connected to a sinusoidal voltage source as shown in Figure 9.19(a).
- Where the source voltage is:

$$e = V_m \sin(\omega t + \psi)$$



(a) (b)

Figure 9.19 Resistor's AC response

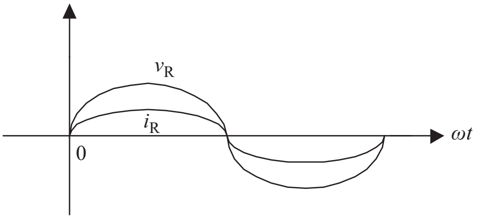


Figure 9.19(c) When $\psi = 0°$

- The sinusoidal current in the circuit can be obtained by applying Ohm's law for AC circuits ($v = Ri$),

  i.e., $i_R = \frac{e}{R} = \frac{V_{R_m}}{R}\sin(\omega t + \psi) = I_{R_m}\sin(\omega t + \psi)$

  – Peak value: $\boxed{I_{R_m} = \frac{V_{R_m}}{R}}$

  – RMS value: $\boxed{I = \frac{V_R}{R}}$

- Voltage across the resistor is the same with the source voltage, i.e., $e = v_R$ or $\boxed{v_R = V_m \sin(\omega t + \psi)}$

9.4.2 Resistor's AC response in time domain

Angular frequency and phase angle of v_R and i_R

- The sinusoidal expressions of resistor voltage v_R and current i_R indicate that voltage and current in the circuit have the same angular frequency (ω) and the same phase angle ψ (or v_R and i_R are in phase). This is also illustrated in Figure 9.19(b).
- Assuming the initial phase angle is zero, i.e., $\psi = 0°$, then,

$$i_R = \frac{v_R}{R} \qquad \boxed{i_R = I_{R_m}\sin\omega t}$$

$$v_R = R\, i_R \qquad \boxed{v_R = V_{R_m}\sin\omega t}$$

- This is illustrated in Figure 9.19(c).

The v_R and i_R in the time domain

The sinusoidal expressions of resistor voltage (v_R) and current (i_R) are in the time domain.

Relationship of voltage and current of a resistor in an AC circuit	– Instantaneous values (time domain): $v_R = V_{R_m}\sin(\omega t + \psi)$ $i_R = I_{R_m}\sin(\omega t + \psi)$ – Ohm's law: Peak value: $V_{R_m} = I_{R_m}\mathrm{R}$ RMS value: $V_R = I_R R$

9.4.3 *Resistor's AC response in phasor domain*

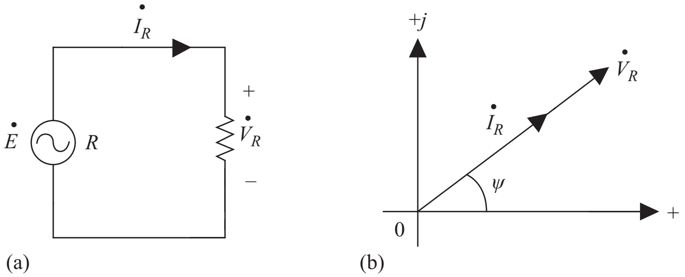
- The peak and RMS values of the resistor voltage and the current in phasor domain also obey the Ohm's law as follows:
  - Peak value: $\dot{I}_{R_m} = \dfrac{\dot{V}_{R_m}}{R}$ or $\boldsymbol{V}_{R_m} = \boldsymbol{I}_{R_m}R$

    A phasor can be represented by the boldface or a little dot on the top of the letter.

  - RMS value: $\dot{I}_R = \dfrac{\dot{V}_R}{R}$ or $\boldsymbol{V}_R = \boldsymbol{I}_R R$
- If it is expressed in terms of conductance, it will give

$$\dot{I}_R = G\dot{V}_R \qquad \left(G = \frac{1}{R}\right)$$

- The relationship of the resistor voltage and current in an AC circuit can be presented by a phasor diagram illustrated in Figure 9.20(b).



(a) (b)

Figure 9.20 The phasor diagram of the AC resistive circuit

Example 9.10: If $R = 10\ \Omega, i_R = 6\sqrt{2}\sin(\omega t - 30^\circ)$ A in Figure 9.20(a), determine the voltage across resistor in the phasor domain.

Solution: $v_R = Ri_R = 10 \times 6\sqrt{2}\sin(\omega t - 30^\circ) = 60\sqrt{2}\sin(\omega t - 30^\circ)$

so $\dot{V}_{R_m} = \boxed{60\sqrt{2}\angle - 30^\circ\ \text{V}}$

Resistor's AC response in phasor domain	Ohm's Law: – Peak value: $\dot{V}_{R_m} = \dot{I}_{R_m} R$ or $\boldsymbol{V}_{R_m} = \boldsymbol{I}_{R_m} R$ – RMS value: $\dot{V}_R = \dot{I}_R R$ or $\boldsymbol{V}_R = \boldsymbol{I}_R R$ – Using conductance: $\dot{I}_R = G\dot{V}_R$ $\left(G = \frac{1}{R}\right)$ Phasor diagram: $\xrightarrow{\dot{I}_R}\xrightarrow{\dot{V}_R}$ (AC resistor voltage and current are in phase)

Note that we can use Ohm's law in AC circuits as long as the circuit quantities are consistently expressed, i.e., both the voltage and current are peak values, RMS values, instantaneous values, etc.

9.4.4 Inductor's AC response

Sinusoidal expression of the i_L and v_L

- If an AC voltage source is applied to an inductor as shown in Figure 9.21(a), the current flowing through the inductor will be

$$\boxed{i_L = I_{L_m}\sin(\omega t + \psi)} \tag{9.4}$$

- The relationship between the voltage across the inductor and the current that flows through it is

$$v_L = L\frac{di}{dt} \tag{9.5}$$

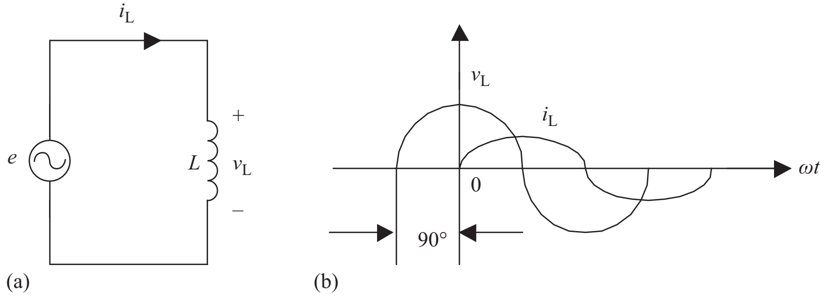


Figure 9.21 Inductor's AC response

- Note: If you have not learned calculus, then just keep in mind that

  $$v_L = \omega L I_{L_m} \sin(\omega t + \psi + 90^\circ)$$

  is the sinusoidal expression of the inductor voltage, and skip the following mathematic derivation process.
- Substituting (9.4) into (9.5) and applying differentiation gives

$$\begin{aligned} v_L &= L\frac{di_L}{dt} = \mathrm{L}\frac{d[I_{L_m}\sin(\omega t + \psi)]}{d\mathrm{t}} = \omega L I_{L_m}\cos(\omega t + \psi) \\ &= \omega L I_{L_m}\sin(\omega t + \psi + 90^\circ) \qquad \because \cos\varphi = \sin(\omega t + 90^\circ) \end{aligned}$$

- Therefore, $$\boxed{v_L \ = \omega L I_{L_m}\sin(\omega t + \psi + 90^\circ)} \tag{9.6}$$

Angular frequency and phase angle of the v_L and i_L

- The sinusoidal expressions of the inductor voltage v_L and current i_L indicate that in an AC inductive circuit, the voltage and current have the same angular frequency (ω) and a phase difference.
- The inductor voltage v_L leads the current i_L by 90° as illustrated in Figure 9.21(b) if we assume that initial phase angle $\psi = 0^\circ$.

9.4.5 The current and voltage in an inductive circuit

Ohm's law for an inductive circuit

- The relationship between the voltage and current in an inductive sinusoidal AC circuit can be obtained from (9.6), which is given by
  - Peak value: $V_{L_m} = \omega L I_{L_m}$
  - RMS value: $V_L = \omega L I_L$
- This is also known as Ohm's law for an inductive circuit.

Inductive reactance and susceptance

- ωL is called inductive reactance and is denoted by X_L,

  i.e., $X_L = \omega L = 2\pi f L \quad (\omega = 2\pi f)$

  - Peak value: $V_{L_m} = X_L I_{L_m}$ or $X_L = \dfrac{V_{L_m}}{I_{L_m}}$
  - RMS value: $V_L = X_L I_L$ or $X_L = \dfrac{V_L}{I_L}$
- X_L is measured in ohms (Ω) (it is the same as resistance R).
- In an inductive circuit, the reciprocal of reactance is called inductive susceptance and is denoted by B_L, i.e., $B_L = \dfrac{1}{X_L}$, and is measured in siemens (S) or mho (℧).

  Recall that the conductance G is the reciprocal of resistance R.

Relationship of voltage and current of inductor in an AC circuit	– Instantaneous values (time domain): $i_L = I_{L_m} \sin(\omega t + \psi)$ $v_L = X_L I_{L_m} \sin(\omega t + \psi + 90°)$ – Ohm's Law: Peak value: $V_{L_m} = X_L I_{L_m}$ RMS value: $V_L = X_L I_L$ – Inductive reactance: $X_L = \omega L = 2\pi f L$ Unit: Ohm (Ω) – Inductive susceptance: $B_L = \dfrac{1}{X_L}$ Unit: Siemens (S) or mho (℧)

9.4.6 Characteristics of an inductor

Angular frequency and inductor voltage

- In an AC inductive circuit, the relationship between the voltage and current is not only determined by the value of inductance L in the circuit but also related to the angular frequency ω.
- If an inductor has a fixed value in the circuit of Figure 9.21(a), inductance L in the circuit is a constant, and the higher the angular frequency ω, the greater the voltage across the inductor.

$$V_L \uparrow = X_L I_L = (\omega \uparrow L)\, I_L$$

- When $\omega \to \infty, \quad V_L \to \omega$

  i.e., when the angular frequency (ω) approaches to infinite, the inductor behaves as an open circuit in which the current is reduced to zero.
- The lower the angular frequency ω, the lower the voltage across the inductor:

$$V_L \downarrow = X_L I_L = (\omega \downarrow L)\, I_L$$

When $\omega = 0, \quad V_L = 0$

- i.e., the AC voltage across the inductor now is equivalent to a DC voltage since the frequency ($\omega = 2\pi f$) does not change any more.

Pass AC and block DC

- Recall that the inductor is equivalent to a short circuit at DC. In this case, the inductor is shortened because of zero voltage across the inductor.
- An inductor can pass the high-frequency signals (pass AC) and block the low-frequency signals (block DC).

Characteristics of an inductor	– An inductor can pass AC (open-circuit equivalent). – An inductor can block DC (short-circuit equivalent).

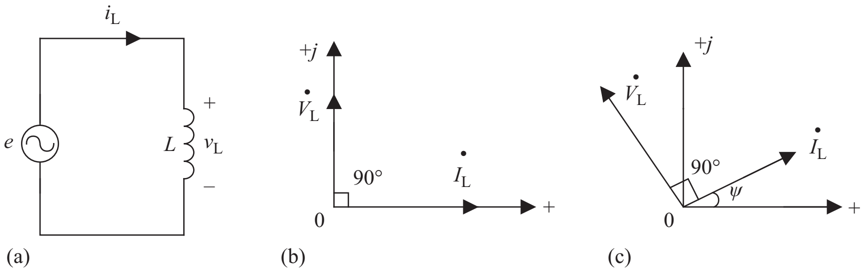


Figure 9.22 The phasor diagram of the AC inductive circuit

9.4.7 Inductor's AC response in phasor domain

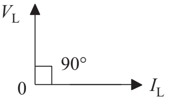
The v_L and i_L in phasor domain

- The sinusoidal expressions of the inductor voltage v_L and current i_L are in the time domain. The peak and RMS values of the inductor voltage and the current in phasor domain also obey Ohm's law as follows:
  - Peak value: $\dot{V}_{L_m} = jX_L\dot{I}_{L_m}$ or $\boldsymbol{V}_{L_m} = jX_L\,\boldsymbol{I}_{L_m}$
  - RMS value: $\dot{V}_L = jX_L\dot{I}_L$ or $\boldsymbol{V}_L = jX_L\,\boldsymbol{I}_L$
- This is because $v_L = L\dfrac{di}{dt} \Leftrightarrow Lj\omega I_L$ Differentiating: multiply by $j\omega$

  So, $\dot{V}_L = (j\omega L)\dot{I}_L$ or $\dot{V}_L = jX_L\dot{I}_L$ $X_L = \omega L$

The phasor diagram of the AC inductive circuit

- The relationship of the inductor voltage and current in an AC circuit can be presented by a phasor diagram illustrated in Figure 9.22(b) and (c).
- Figure 9.22(b) is when the initial phase angle is zero, i.e., $\psi = 0°$, and Figure 9.22(c) is when $\psi \neq 0°$ (the inductor current lags voltage by 90°.)

Inductor's AC response in phasor domain	Ohm's law: – Peak value: $\dot{V}_{L_m} = jX_L\dot{I}_{L_m}$ or $\boldsymbol{V}_{L_m} = jX_L\boldsymbol{I}_{L_m}$ – RMS value $\dot{V}_L = jX_L\dot{I}_L$ or $\boldsymbol{V}_L = jX_L\boldsymbol{I}_L$ Phasor diagram:  Inductor voltage leads the current by 90°.

Example 9.11: In an AC inductive circuit, given $v_L = 6\sqrt{2}\sin(60t + 35^\circ)$ V, and L is 0.2 H, determine the current through the inductor in time domain.

Solution: Inductive reactance $X_L = \omega L = (60 \text{ rad/s})(0.2 \text{ H}) = 12\ \Omega$

$$\dot{I}_{L_m} = \frac{\dot{V}_{L_m}}{jX_L} = \frac{6\sqrt{2}\angle 35^\circ \text{ V}}{j\,12\ \Omega} = \frac{6\sqrt{2}\angle 35^\circ \text{ V}}{12\angle 90^\circ\ \Omega} = 0.5\sqrt{2}\angle -55^\circ \text{ A} \qquad j = 90^\circ$$

Convert the inductor current from the phasor domain to the time domain

$$\boxed{i_L = -0.5\sqrt{2}\sin(60t - 55^\circ) \text{ A}}$$

9.4.8 Capacitor's AC response

Sinusoidal expression of the i_C and v_C

- If an AC voltage source is applied to a capacitor as shown in Figure 9.23(a), the voltage across the capacitor will be

$$\boxed{v_C = V_{C_m}\sin(\omega t + \psi)}$$

- The relationship between the voltage across the capacitor and the current through it is

$$i_C = C\frac{dv_C}{dt}$$

- Substituting v_C into the above expression and applying differentiation gives

$$i_C = C\frac{d[V_{C_m}\sin(\omega t + \psi)]}{dt} = \omega C V_{C_m}\sin(\omega t + \psi + 90^\circ)$$

That is

$$i_C = \omega C V_{C_m}\sin(\omega t + \psi + 90^\circ) \qquad (9.7)$$

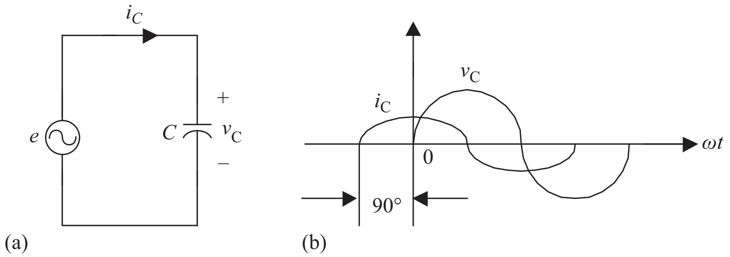


Figure 9.23 Capacitor's AC response

Angular frequency and phase angle of the v_C and i_C

- The sinusoidal expressions of the capacitor voltage v_C and current i_C indicated that in an AC capacitive circuit, the voltage and current have the same angular frequency (ω) and a phase difference.
- The capacitor current leads the voltage by 90° as illustrated in Figure 9.23(b), if we assume that the initial phase angle $\psi = 0°$.

9.4.9 The current and voltage in a capacitive circuit

Ohm's law for an capacitive circuit

- The relationship between voltage and current in a capacitive sinusoidal AC circuit can be obtained from (9.7), which is given by
  - Peak value: $I_{C_m} = (\omega C)\, V_{C_m}$
  - RMS value: $I_C = (\omega C)\, V_C$
- This is also known as Ohm's law for a capacitive circuit.

Capacitive reactance and susceptance

- ωC is called capacitive reactance that is denoted by the reciprocal of X_C,

  i.e., $$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \qquad \omega = 2\pi f C$$

  - Peak value: $X_C = \dfrac{v_{C_m}}{I_{C_m}}$
  - RMS value: $X_C = \dfrac{v_C}{I_C}$
- X_C is measured in ohms (Ω).

  That is the same as resistance R and inductive reactance X_L.
- Recall that the inductive susceptance B_L is the reciprocal of the inductive reactance X_L. The reciprocal of capacitive reactance is called capacitive susceptance and is denoted by B_C, i.e., $B_C = \dfrac{1}{x_C}$, and it is also measured in siemens (S) or mho (℧) (the same as B_L).

The relationship of voltage and current of capacitor in an AC circuit	– Instantaneous values (time domain): $v_C = V_{C_m} \sin(\omega t + \psi)$ $i_C = \omega C V_{C_m} \sin(\omega t + \psi + 90°)$ – Ohm's law: Peak value: $V_{C_m} = X_C\, I_{C_m}$ RMS value: $V_C = X_C\, I_C$ – Capacitive reactance: $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$ Unit: Ohm (Ω) – Capacitive susceptance: $B_C = \dfrac{1}{X_C}$ Unit: Siemens (S) or mho (℧)

9.4.10 Characteristics of a capacitor

Angular frequency and capacitor voltage

- Similar to an inductor, in an AC capacitive circuit not only is the relationship between voltage and current determined by the value of capacitance C in the circuit but it is also related to angular frequency ω.
- If there is a fixed capacitor in Figure 9.23(a), the capacitance C in the circuit is a constant, and the higher the angular frequency ω, the lower the voltage across the capacitor:

$$V_C \downarrow = X_C I_C = \frac{I_C}{\omega \uparrow C}$$

When $\omega \to \infty, \quad V_C \to 0$

- i.e., when the angular frequency ω approaches infinite, the capacitor behaves as a short circuit in which the voltage across it will be reduced to zero.
- The lower the angular frequency ω, the higher the voltage across capacitor.

$$V_C \uparrow = \frac{I_C}{\omega \downarrow C}$$

When $\omega \to 0, \quad V_C \to \infty$

- i.e., the AC voltage across the capacitor now is equivalent to a DC voltage since the frequency ($\omega = 2\pi f$) does not change any more.

Pass DC and block AC

- Recall that a capacitor is equivalent to an open circuit at DC. In this case, the capacitor is open because there will be no current flowing through the capacitor.
- This indicates that a capacitor can block the high-frequency signal (block AC) and pass the low-frequency signal (pass DC).

(The characteristics of a capacitor are opposite to those of an inductor.)

Characteristics of a capacitor	– A capacitor can pass DC (short-circuit equivalent). – A capacitor can block AC (open-circuit equivalent).

9.4.11 Capacitor's AC response in phasor domain

The v_C and i_C in phasor domain

- The sinusoidal expressions of the capacitor voltage v_C and current i_C are in the time domain.
- The peak and RMS values of the capacitor voltage and the current in phasor domain also obey Ohm's law as follows:
  - Peak value: $\dot{V}_{Cm} = -jX_C \dot{I}_{Cm}$ or $\boldsymbol{V}_C = -jX_C \boldsymbol{I}_{Cm}$
  - RMS value: $\dot{V}_C = -jX_C \dot{I}_C$ or $\boldsymbol{V}_C = -jX_C \boldsymbol{I}_C$

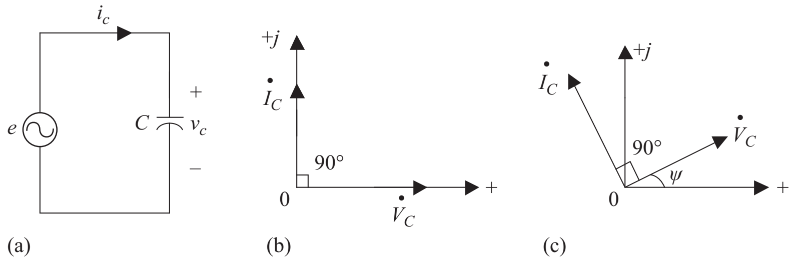


Figure 9.24 The phasor diagram of an AC capacitive circuit

- This is because $i_C = C\frac{dv_C}{dt} \Leftrightarrow C\,j\omega V_C$ Differentiating: multiply by $j\omega$

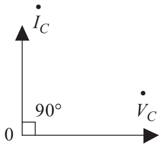
  So $\dot I_C = j\omega C\dot V_C = j\frac{1}{X_C}\dot V_C$ $X_C = \frac{1}{\omega C}$

  or $\dot V_C = -jX_C\dot I_C$ $\left(\frac{1}{j} = -j\right)$

The phasor diagram of the AC capacitive circuit

The relationship of the capacitor voltage and current in an AC circuit can be presented by a phasor diagram and is illustrated in Figure 9.24(b) and (c).

- Figure 9.24(b) is when the initial phase angle is zero, i.e., $\psi = 0°$ (capacitor voltages lags current by 90°).
- Figure 9.24(c) is when $\psi \neq 0°$

Capacitor's AC response in phasor domain	Ohm's law: – Peak value: $\dot V_{Cm} = -jX_C\,\dot I_{Cm}$ or $\boldsymbol{V}_{Cm} = -jX_C\,\boldsymbol{I}_{Cm}$ – RMS value: $\dot V_C = -jX_C\,\dot I_C$ or $\boldsymbol{V}_C = -jX_C\,\boldsymbol{I}_C$ Phasor diagram:  Capacitor current leads voltage by 90°.

Example 9.12: Given a capacitive circuit in which $v_C = 50\sqrt{2}\sin(\omega t - 20^\circ)$ V, capacitance is 5 μF, and frequency is 500 Hz, determine the capacitor current in the time domain.

Solution: $\omega = 2\pi f = 2\pi(500\,\text{Hz}) \approx 3{,}142\,\text{rad/s}$

$$X_C = \frac{1}{\omega C} = \frac{1}{(3142\,\text{rad/s})(5 \times 10^{-6}\,\text{F})} \approx 63.65\,\Omega$$

$$\therefore I_{Cm} = \frac{V_{cm}}{X_C} = \frac{50\sqrt{2}\text{V}}{63.65\,\Omega} \approx 786\,\sqrt{2}\,\text{mA}$$

$$i_C = \boxed{786\sqrt{2}\sin(\omega t - 20^\circ + 90^\circ)}$$

$$= \boxed{786\sqrt{2}\sin(\omega t + 70^\circ)\,\text{mA}}$$

Summary

Direct current (DC)

- The polarity of DC voltage and direction of DC current do not change.
- The pulsing DC changes the amplitude of the pulse, but does not change the polarity.

Alternating current (AC)

- The voltage and current periodically change polarity with time (such as sine wave, square wave, saw-tooth wave, etc.).
- Sine AC varies over time according to the sine function, and is the most widely used AC.

Period and frequency

- Period T is the time to complete one full cycle of the waveform.
- Frequency f is the number of cycles of waveforms within one second $\left(f = \frac{1}{T}\right)$.

Three important components of the sinusoidal function $f(t) = F_m \sin(\omega t + \psi)$

- F_m: Peak value (amplitude)
- ω: Angular velocity (or angular frequency) $\qquad \omega = \frac{2\pi}{T} = 2\pi f$
- ψ: Phase or phase shift
  - $\psi > 0$: Waveform shifted to the left side of 0°
  - $\psi < 0$: Waveform shifted to the right side of 0°

Phase difference φ: For two waves with the same frequency such as

$$v(t) = V_m \sin(\omega t + \psi_v) \quad \text{and} \qquad i(t) = I_m \sin(\omega t + \psi_i) \qquad \varphi = \psi_v - \psi_i$$

- If $\varphi = 0$: v and i in phase
- If $\varphi > 0$: v leads i
- If $\varphi < 0$: v lags i
- If $\varphi = \pm\frac{\pi}{2}$: v and i are orthogonal
- If $\varphi = \pm\pi$: v and i are out of phase by 180 degrees

Peak value, peak–peak value, instantaneous value, and average value of sine waveform

- Peak value $F_{\text{pk}} = F_{\text{m}}$: the amplitude
- Peak–peak value $F_{\text{p-p}}$: $F_{\text{p-p}} = 2F_{\text{pk}}$
- Instantaneous value $f(t)$: value at any time at any particular point of the waveform.
- Average value: average value of a half-cycle of the sine waveform $F_{\text{Avg}} = 0.637\,F_{\text{m}}$

RMS value (or effective value) of AC sinusoidal function

- If an AC source delivers the equivalent amount of power to a resistor as a DC source, which is the effective or RMS value of AC.

$$V = \frac{V_{\text{m}}}{\sqrt{2}} = 0.707V_{\text{m}}, \qquad I = \frac{I_{\text{m}}}{\sqrt{2}} = 0.707I_{\text{m}}$$

- The general formula to calculate RMS value is $F = \sqrt{\frac{1}{T}\int_0^T f^2(t)\mathrm{d}t}$

Complex numbers

- Rectangular form: $\boldsymbol{A} = x + jy$
- Polar form: $\boldsymbol{A} = a\ \angle\psi$
- Conversion between rectangular and polar forms:

$$A = x + jy = \sqrt{x^2 + y^2}\tan^{-1}\frac{y}{x} = a\angle\psi$$
$$A = a\angle\psi = x + j\,y = a(\cos\psi + j\sin\ \psi)$$

- Addition and subtraction: $\boldsymbol{A}_1 \pm \boldsymbol{A}_2 = (x_1 \pm x_2) + j\ (y_1 \pm y_2)$
- Multiplication: $\boldsymbol{A}_1 \cdot \boldsymbol{A}_2 = a_1 \cdot a_2\ \angle(\psi_1 + \psi_2) = (x_1 + jy_1)\ (x_2 + jy_2)$
- Division: $\frac{A_1}{A_2} = \frac{a_1}{a_2}\angle(\psi_1 - \psi_2) = \frac{x_1 + j\,y_1}{x_2 + j\,y_2}$

Phasor

- A phasor is a vector that contains both amplitude and angle information, and can be represented as a complex number.
- The phasor notation is a method that uses complex numbers to represent the sinusoidal quantities for analyzing AC circuits when all quantities have the same frequency.

	Time domain	Phasor domain
Phasor	- $f(t)=F_{\mathrm{m}}\sin(\omega t+\psi)$	Peak value: $\boldsymbol{F}_{\mathbf{m}}=F_{\mathrm{m}}\angle\psi$ (or $\dot{F}_m=F_{\mathrm{m}}\angle\psi$)
		RMS value: $\boldsymbol{F}=F\angle\psi$ (or $\dot{F}=F\angle\psi$)
	- $v(t)=V_{\mathrm{m}}\sin(\omega t+\psi)$	Peak value: $\dot{V}_m=V_{\mathrm{m}}\angle\psi_{\mathrm{v}}$
		RMS value: $\dot{V}=V\angle\psi_{\mathrm{v}}$
	- $i(t)=I_{\mathrm{m}}\sin(\omega t+\psi)$	Peak value: $\dot{I}_m=I_{\mathrm{m}}\angle\psi_{\mathrm{i}}$
		RMS value: $\dot{I}=I\angle\psi_{\mathrm{i}}$

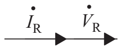
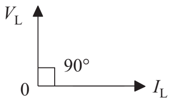
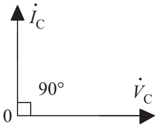
Differentiation and integration of the sinusoidal function in phasor notation

- Differentiation: $\frac{\mathrm{d}f(t)}{\mathrm{d}t}=j\omega\boldsymbol{F}$ or $j\omega\dot{F}$ $\qquad +j=+90^{\circ}$
- Integration: $\int f(t)\mathrm{d}t=\frac{\boldsymbol{F}}{j\omega}$ or $\frac{1}{j\omega}\dot{F}$ $\qquad \frac{1}{j}=-j=-90^{\circ}$
- Rotation factor: $\mathrm{e}^{j\omega t}$ or $\pm j=\pm 90^{\circ}$

Characteristics of the inductor and capacitor

Element	DC: when $\omega=0$	AC: when $\omega\to\infty$	Characteristics
Inductor L	Short circuit	Open circuit	Pass DC and block AC
Capacitor C	Open circuit	Short circuit	Pass AC and block DC

Three basic elements in an AC circuit

Element	Time domain	Phasor domain	Resistance and reactance	Conductance and susceptance	Phasor diagram
Resistor	$V_{\mathrm{R}}=Ri_{\mathrm{R}}$	$\dot{V}_{\mathrm{R}}=\dot{I}_{\mathrm{R}}R$	R	$G=\frac{1}{R}$	
Inductor	$v_{\mathrm{L}}=L\frac{\mathrm{d}i}{\mathrm{d}t}$	$\dot{V}_{\mathrm{L}}=jX_{\mathrm{L}}\dot{I}_{\mathrm{L}}$	$X_{\mathrm{L}}=\omega L$	$B_{\mathrm{L}}=\frac{1}{X_{\mathrm{L}}}$	
Capacitor	$i_{\mathrm{C}}=C\frac{\mathrm{d}v_{\mathrm{C}}}{\mathrm{d}t}$	$\dot{V}_{\mathrm{C}}=-jX_{\mathrm{C}}\dot{I}_{\mathrm{C}}$	$X_{\mathrm{C}}=\frac{1}{\omega C}$	$B_{\mathrm{C}}=\frac{1}{X_{\mathrm{C}}}$	 $\dot{I}_{\mathrm{C}}$ 90° $\dot{V}_{\mathrm{C}}$ 0

Practice problems

9.1

1. The difference between the AC and DC is that AC changes ( ), and DC does not.
2. When the time to complete a full cycle of a sinusoidal waveform is 2 ms, the frequency of this waveform will be ( ).
3. Determine the peak value, phase angle, angular frequency, period, and frequency of the sinusoidal current $i\ (t) = 20\ \sin(30t + 45°)$ A, and also plot the waveform of this current.
4. Determine the phase differences of the voltages and currents for the expressions in (a) and (b), and also determine their relationships of leading or lagging.

   (a) $v(t) = 5\sin(\omega t + 40^{\circ})$ V, $i(t) = 20\sin(30t + 30^{\circ})$ A
   (b) $v(t) = 20\sin(\omega t - 60^{\circ})$ V, $i(t) = 15\sin(\omega t - 30^{\circ})$ A

5.
   (a) v and i are ( ) if the phase difference φ is 180 degrees;
   (b) v and i are ( ) if the phase difference φ is $-90°$.

9.2

6. Write the sinusoidal expressions (instantaneous expressions) for values in (a) and (b).

   (a) $I_m = 50$ mA, $\omega t = 30°$
   (b) $V_m = 15$ V, $T = 20$ ms

7. Determine the average value for a half-cycle of sinusoidal waveform that has the peak value of 15 V.
8. Determine the average value for a full cycle of sinusoidal waveform that has the peak value of 10 V.
9. Determine the peak value, peak–peak value, average value, and RMS value of the sinusoidal voltage $v\ (t) = 20\ \sin(\omega t + 30°)$ V.

9.3

10. Perform the following operations and express the result in rectangular form:

$$3 - \frac{2 + j_1}{3 - j_2}$$

11. Convert the following sinusoidal expressions to polar forms:

$$v(t) = 30\sin(\omega t - 45^{\circ})\ \text{V}, \qquad i(t) = 15\sin(30t + 35^{\circ})$$

12. Convert the following polar forms to instantaneous forms (30 and 15 are RMS values):

$$\dot{V} = 30\angle 45^{\circ}\ \text{V}, \quad \dot{I} = 15\angle -60^{\circ}\ \text{A}$$

13. Convert the following polar forms to sinusoidal forms (10, −10, and 20 are peak values):
    (a) $\dot{V} = 10\angle{-45^\circ}$ V
    (b) $\dot{I} = -10\angle 5^\circ + 20\angle 10^\circ$ A
14. Convert the following equation to polar form and sinusoidal form:

$$3\frac{di}{dt} + 4i(t) = 5\sin(3t - 30^\circ)\ \mathrm{A}$$

15. Determine the difference of the following two sinusoidal expressions $(i_1 - i_2)$:

$$i_1 = 5\sin(\omega t + 45^\circ)\ \mathrm{A}, \quad i_2 = 20\sin(\omega t - 10^\circ)\ \mathrm{A}$$

9.4

16. The resistance R is 3Ω in a purely resistive circuit, and the current is $i_{\mathrm{R}} = 5\sqrt{2}\sin(\omega t + 45^\circ)$ A. Determine the phasor form of the resistor voltage $\dot{V}_{\mathrm{R}m}$.
17. In a purely inductive circuit, the inductance is 0.008 H, and the current is $i_{\mathrm{L}} = 5\sin(60t + 30^\circ)$ A. Determine the instantaneous expression of the inductor voltage v_{L} in this circuit.
18. In a purely capacitive circuit, the capacitance is 0.09 μF, and the current is $i_{\mathrm{C}} = 0.08\sin(120t + 45^\circ)$. Determine the instantaneous expression of the capacitor voltage v_{C} in this circuit.

Chapter 10

Methods of AC circuit analysis

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10.1 Impedance and admittance

10.1.1 Impedance

Ohm's law of AC circuits

- The phasor forms of relationship between voltage and current for resistor, inductor, and capacitor in an AC circuit are as follows:

$$\dot{V}_{\mathrm{R}} = \dot{I}_{\mathrm{R}}R, \qquad \dot{V}_{\mathrm{L}} = j\dot{I}_{\mathrm{L}}X_{\mathrm{L}}, \qquad \dot{V}_{\mathrm{C}} = -j\dot{I}_{\mathrm{C}}X_{\mathrm{C}}$$

- The above equations can be changed to a ratio of voltage and current:

$$\frac{\dot{V}_{\mathrm{R}}}{\dot{I}_{R}} = R, \qquad \frac{\dot{V}_{\mathrm{L}}}{\dot{I}_{\mathrm{L}}} = jX_{\mathrm{L}}, \qquad \frac{\dot{V}_{\mathrm{C}}}{\dot{I}_{\mathrm{C}}} = -jX_{\mathrm{C}}$$

- The ratio of voltage and current is the impedance of an AC circuit, and it can be generally expressed as $= \frac{\dot{V}}{\dot{I}}$. This equation is also known as Ohm's law of AC circuits.

The impedance of R, L, and C

- The physical meaning of the impedance ($\boldsymbol{Z}$) is that it is a measure of the opposition to AC current in an AC circuit. It is similar to the concept of resistance in DC circuits, so the impedance is also measured in ohms (Ω).
- The impedance can be extended to the inductor and capacitor in an AC circuit. It is a complex number that describes both the amplitude and phase characteristics.
- The impedance of resistor (R), inductor (L), and capacitor (C) are as follows:

$$\boldsymbol{Z}_{\mathrm{R}} = R = \frac{\dot{V}_{R}}{\dot{I}_{R}}, \qquad \boldsymbol{Z}_{\mathrm{L}} = jX_{\mathrm{L}} = \frac{\dot{V}_{\mathrm{L}}}{\dot{I}_{\mathrm{L}}}, \qquad \boldsymbol{Z}_{\mathrm{C}} = -jX_{\mathrm{C}} = \frac{\dot{V}_{\mathrm{C}}}{\dot{I}_{\mathrm{C}}}$$

Impedance ($\boldsymbol{Z}$)	– $\boldsymbol{Z}$ is a measure of the opposition to AC current in an AC circuit. – Ohm's law in AC circuits: $\boldsymbol{Z} = \frac{\dot{V}}{\dot{I}}$ – Unit of $\boldsymbol{Z}$: ohm (Ω)

10.1.2 Admittance

Admittance *Y*

- Recall that the conductance G is the inverse of resistance R, and it is a measure of how easily current flows in a DC circuit. It is more convenient to use the conductance G in a parallel DC circuit.

- The admittance is the inverse of impedance $\boldsymbol{Z}$, it is denoted by $\boldsymbol{Y}$,

$$\boxed{\boldsymbol{Y} = \frac{1}{\boldsymbol{Z}}}$$

- The admittance $\boldsymbol{Y}$ is measured in siemens (S) or mho (℧).
- The admittance $\boldsymbol{Y}$ is a measure of how easily a current can flow in an AC circuit. It can be expressed as the ratio of current and voltage of an AC circuit, i.e., $\boxed{\boldsymbol{Y} = \frac{\dot{I}}{\dot{V}}}$.

The admittance of R, L, and C

- It is more convenient to use the admittance in a parallel AC circuit.
- The admittance of resistor (R), inductor (L), and capacitor (C) are as follows:
  - The admittance of R: $Y_R = \frac{1}{R}$
  - The admittance of L: $Y_L = \frac{1}{jX_L} = -j\frac{1}{X_L}$ $\left(j = \frac{1}{-j}\right)$
  - The admittance of C: $Y_C = \frac{1}{-jX_C} = j\frac{1}{X_C}$

Admittance (*Y*)	– ***Y*** is a measure of how easily current can flow in an AC circuit. – ***Y*** is the inverse of impedance ***Z***: $Y = \frac{1}{Z}$ – Ohm's law in AC circuits: $\dot{I} = \dot{V}Y$ – The unit of ***Y***: siemens (S) or mho (℧)

10.1.3 Characteristics of the impedance

Impedance *Z*

- The impedance $\boldsymbol{Z}$ is a vector quantity; it can be expressed in both polar form and rectangular form (complex number) as follows:

$$\boxed{\boldsymbol{Z} = z\angle\varphi = R + jX = z(\cos\varphi + j\sin\varphi)}$$

  - Polar form: $\boldsymbol{Z} = z\angle\varphi$
  - Rectangular form: $\boldsymbol{Z} = R + jX = z\ (\cos\varphi + j\sin\varphi)$
- The rectangular form is the sum of the *real part* and the imaginary part, where
  - the real part of the complex is the resistance R,
  - the imaginary part of the complex is the reactance X.
- The reactance X is the difference of inductive reactance X_L and capacitive reactance X_C,

  i.e., $X = X_L - X_C$

- The lower case letter z in $z\angle\varphi$ is the magnitude of the impedance, which is $\boxed{z=\sqrt{R^2+X^2}}$
- The corresponding angle φ between the resistance R and reactance X is called the impedance angle and can be expressed as follows: $\boxed{\varphi=\tan^{-1}\frac{X}{R}}$

Impedance, voltage, and current triangles

- The relationship between R, X, and Z in the equation of the impedance is a right triangle, and can be described using the Pythagoras' theorem. This can be illustrated in Figure 10.1(a).
- Figure 10.1(a) is an impedance triangle. If we multiply each side of the quantity in the impedance triangle by current $\dot{I}$ the following equations will be obtained:

$$\dot{V}_R=Z\dot{I}_R, \qquad \dot{V}_X=\dot{I}_X X, \qquad \dot{V}_Z=\dot{I}_Z Z$$

- These can form another triangle that is called the voltage triangle, which is illustrated in Figure 10.1(b).
- If we divide each side of the value by voltage $\dot{V}$ in the impedance triangle, the following equations will be obtained:

$$\dot{I}_Z=\frac{\dot{V}_Z}{Z}, \qquad \dot{I}_X=\frac{\dot{V}_X}{X}, \qquad \dot{I}_R=\frac{\dot{V}_R}{R}$$

- The above equations can form another triangle that is called the current triangle, and it is illustrated in Figure 10.1(c).

The characteristics of the impedance triangle

The characteristics of the impedance triangle in Figure 10.1(a) can be summarized as follows:

- If $X>0$ or $X=X_L-X_C>0, X_L>X_C$: The reactance X is above the horizontal axis, and the impedance angle $\varphi>0$. The circuit is more inductive as shown in Figure 10.2(a).
- If $X<0$ or $X=X_L-X_C<0, X_C>X_L$: The reactance X is below the horizontal axis, and the impedance angle $\varphi<0$. The circuit is more capacitive as shown in Figure 10.2(b).

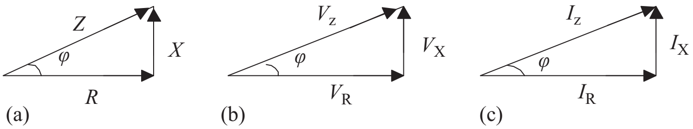


Figure 10.1 Impedance, voltage, and current triangles

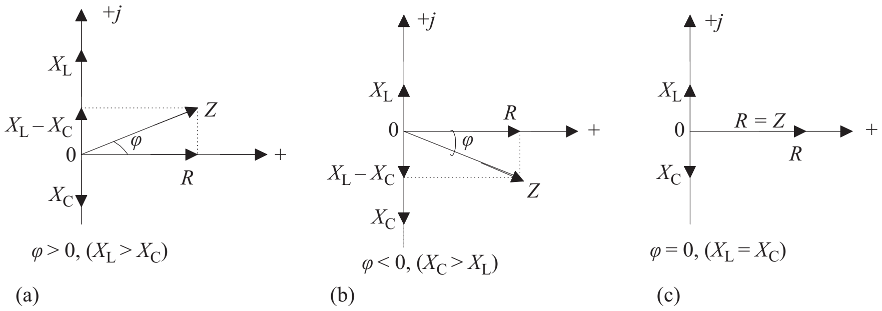


Figure 10.2 The phasor diagrams of the impedance

- If $X = 0$ or $X = X_L - X_C = 0$, $X_C = X_L$: the impedance angle $\varphi = 0$, the circuit will look like a purely resistive circuit ($z = R$) as shown in Figure 10.2(c).

10.1.4 Impedance examples

Example 10.1: Determine the impedance $\boldsymbol{Z}$ in the circuit of Figure 10.3 and plot the phasor diagram of the impedance.

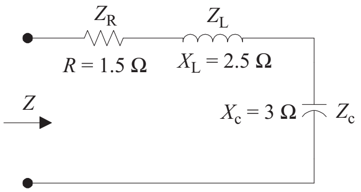


Figure 10.3 Figure for Example 10.1

Solution:

- The impedances in series in an AC circuit behave like resistors in series.

$$\text{So, } \boldsymbol{Z} = Z_R + Z_L + Z_C = R + jX = R + j(X_L - X_C)$$

$$= 1.5\ \Omega + j(2.5 - 3)\ \Omega = 1.5\ \Omega - j0.5\ \Omega$$

$$= \sqrt{1.5^2 + (-0.5)^2}\tan^{-1}\frac{-0.5}{1.5} \approx \boxed{1.58\ \angle - 18.44°\Omega}$$

$$\boldsymbol{Z} = z\angle\varphi, \qquad \varphi = \tan^{-1}\frac{X}{R} \qquad z = \sqrt{R^2 + X^2}$$

Note: Since the imaginary term is −0.5 on the y-coordinate, the real term is +1.5 on the x-coordinate, the impedance angle for this circuit is located in the 4th quadrant.

- The circuit for Example 10.1 is more capacitive since $X_C > X_L$, and $\varphi < 0$ as shown in Figure 10.4:

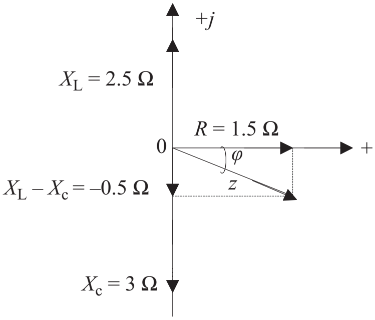


Figure 10.4 Impedance angle for Example 10.10

10.1.5 Characteristics of the admittance

Admittance *Y*

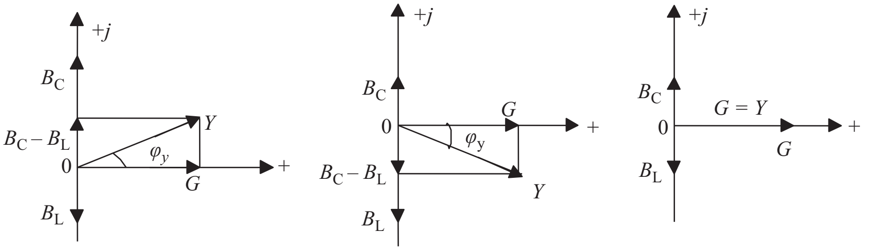
- The admittance is also a complex number; it can be expressed in both polar and rectangular forms as follows:

$$\boldsymbol{Y} = y\angle\varphi_y = G + jB = y(\cos\varphi + j\sin\varphi)$$

  - Polar form: $\boldsymbol{Y} = y\angle\varphi_y$
  - Rectangular form: $\boldsymbol{Y} = G + jB = y\ (\cos\varphi + j\sin\varphi)$
- The real part of the complex is the conductance *G*, and the imaginary part is called the susceptance *B*.
- The susceptance is measured in the same way as the admittance, i.e., siemens (S) or mho (℧).
- The susceptance is the difference of the capacitive susceptance B_C and inductive susceptance B_L, i.e., $B = B_C - B_L$
- The lower case letter *y* in $\boldsymbol{Y} = y\angle\varphi_y$ is the magnitude of the admittance, i.e.,

$$Y = \sqrt{G^2 + B^2}$$

- The corresponding angle φ between the conductance *G* and susceptance *B* is called the admittance angle and can be expressed as $\varphi_y = \tan^{-1}\dfrac{B}{G}$



$\varphi_y > 0, (B_C > B_L)$
(a)

$\varphi_y < 0, (B_L > B_C)$
(b)

$\varphi_y = 0, (B_L = B_C)$
(c)

Figure 10.5 The phasor diagrams of the admittance

The characteristics of the admittance triangle

The characteristics of the admittance triangle in Figure 10.5 can be summarized as follows:

- If $B > 0$ or $B = B_C - B_L > 0$, $B_C > B_L$: The susceptance B is above the horizontal axis, the admittance angle $\varphi_y > 0$, and the circuit is more capacitive as shown in Figure 10.5(a).
- If $B < 0$ or $B = B_C - B_L < 0$, $B_L > B_C$: The susceptance B is below the horizontal axis, the admittance angle $\varphi_y < 0$, and the circuit is more inductive as shown in Figure 10.5(b).
- If $B = 0$, or $B = B_C - B_L = 0$, $B_L = B_C$: The admittance angle $\varphi_y = 0$, the circuit will look like a purely resistive circuit ($Y = G$) as shown in Figure 10.5(c).

The characteristics of impedance and admittance

$X > 0$	– If $X > 0$, $\varphi > 0$, $B < 0$, $\varphi_y < 0$: The circuit is more inductive.
$X < 0$	– If $X < 0$, $\varphi < 0$, $B > 0$, $\varphi_y > 0$: The circuit is more capacitive.
$X = 0$	– If $X = 0$, $\varphi = 0$, $B = 0$, $\varphi_y = 0$: The circuit is purely resistive.

Impedance, admittance, and susceptance

- The admittance of resistor (R), inductor (L),and capacitor (C) are as follows:

$$Y_R = \frac{1}{R} = G \qquad Y_L = \frac{1}{jX_L} = -j\frac{1}{X_L} \qquad Y_C = \frac{1}{-jX_C} = j\frac{1}{X_C}$$

- The impedance (Z), admittance (Y), susceptance (B), and their relationship can be summarized as given in Table 10.1.

Table 10.1 Relationship between the impedance (Z), admittance (Y), and susceptance (B)

Component	**Impedance** $\boldsymbol{Z = \frac{\dot{V}}{\dot{I}}}$	**Admittance** $\boldsymbol{Y = \frac{1}{Z}}$	**Conductance (*G*) and susceptance (*B*)**
Resistor (R)	$Z_R = R$	$Y_R = G$	Conductance: $G = \frac{1}{R}$
Inductor (L)	$Z_L = jX_L$	$Y_L = -jB_L$	Inductive susceptance: $B_L = \frac{1}{X_L}$
Capacitor (C)	$Z_C = -jX_C \quad j = \frac{1}{-j}$	$Y_C = jB_C$	Capacitive susceptance: $B_C = \frac{1}{X_C}$
Z, Y, X,* and *B	$\boldsymbol{Z} = z\angle\phi = R + jX$ $z = \sqrt{R^2 + X^2}$ $\varphi = \tan^{-1}\frac{X}{R}$	$\boldsymbol{Y} = y\angle\varphi_y = G + jB$ $y = \sqrt{G^2 + B^2}$ $\varphi_y = \tan^{-1}\frac{B}{G}$	Reactance: $X = X_L - X_C$ Susceptance: $B = B_C - B_L$ $\left(X_L = \omega L, \quad X_C = \frac{1}{\omega C}\right)$

10.1.6 Admittance examples

Example 10.2: Determine the admittance in the circuit of Figure 10.6 and plot the phasor diagrams of the admittance.

Solution:

- The admittances in parallel in AC circuits behave like the conductances in parallel in DC circuits.

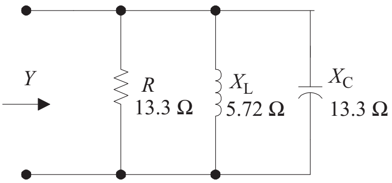


Figure 10.6 Figure for Example 10.2

So, $Y = Y_R + Y_L + Y_C = G + j\mathrm{B} = G + j(B_C - B_L)$ $\qquad G = \frac{1}{R}$

$$= \frac{1}{13.3\ \Omega} + j\left(\frac{1}{13.3\ \Omega} - \frac{1}{5.72\ \Omega}\right) \qquad B_C = \frac{1}{X_C}, B_L = \frac{1}{X_L}$$

$$\approx 0.075\ \mathrm{S} + j(0.075 - 0.175)\mathrm{S}$$

$$= 0.075\mathrm{S} - j0.1\mathrm{S} = \sqrt{0.075^2 + (-0.1)^2}\tan^{-1}\frac{-0.1}{0.075} = 0.125\angle - 53.13°\mathrm{S}$$

$$\boldsymbol{Y} = G + jB = y\angle\varphi_y,\ y = \sqrt{G^2 + B^2},\ \varphi_y = \tan^{-1}\frac{B}{G}$$

Note: The admittance angle for this circuit is located in the fourth quadrant since

- the imaginary term is negative 0.1 on the *y*-coordinate,
- the real term is positive 0.075 on the *x*-coordinate.

- Since $B_L > B_C$ ($B_L = 0.175$, $B_C = 0.075$) and $\varphi_y < 0$, the circuit is more inductive as shown in Figure 10.7.

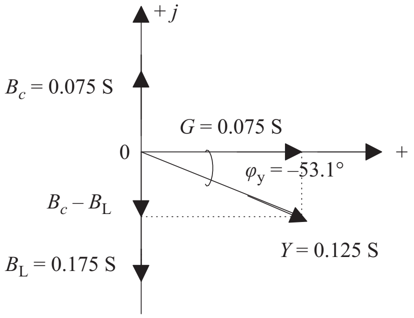

+j
$B_c = 0.075$ S
$G = 0.075$ S
0
+
$\varphi_y = -53.1°$
$B_c - B_L$
$Y = 0.125$ S
$B_L = 0.175$ S

Figure 10.7 Admittance angle for Example 10.2

10.2 Impedance in series and parallel

10.2.1 Equivalent impedance

Impedance of a series circuit

- The impedances in series and parallel AC circuits behave like resistors in series and parallel DC circuits, except the phasor form (complex number) is used.
- The equivalent impedance (or total impedance) for a series circuit in Figure 10.8 is given:

$$Z_{eq} = Z_1 + Z_2 + \dots + Z_n$$

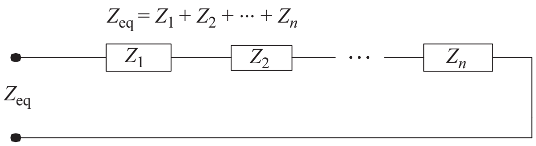

$Z_{eq} = Z_1 + Z_2 + \cdots + Z_n$
Z_1
Z_2
$\cdots$
Z_n
Z_{eq}

Figure 10.8 Impedance of a series circuit

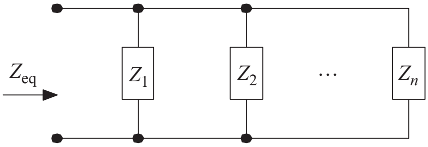


Figure 10.9 Impedance of a parallel circuit

Impedance of a parallel circuit

- The equivalent impedance (or total impedance) for a parallel circuit in Figure 10.9 is given as

$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}} = Z_1//Z_2 \ldots //Z_n$$

$$Y_{eq} = Y_1 + Y_2 + \ldots + Y_n$$

- The equivalent impedance is the reciprocal of equivalent admittance, $Z_{eq} = \frac{1}{Y_{eq}}$.
- If only have two impedances in parallel, the equivalent impedance is given as

$$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2} = Z_1//Z_2$$

Impedances in series and parallel	– Impedances in series: $Z_{eq} = Z_1 + Z_2 + \ldots + Z_n$ – Impedances in parallel: $Z_{eq} = Z_1//Z_2 // \ldots //Z_n$, $Y_{eq} = Y_1 + Y_2 + \ldots + Y_n$ – Two impedances in parallel: $Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$

10.2.2 The phasor forms of KVL, KCL, VDR, and CDR

The phasor forms of VDR and CDR

- The voltage divider and current divider rules in phasor form in AC circuits are very similar to the DC circuits (Figure 10.10).
- Voltage divider rule (VDR): $\dot{V}_1 = \frac{z_1}{z_1 + z_2}\dot{E}, \quad \dot{V}_2 = \frac{z_2}{z_1 + z_2}\dot{E}$
- Current divider rule (CDR): $\dot{I}_1 = \frac{z_2}{z_1 + z_2}\dot{I}_T, \quad \dot{I}_2 = \frac{z_1}{z_1 + z_2}\dot{I}_T$

VDR and CDR in DC circuits: $V_1 = \frac{R_1}{R_1 + R_2}E, \quad V_2 = \frac{R_2}{R_1 + R_2}E,$

$$I_1 = \frac{R_2}{R_1 + R_2}\dot{I}_T, \quad I_2 = \frac{R_1}{R_1 + R_2}\dot{I}_T$$

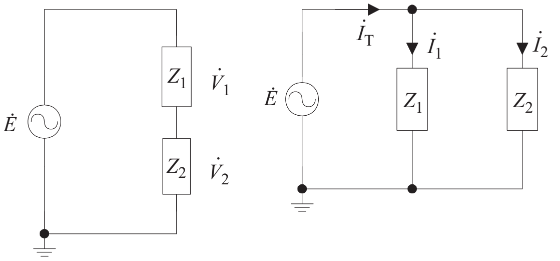

Z_1
$\dot{V}_1$
$\dot{E}$
Z_2
$\dot{V}_2$
$\dot{I}_T$
$\dot{I}_1$
$\dot{I}_2$
$\dot{E}$
Z_1
Z_2

Figure 10.10 Voltage and current dividers

The phasor forms of KVL and KCL

The phasor forms of KVL and KCL also hold true in AC circuits.

- KVL: $\Sigma\dot{V} = 0$ or $\dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n = \dot{E}$
- KCL: $\Sigma\dot{I} = 0$ or $\dot{I}_{\text{in}} = \dot{I}_{\text{out}}$

Phasor forms of VDR, CDR, KVL, and KCL	– VDR: $\dot{V}_1 = \frac{z_1}{z_1 + z_2}\dot{E}$, $\dot{V}_2 = \frac{z_2}{z_1 + z_2}\dot{E}$ – CDR: $\dot{I}_1 = \frac{z_2}{z_1 + z_2}\dot{I}_T$, $\dot{I}_2 = \frac{z_1}{z_1 + z_2}\dot{I}_T$ – KVL: $\Sigma\dot{V} = 0$ or $\dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_n = \dot{E}$ – KCL: $\Sigma\dot{I} = 0$ or $\dot{I}_{\text{in}} = \dot{I}_{\text{out}}$

10.2.3 Equivalent impedance examples

To determine the equivalent impedance, currents, and voltages in series and parallel AC circuits, use the same method that determines the equivalent resistance in series and parallel DC circuits.

Example 10.3: Determine the following values for the circuit in Figure 10.11.

(a) The input equivalent impedance Z_{eq}
(b) The current $\dot{I}_3$ in the branch of R_L and X_L

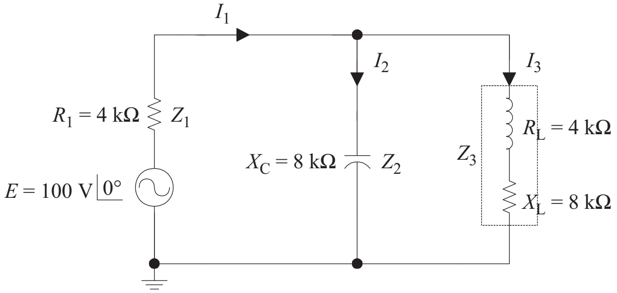

I_1
I_2
I_3
$R_1 = 4\ \text{k}\Omega$
Z_1
$E = 100\ \text{V}\angle 0°$
$X_C = 8\ \text{k}\Omega$
Z_2
Z_3
$R_L = 4\ \text{k}\Omega$
$X_L = 8\ \text{k}\Omega$

Figure 10.11 Figure for Example 10.3

Solution:

(a)
$$Z_{\mathrm{eq}} = Z_1 + Z_2//Z_3$$

$$Z_1 = R_1 = 4\ \mathrm{k\Omega}$$

$$Z_2 = -jX_{\mathrm{C}} = -j8\ \mathrm{k\Omega}$$

$$Z_3 = R_{\mathrm{L}} + jX_{\mathrm{L}} = 4\ \mathrm{k\Omega} + j8\ \mathrm{k\Omega} \approx 8.94\angle 63.44^\circ\ \mathrm{k\Omega}$$

$$Z_2//Z_3 = \frac{Z_2Z_3}{Z_2+Z_3} = \frac{(-j8)(4+j8)}{-j8+4+j8}\mathrm{k\Omega} = \frac{64-\mathrm{j}32}{4}\ \mathrm{k\Omega} \approx \frac{71.55\angle -26.57}{4\angle 0^\circ}\ \mathrm{k\Omega}$$

$$\boldsymbol{Z} = R + jX = z\angle\varphi$$

$$= 17.9\ \angle -26.57^\circ\ \mathrm{k\Omega} = 17.9[\cos(-26.57^\circ) + j\sin(-26.57^\circ)]\ \mathrm{k\Omega}$$

$$= (16 - j8)\ \mathrm{k\Omega}$$

$$\boldsymbol{Z} = R + jX = z(\cos\varphi + j\sin\varphi)$$

$$Z_{\mathrm{eq}} = Z_1 + Z_2//Z_3$$

$$= [4 + (16 - j8)]\ \mathrm{k\Omega} = (20 - j8)\ \mathrm{k\Omega} \approx \boxed{21.54\angle -21.8^\circ\ \mathrm{k\Omega}}$$

(b)
$$\dot{I}_3 = \frac{z_2}{z_2+z_3}\dot{I}_1$$

$$\dot{I}_1 = \frac{\dot{E}}{z_{\mathrm{eq}}} = \frac{100\angle 0^\circ\ \mathrm{V}}{21.54\angle -21.8^\circ\ \Omega} \approx 4.64\angle 21.8^\circ\ \mathrm{mA}$$

$$\therefore\ \dot{I}_3 = \frac{z_2}{z_2+z_3}\dot{I}_1$$

$$= 4.64\angle 21.8^\circ\ \mathrm{mA}\frac{8\angle -90^\circ\ \mathrm{k\Omega}}{(-j8+4+j8)\ \mathrm{k\Omega}} = \frac{37.12\angle -68.2^\circ}{4\angle 0^\circ}\ \mathrm{mA}$$

$$= \boxed{9.28\angle -68.2^\circ\ \mathrm{mA}}$$

Example 10.4: Determine the voltage across the inductor L for the circuit in Figure 10.12(a).

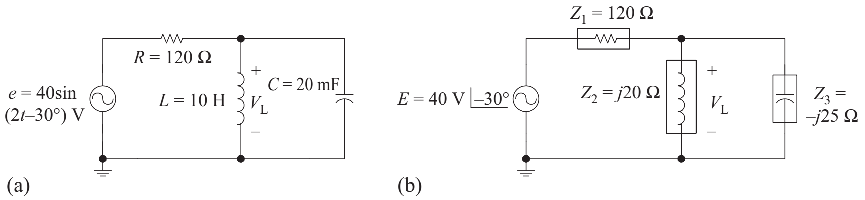


Figure 10.12 Figure for Example 10.4

Solution:

- Convert the time domain to the phasor domain as shown in Figure 10.12(b) first.

$$Z_1 = R = 120\ \Omega$$

$$Z_2 = jX_L = j(\omega L) = j(2\times 10\ \text{H}) = j20\ \Omega$$

$$e = 40\sin(\overset{\omega}{2}t - 30^\circ)\ \text{V}$$

$$Z_3 = -jX_C = -j\frac{1}{\omega C} = -j\frac{1}{2\times 20\text{mF}} = -j\,25\ \Omega$$

$$e = 40\sin(2t - 30^\circ)\ \text{V} \quad \rightarrow \quad \dot{E} = 40\angle -30^\circ\ \text{V}$$

$$\dot{V}_L = \dot{E}\frac{Z_2//Z_3}{Z_1 + Z_2//Z_3} \qquad (Z_2//Z_3 = ?)$$

$$\dot{V}_2 = \frac{z_2}{z_1 + z_2}\dot{E}$$

$$Z_2//Z_3 = \frac{Z_2Z_3}{Z_2 + Z_3} = \frac{j20\,(-j25)}{j20 - j25}\Omega = \frac{500}{-j5}\Omega = j100\Omega$$

$$j^2 = -1, \quad \frac{1}{-j} = j$$

$$\therefore\ \dot{V}_L = \dot{E}\frac{Z_2//Z_3}{Z_1 + Z_2//Z_3} = (40\angle -30^\circ)\ \text{V}\cdot\frac{j100}{120 + j100}\Omega$$

$$j = 90^\circ$$

$$\approx \frac{4{,}000\ \angle 60^\circ}{156.2\ \angle 39.8^\circ}\text{V} \approx 25.61\ \angle 20.2^\circ\ \text{V}$$

- After converting the phasor form to the time form gives

$$\boxed{V_L = 25.61\sin(2t + 20.2^\circ)\ \text{V}}$$

10.3 Power in AC circuits

10.3.1 Instantaneous power

Instantaneous power *p*

- There are different types of power in AC circuits such as instantaneous power, active power, reactive power, and apparent power.
- The instantaneous power p is the power dissipated in a component of an AC circuit at any instant time.
- The instantaneous power p is the product of instantaneous voltage v and current i at that particular moment (Figure 10.13), i.e., instantaneous power can be expressed as:

$$\boxed{p = vi}$$

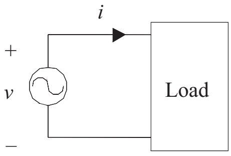


Figure 10.13 Instantaneous power

Derive the equation of the instantaneous power

- If $v = V_m \sin(\omega t + \varphi)$ and $i = I_m \sin\omega t$

  Then, $p = v\,i = V_m I_m \sin\omega t \sin(\omega t + \varphi)$

$$\because \qquad -\sin x \sin y = \frac{1}{2}[\cos(x+y) - \cos(x-y)]$$

$$\therefore \quad p = -\frac{1}{2} V_m I_m [\cos(2\omega t + \varphi) - \cos\varphi] \qquad x = \omega t, \quad y = \varphi$$

$$= VI\cos\varphi - VI\cos(2\omega t + \varphi) \qquad (V_m = \sqrt{2}V, I_m = \sqrt{2}I)$$

$$= VI\cos\varphi - VI(\cos 2\omega t \cos\varphi - \sin 2\omega t \sin\varphi) \qquad x = 2\omega t, \quad y = \varphi$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

- Therefore, instantaneous power is given as

$$p = VI\cos\varphi\,(1 - \cos 2\omega t) + VI\sin\varphi \sin 2\omega t$$

10.3.2 The waveform of instantaneous power

The waveform of instantaneous power

- The waveform of the instantaneous power p can be obtained from the product of instantaneous voltage v and current i at each point on their waveforms as shown in Figure 10.14.
- Such as:
  - at time $t = 0$: $i = 0, \quad p = vi = 0$
  - at time $t = t_1$: $v = 0, \quad p = vi = 0$
  - between time $0 \sim t_1$: $v > 0$ and $i > 0$, $\therefore p = vi > 0$
  - between time $t_1 \sim t_2$: $v < 0$ and $i > 0$, $\therefore p = vi < 0$
  - between time $t_2 \sim t_3$: $v < 0$ and $i < 0$, $\therefore p = vi > 0$

Instantaneous power and energy

- When instantaneous power $p > 0$ (p is positive), the component stores energy provided by the source.
- When instantaneous power $p < 0$ (p is negative), the component returns the stored energy to the source.

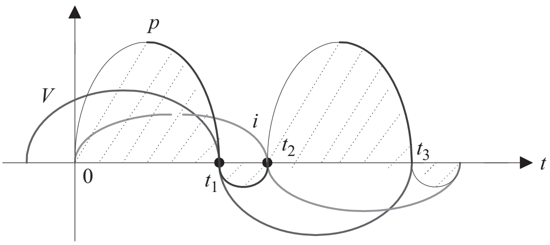


Figure 10.14 The waveform of instantaneous power

Instantaneous power p	p is the product of instantaneous voltage and current at any instant time. $p = vi = VI\cos\varphi\,(1 - \cos 2\omega t) + VI\sin\varphi \sin 2\omega t$. $p > 0$: the component absorbs (stores) energy. $p < 0$: the component returns (releases) energy.

10.3.3 *Instantaneous power for a resistive component*

The formula of instantaneous power for a resistive load

- Since voltage and current in a purely resistive circuit is in phase, i.e., $\varphi = 0$, substituting this into the equation of the instantaneous power gives

$$\begin{aligned} p_R &= vi \\ &= VI\cos\varphi(1 - \cos 2\omega t) + VI\sin\varphi \sin 2\omega t \\ &= VI\cos 0^\circ(1 - \cos 2\omega t) + VI\sin 0^\circ \sin 2\omega t && \varphi = 0 \\ &= VI - VI\cos 2\omega t && \cos 0^\circ = 1, \sin 0^\circ = 0 \end{aligned}$$

- Instantaneous power for a resistive load (p_R):

$$\boxed{p_R = VI(1 - \cos 2\omega t)} \tag{10.1}$$

The waveform of instantaneous power for a resistive load

- The first part VI in (10.1) is average power dissipated in the resistive load ($p > 0$, the load absorbs power).
- The second part in (10.1) is a sinusoidal quantity with a double frequency 2ω, this indicates that when voltage and current waveforms oscillate one full cycle in one period of time, power waveform will oscillate two cycles as illustrated in Figure 10.15.
- The mathematic equation and the waveform all show that instantaneous power of a resistive load is always positive, or a resistor always dissipates power, indicating that the resistor is an energy-consuming element.

(The current is always in phase with the voltage, and the generator is delivering positive power.)

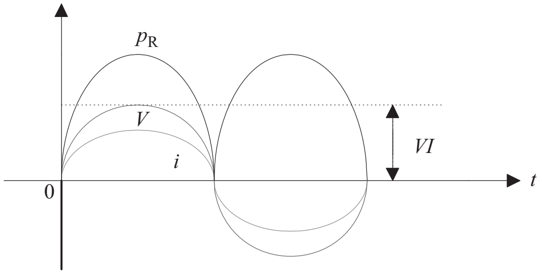


Figure 10.15 The waveform of instantaneous power for an R load

10.3.4 Instantaneous power for inductive/capacitive components

Formulas of the instantaneous power for inductive/capacitive loads

- In a purely inductive load circuit, voltage leads current by 90°.
- In a purely capacitive circuit, voltage lags current by 90°.
- Substituting $\varphi = \pm 90^\circ$ into the equation of instantaneous power gives

$$\begin{aligned} p &= VI\cos\varphi(1-\cos 2\omega t) + VI\sin\varphi\sin 2\omega t \\ &= VI\cos(\pm 90^\circ)(1-\cos 2\omega t) + VI\sin(\pm 90^\circ)\sin 2\omega t && \varphi = \pm 90^\circ \\ &= \pm VI\sin 2\omega t && (10.2) \end{aligned}$$

$$\cos 90^\circ = 0, \sin(\pm 90^\circ) = \pm 1$$

- The instantaneous power for inductive and capacitive loads can be obtained from (10.2):
  - Instantaneous power for an inductive load: $p_L = VI\sin 2\omega t$
  - Instantaneous power for a capacitive load: $p_C = -VI\sin 2\omega t$

The waveforms of instantaneous power for inductive/capacitive loads

- The diagrams of instantaneous power for inductive and capacitive loads are illustrated in Figure 10.16.
- As seen from the formula (10.2) and waveforms in Figure 10.16, both the instantaneous powers of inductive and capacitive loads are sinusoidal quantities with a double frequency 2ω. They have an average value of zero over a complete cycle since the positive and negative waveforms will cancel each other out.
- When instantaneous power is positive, the component stores energy; when instantaneous power is negative, the component releases energy.
- Therefore, the inductor and capacitor do not absorb power, they convert or transfer energy between the source and elements. This also indicates that the inductor and capacitor are energy storage elements.

Instantaneous power for R, L, and C components	– $p_R = VI(1-\cos 2\omega t)$ – $p_L = VI\sin 2\omega t$ – $p_C = -VI\sin 2\omega t$

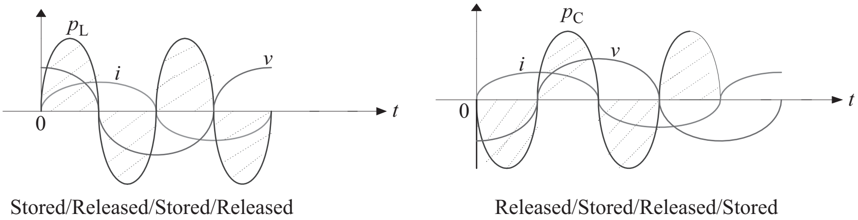


Figure 10.16 The waveforms of instantaneous power for L and C loads

10.3.5 Active power (or average power)

The active power *P*

- The active power is also known as average power, which is the product of the RMS voltage and RMS current in an AC circuit.
- The active power is actually the average power dissipation on the resistive load, i.e., the average power within one period of time (one full cycle) for a sinusoidal power waveform in an AC circuit.
- The active power is also called true or real power since the power is really dissipated by the load resistor, and it can be converted to useful energy such as heat or light energy, etc. (Electric stoves and lamps are examples of this kind of resistive load.)
- The instantaneous power always varies with time and is difficult to measure, so it is not very practical to use. Since it is the actual power dissipated in the load, average or active power P is used more often in AC sinusoidal circuits.
- Average power is easy to measure by an AC power meter (an instrument to measure AC power) in an AC circuit.
- Average power is the average value of instantaneous power in one period of time. It can be obtained from integrating for instantaneous power in one period of time.

Derive active power

- Note: If you have not learned calculus, then just keep in mind that $P = VI\cos\varphi$ is the equation for average power, and skip the following mathematical derivation process.

$$P = VI\cos\varphi(1 - \cos 2\omega t) + VI\sin\varphi\sin 2\omega t$$

$$P = \frac{1}{\mathrm{T}}\int_0^T p(t)dt = \frac{1}{\mathrm{T}}\int_0^T [VI\cos\varphi(1 - \cos 2\omega t) + VI\sin\varphi\sin 2\omega t]d\omega t$$

$$P = \frac{1}{\mathrm{T}} VI\cos\varphi(\omega t)|_0^T + VI\sin\varphi\frac{1}{\mathrm{T}}\int_0^T \sin 2\omega t\, d\omega t = VI\cos\varphi \tag{10.3}$$

- Where φ is a constant, and ωt is a variable, so the first part of the integration is a constant $VI\cos\varphi$. The integration of the second part is zero (integrating for sine function), since the average value for a sine wave in one period of time is zero.
- The active power or average power: $P = VI\cos\varphi$

10.3.6 Active power and φ

Absorbs or release active power

- Active or average power P is a constant. It consists of the product of RMS values of voltage and current VI and $\cos\varphi$, where $\cos\varphi$ is called power factor (it will be discussed at the end of this section).
- When active power $P > 0$, the element absorbs power;

When active power $P < 0$, the element releases power.

Active power and φ

- When $\varphi = 0°$, the voltage and current are in phase, the circuit is a purely resistive circuit, and $P_R = VI\cos 0° = VI$ $\qquad \cos 0° = 1$

  Therefore, $P_R = VI = I^2R = \dfrac{V^2}{R}$

  or $P_R = VI = \dfrac{V_m}{\sqrt{2}}\dfrac{I_m}{\sqrt{2}} = \dfrac{1}{2}V_mI_m$ $\qquad V = \dfrac{V_m}{\sqrt{2}}, I = \dfrac{I_m}{\sqrt{2}}$
- When $\varphi = 90°$, the voltage leads the current by 90°, the circuit is a purely inductive circuit, and $P_L = VI\cos 90° = 0$ $\qquad \cos 90° = 0$

  i.e., $P_L = 0$
- When $\varphi = -90°$, the current leads the voltage by 90°, the circuit is a purely capacitive circuit, and $P_C = VI\cos(-90°) = 0$ $\qquad \cos(-90°) = 0$

  i.e., $P_C = 0$

Active power *P* (or average power, real power, and true power)	The active power is the average value of the instantaneous power that is actually dissipated by the load. $P = VI\cos\varphi$ The unit of P: Watt (W) – When $\varphi = 0°$ $P_R = VI = \frac{1}{2}V_mI_m = I^2R = \frac{V^2}{R}$ – When $\varphi = 90°$ $P_L = 0$ – When $\varphi = -90°$ $P_C = 0$

10.3.7 Reactive power

Reactive power *Q*

- Since the effect of charging/discharging in a capacitor C and storing/releasing energy in an inductor L is that energy is only exchanged or transferred back and forth between the source and the component and will not do any real work for the load.
- The reactive power Q can describe the maximum velocity of energy transferring between the source and the storage element L or C.

Calculating reactive power

- The first part in (10.3) is active or average power. The integration of the second part of (10.3) is zero, and that is the reactive power.

$$P = \frac{1}{T}VI\cos\varphi(\omega t)|_0^T + VI\sin\varphi\frac{1}{\mathrm{T}}\int_0^T \sin 2\omega t\, d\omega t = VI\cos\varphi \qquad (10.3)$$

- While energy is converting between the source and energy store elements, the load will do not do any actual work, and average power dissipated on the load will be zero.

- Because the physical meaning of the reactive power is the *maximum* velocity of energy conversion between the energy storing element and the source, the peak value of the second part is reactive power, denoted as Q.
- Reactive power can be expressed mathematically as $Q = VI \sin\varphi$.
- The reactive power Q is measured in volt-ampere reactive (VAR).

Reactive power and φ

- When $\varphi = 0°$, the circuit is a purely resistive circuit:

$$Q_R = VI \sin 0° = 0 \qquad \sin 0° = 0$$

- When $\varphi = 90°$, the circuit is a purely inductive circuit:

$$Q_L = VI \sin 90° = VI \qquad \sin 90° = 1$$

Substituting $V = I X_L$ or $I = \frac{V^2}{X_L}$ into Q_L gives

$$Q_L = VI = I^2 X_L = \frac{V^2}{X_L}$$

- When $\varphi = -90°$, the circuit is a purely capacitive circuit:

$$Q_C = VI \sin(-90°) = -VI \qquad \sin(-90°) = -1$$

Substituting $V = IX_C$ or $I = \frac{V}{X_C}$ into Q_C gives

$$Q_C = -VI = -I^2 X_C = -\frac{V^2}{X_C}$$

Reactive power Q	Q is the maximum velocity of energy conversion between the source and energy-storing element. $Q = VI \sin\varphi$ The unit of Q: volt-ampere reactive (VAR) – $\varphi = 0°$: $Q_R = 0$ – $\varphi = 90°$: $Q_L = VI = I^2 X_L = \frac{V^2}{X_L}$ – $\varphi = -90°$: $Q_C = -VI = -I^2 X_C = -\frac{V^2}{X_C}$

Absorbs or release reactive power

- When power > 0, the element absorbs power;
  When power < 0, the element releases power.
- Since Q_L is positive ($Q_L > 0$) and Q_C is negative ($Q_C < 0$),
  - the inductor absorbs (consumes) reactive power,
  - the capacitor produces (releases) reactive power.

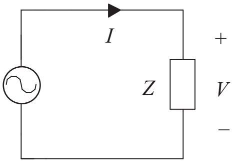


Figure 10.17 Apparent power

10.3.8 Apparent power

Apparent power *S*

- When the voltage V across a load produces a current I in the circuit of Figure 10.17, the power produced in the load is the product of voltage and current VI.
- If the load Z includes both the resistor and storage element inductor or capacitor, then VI will be neither a purely active power nor a purely reactive power.
- Since VI is the expression of the power equation, it is called apparent power.
- Apparent power is the maximum average power rating that a source can provide to the load or maximum capacity of an AC source and is denoted as S.

Calculating apparent power

- The mathematical formula of apparent power is the product of the source current and voltage, i.e.,

$$\boxed{S = IV}$$

- Apparent power S is measured in volt-amperes (VA).
- Substituting $I = \frac{V}{Z}$ or $V = IZ$ into apparent power $S = IV$ gives

$$S = I^2 Z = \frac{V^2}{Z}$$

Usually the power listed on the nameplates of electrical equipment is the apparent power.

Apparent power *S*	– S is the maximum average power rating that a source can provide to an AC circuit. – $S = IV = I^2 Z = \frac{V^2}{Z}$ – The unit of S: volt-amperes (VA)

10.3.9 Power triangle

The relationships between *P*, *Q*, and *S*

- What are the relationships between the active power P, reactive power Q and apparent power S in AC circuits?
- These three powers (P, Q, and S) related to one another in a right triangle, is called the power triangle, and can be derived as follows.

Circuit triangles

- For a series resistor, inductor and capacitor circuit, if the circuit is more inductive ($X = X_L - X_C > 0$, $\varphi > 0$), then the impedance triangle, voltage triangle and current triangle can be illustrated as shown in Figure 10.18(a)–(c) (refer to Section 10.1).
- If we multiply all quantities on each side of the voltage triangle by the current I, it will yield

$$VI = S, \qquad IV_X = Q, \qquad \text{and} \qquad V_R I = P$$

- This can be illustrated as a power triangle as shown in Figure 10.18(d).
- If the circuit load is more capacitive ($X_C > X_L$, $\varphi < 0$), the circuit triangles will be opposite to the inductive circuit triangles as shown in Figure 10.19.

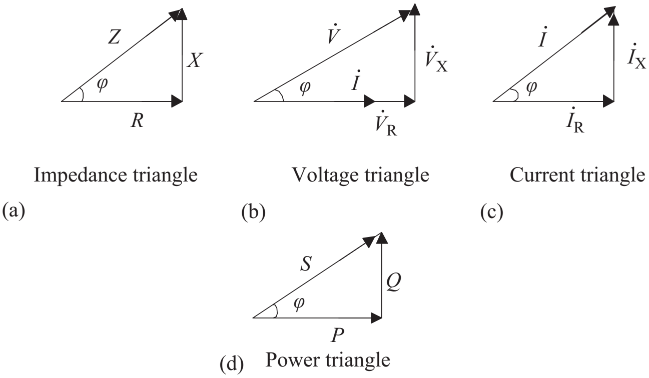

Z
X
φ
R
Impedance triangle
(a)
$\dot{V}$
$\dot{V}_X$
φ
$\dot{I}$
$\dot{V}_R$
Voltage triangle
(b)
$\dot{I}$
$\dot{I}_X$
φ
$\dot{I}_R$
Current triangle
(c)
S
Q
φ
P
(d) Power triangle

Figure 10.18 Circuit triangles for an more inductive circuit

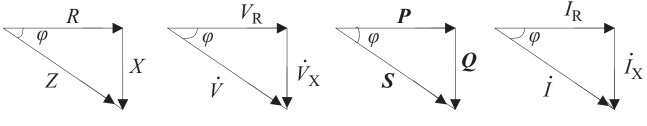

R
φ
Z
X
$\dot{V}_R$
φ
$\dot{V}$
$\dot{V}_X$
P
φ
S
Q
$\dot{I}_R$
φ
$\dot{I}$
$\dot{I}_X$

Figure 10.19 Circuit triangles for a more capacitive circuit

10.3.10 Impedance angle and phasor power

Impedance angle φ

- The impedance triangle indicates that it has an angle φ between resistance R and impedance Z of the circuit. It is called the impedance angle; φ is also in the power triangle.
- φ is also called the power factor angle. (Later on we will introduce the power factor $\cos\varphi$.)
- The impedance angle φ can be obtained from circuit triangles (either inductive or capacitive circuit in Figure 10.18 or Figure 10.19) and can be expressed as

$$\varphi = \tan^{-1}\frac{Q}{P} = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{\dot{V}_{\mathrm{X}}}{\dot{V}_{\mathrm{R}}} = \tan^{-1}\frac{\dot{I}_{\mathrm{X}}}{\dot{I}_{\mathrm{R}}}$$

The relationship between different powers in the power triangle

- The relationship between different powers in the power triangle can be obtained from the Pythagoras theorem, i.e.,

$$S = \sqrt{P^2 + Q^2}$$

- Active power P and reactive power Q can be expressed with the impedance angle φ and obtained from the power triangle in Figure 10.18 or Figure 10.19 as

$$P = S\cos\varphi$$

and $$Q = S\sin\varphi$$

Phasor power

- If express by complex numbers, apparent power S is

$$\dot{S} = P + jQ$$

  This is known as the phasor power.
- The phasor apparent power can also be expressed as

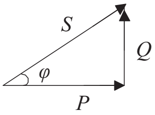
$$\dot{S} = \dot{V}\dot{I} = \dot{I}^2 Z = \frac{\dot{V}^2}{Z}$$

10.3.11 Power in AC circuits

Different types of power in AC circuits are summarized in Table 10.2.

Table 10.2 Power in AC circuits

Power	**General expression**	**R**	**L**	**C**
Instantaneous power	$p = VI\cos\varphi\,(1 - \cos 2\omega t) + VI\sin\varphi\sin 2\omega t$	$p_R = VI\,(1 - \cos 2\omega t)$	$p_L = VI\sin 2\omega t$	$p_C = -VI\sin 2\omega t$
Active power	$p = VI\cos\varphi$	$P_R = VI = \frac{1}{2}V_m I_m = I^2 R = \frac{V^2}{R}$	$P_L = 0$	$P_C = 0$
Reactive power	$Q = VI\sin\varphi$	$Q_R = 0$	$Q_L = VI = I^2 X_L = \frac{V^2}{X_L}$	$Q_C = -VI = -I^2 X_C = -\frac{V^2}{X_C}$
Apparent power	$S = IV = I^2 Z = \frac{V^2}{Z}$			

Power triangle	 $P = S\cos\varphi, Q = S\sin\varphi, S = \sqrt{P^2 + Q^2}$ Impedance angle: $Q = \tan^{-1}\frac{Q}{P} = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{\dot{V}_X}{\dot{V}_R} = \tan^{-1}\frac{\dot{I}_X}{\dot{I}_R}$ Phasor power: $\dot{S} = \dot{V}\dot{I} = \dot{I}^2 Z = \frac{\dot{V}^2}{Z}, \dot{S} = P + jQ$

10.3.12 Power factor

Power factor PF

- The ratio of active power P and apparent power S is called the power factor PF, and represented by $\cos\varphi$.
- The power factor also can be obtained from the power triangle as $\mathrm{PF} = \frac{P}{S} = \cos\varphi$

Resistive and reactive circuit

- For a purely resistive circuit ($\varphi = 0°$), the reactive power Q is zero, so
  - the apparent power S is equal to the active power P, i.e.,

$$S = \sqrt{P^2 + Q^2} = \sqrt{P^2 + 0^2} = P$$

  - the power factor is 1, i.e., $\cos\varphi = \frac{P}{S} = \frac{P}{P} = 1$
  - 1 is the maximum value for the power factor $\cos\varphi$.
- For a purely reactive load ($\varphi = \pm 90°$), active power P in the circuit is zero, so the power factor is also zero, i.e., $\cos\varphi = \frac{P}{S} = \frac{O}{P} = 0$
- The range of the power factor $\cos\varphi$:
  - $\cos\varphi$ is between 0 and 1.
  - The impedance angle φ is between 0° and ±90°. $\cos(\pm 90°) = 0$, $\cos 0° = 1$

The power factor is an important factor in circuit analysis

- The circuit source will produce active power P to the load, and the amount of the active power P can be determined by the power factor $\cos\varphi$. (This is indicated in the equation of $P = S\cos\varphi$)
- If the power factor $\cos\varphi$ of the load is the maximum value of 1, the active power produced by the source is the maximum capacity of the source, and all the energy supplied by the source will be consumed by the load ($P = S$, $\cos\varphi = 1$).
- If the power factor $\cos\varphi$ decreases, the active power P produced by the source will also decrease accordingly ($P\downarrow = S\cos\varphi\downarrow$).

10.3.13 Power factor correction

Power-factor correction can increase the power factor

- Increasing the power factor can increase the real power in a circuit. But how to increase the power factor of a circuit? A method called power-factor correction can be used.
- Power-factor correction can increase the power factor and does not affect the load voltage and current.
- Since most of the loads of the electrical systems are inductive loads (such as the loads that are driven by a motor), an inductive load in parallel with a capacitor (Figure 10.20(b)) can increase the power factor of the load.

Power-factor correction can reduce the line power loss

- The power triangle in Figure 10.20(c) indicates that when a capacitor C is in parallel with the inductive load, the reactive power Q in the circuit will be reduced to Q' ($Q' = Q - Q_C$). Therefore,
  - the impedance angle will reduce from φ to φ',
  - the power factor $\cos\varphi$ will increase to $\cos\varphi'$.

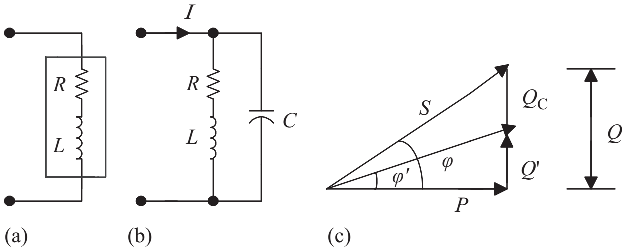


Figure 10.20 Increasing the power factor

- Since $\varphi\downarrow \rightarrow \cos\varphi\uparrow$, for instance, $\cos 30° = 0.866$ is $> \cos 60° = 0.5$, the total current I will also decrease, since $I\downarrow = \dfrac{P}{V\cos\varphi\uparrow}$. $(P = S\cos\varphi = VI\cos\varphi)$
- This can reduce the source current and line power loss (I^2R). This is why increasing the power factor has a significant meaning.

10.3.14 Total power

P_T, Q_T, and S_T

- When calculating the total power in a complicated series–parallel circuit, determine the active power P and reactive power Q in each branch first.
- P_T: The sum of all the active powers is the total active power P_T.

$$P_T = P_1 + P_2 + \ldots + P_n$$

- Q_T: The difference between Q_{LT} and Q_{CT} is the total reactive power Q_T.

$$Q_T = Q_{LT} - Q_{CT} = (Q_{L_1} + Q_{L_2} + \cdots) - (Q_{C_1} - Q_{C_2} + \ldots)$$

  - Q_{LT} is the sum of all reactive powers for the inductors.
  - Q_{CT} is the sum of all reactive powers for the capacitors.

- S_T: The total apparent power S_T can be determined by using Q_T and P_T using the Pythagoras' theorem, i.e., $S_T = \sqrt{P_T{}^2 + Q_T{}^2}$

Total power factor PF_T

- PF_T: The total power factor PF_T can be determined by using the total active and reactive power.
- Calculating PF_T: $\text{PF}_\text{T} = \cos\varphi_T = \dfrac{P_T}{S_T}$

Power factor (cos φ)	– Power factor PF: $\cos\varphi = \dfrac{P}{S}$ $(0 \le \cos\varphi \le 1, \cos\varphi$—without unit) – When $\cos\varphi = 1$: All energy supplied by the source is consumed by the load. – Power factor correction: An inductive load in parallel with a capacitor can increase $\cos\varphi$.

Total power	– Total active power: $P_T = P_1 + P_2 + \ldots + P_n$ – Total reactive power: $Q_T = Q_{LT} - Q_{CT} = (Q_{L_1} + Q_{L_2} + \ldots) - (Q_{C_1} + Q_{C_2} + \ldots)$ Q_{LT}: the total reactive power for inductors. Q_{CT}: the total reactive power for capacitors. – Total apparent power: $S_T = \sqrt{P_T{}^2 + Q_T{}^2}$ – Total power factor: $\mathrm{PF_T} = \cos\varphi_T = \dfrac{P_T}{S_T}$

10.3.15 Power factor examples

Example 10.5: Determine the total power factor cos φ in the circuit of Figure 10.21 and plot the power triangle for this circuit.

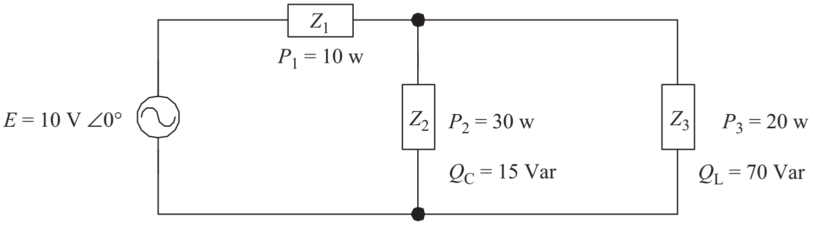


Figure 10.21 Figure for Example 9.5

Solution:

- Total power factor: $\mathrm{PF_T} = \cos\varphi_T = \dfrac{P_T?}{S_T?}$, $\quad S_T = \sqrt{P_T?^2 + Q_T?^2}$

  (The symbol "?" indicates an unknown.)

  – Total active power: $P_T = P_1 + P_2 + P_3 = 10\text{ W} + 30\text{ W} + 20\text{W} = 60\text{ W}$
  – Total reactive power: $Q_T = Q_{LT} - Q_{CT} = 70\text{ Var} - 15\text{ Var} = 55\text{ Var}$
  – Total apparent power: $S_T = \sqrt{P_T{}^2 + Q_T{}^2} = \sqrt{60^2 + 55^2} = 81.39\text{ VA}$
  – Total power factor:

$$\mathrm{PF_T} = \cos\varphi_T = \frac{P_T}{S_T} = \frac{60\text{ W}}{81.39\text{ VA}} \approx 0.74 \qquad \cos\varphi \text{ – no unit}$$

- Impedance angle: $\varphi = \cos^{-1}\varphi_T = \cos^{-1}0.74 \approx 42.3°$
- Power triangle Figure 10.22:

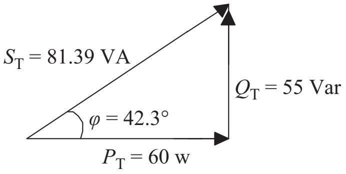

S_T = 81.39 VA
Q_T = 55 Var
φ = 42.3°
P_T = 60 w

Figure 10.22 The power triangle for Example 10.5

Example 10.6: Determine the following values in the circuit shown in Figure 10.23:

(a) the total power P_T, Q_T, and S_T for the circuit
(b) power factor $\cos\varphi$
(c) power triangle
(d) source current I
(e) the capacitance C needed to increase the power factor $\cos\varphi$ to 0.87
(f) the source current I' after increasing the power factor

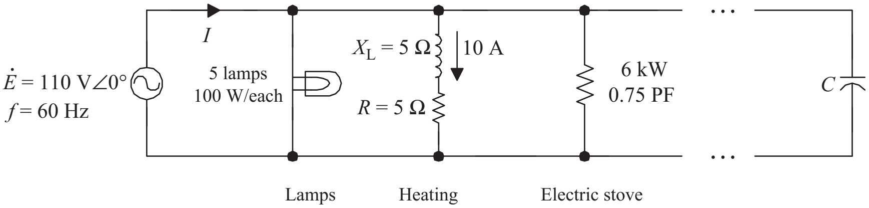

I
$\dot{E}$ = 110 V∠0°
f = 60 Hz
5 lamps
100 W/each
X_L = 5 Ω
10 A
R = 5 Ω
6 kW
0.75 PF
C
Lamps
Heating
Electric stove

Figure 10.23 Figure for Example 10.6

Solution:

- Lamp: $P_1 = 5 \times 100\text{ W} = 500\text{ W}$ 5 lamps
- Heating: $P_2 = I^2R = (10\text{ A})^2(5\ \Omega) = 500\text{ W}$

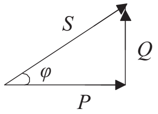
$$Q_2 = I^2X_L = (10\text{ A})^2(5\ \Omega) = 500\text{ Var}$$

– Electric stove: $P_3 = 6$ kW $= 6{,}000$ W

$$\varphi = \cos^{-1}0.75 \approx 41.4^\circ$$
$$Q_3 = P_3 \tan\varphi = (6{,}000\ \mathrm{W})(\tan 41.4^\circ) \approx 5{,}290\ \mathrm{Var}$$

$\mathrm{PF} = \cos\varphi = 0.75$

$\varphi = \tan^{-1}Q/P$



– Total power:

$$P_\mathrm{T} = P_1 + P_2 + P_3 = 500\ \mathrm{W} + 500\ \mathrm{W} + 6{,}000\ \mathrm{W} = 7{,}000\ \mathrm{W}$$
$$Q_\mathrm{T} = Q_1 + Q_2 + Q_3 = 0 + 500\ \mathrm{Var} + 5{,}290\ \mathrm{Var} = 5{,}790\ \mathrm{Var}$$
$$S_\mathrm{T} = \sqrt{P_\mathrm{T}{}^2 + Q_\mathrm{T}{}^2} = \sqrt{7{,}000^2 + 5{,}790^2} \approx 9{,}084.3\ \mathrm{VA}$$

(a) Power factor: $\mathrm{PF_T} = \cos\varphi_\mathrm{T} = \dfrac{P_\mathrm{T}}{S_\mathrm{T}} = \dfrac{7{,}000}{9{,}084.3} \approx 0.77$

(b) Power triangle (as shown in Figure 10.24):

$$\varphi = \cos^{-1}0.77 \approx 39.7^\circ$$

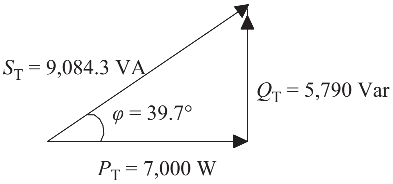


Figure 10.24 Power triangle for Example 10.6

(c) Source current I: $\because\ S_\mathrm{T} = EI$

$$\therefore\ I = \frac{S_\mathrm{T}}{E} = \frac{9{,}084.3\ \mathrm{VA}}{110\ \mathrm{V}} \approx 82.6\ \mathrm{A}$$

Therefore $\dot{I} = 82.6\angle{-39.7^\circ}$ A $\qquad \dot{I} = I\angle\varphi$

(Voltage leads current or current lags voltage in the inductive load, so $\varphi = -39.7^\circ$)

(d) The capacitance C that needs to increase the power factor to 0.87 can be determined by the following way:

$$C = \frac{1}{2\pi f X_\mathrm{C}?} \Rightarrow Q_\mathrm{C} = -\frac{V^2}{X_\mathrm{C}} \Rightarrow X_\mathrm{C} = -\frac{V^2}{Q_\mathrm{C}?} \Rightarrow \qquad \left(X_\mathrm{C} = \frac{1}{2\pi fC}\right)$$
$$Q_\mathrm{T} = Q_\mathrm{C} + Q_\mathrm{T}'? \quad \Rightarrow \quad Q_\mathrm{T}' = P_\mathrm{T}\tan\varphi'? \quad \Rightarrow \quad \varphi' = \cos^{-1}0.87$$

(as shown in Figure 10.25)

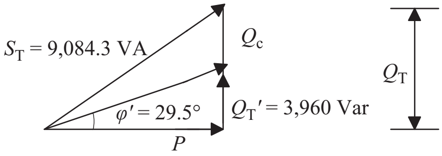


Figure 10.25 New power factor angle

- To increase the power factor to 0.87, the power factor angle should be reduced to
  $\varphi' = \cos^{-1}0.87 \approx 29.5°$
- The new power factor angle φ' is shown in Figure 10.25.
- The reactive power can be determined from the above equation as

$$Q_T' = P_T \tan \varphi' = 7{,}000 \tan 29.5° \approx 3{,}960 \text{ Var}$$

- Q_C can be obtained from Figure 10.25:

$$Q_C = Q_T - Q_T' = 5{,}790 - 3{,}960 = 1{,}830 \text{ Var}$$

$$X_C = -\frac{V^2}{Q_C} = -\frac{E^2}{Q_C} = -\frac{110^2 \text{ V}}{1{,}830 \text{ Var}} \approx 6.61 \ \Omega$$

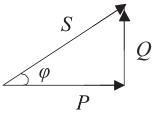
(the voltage across X_L and R is equal to E)

- C: $C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{2\pi(60 \text{ Hz})(6.61 \ \Omega)} \approx 0.0004 \text{ F} = 400 \ \mu\text{F}$

The capacitance C needed to increase the power factor to 0.87 should be 400 μF.

(e) The source current I' after increasing the power factor can be determined by the following equation:

$$P = S \cos\varphi = IE \cos\varphi$$



$$\text{So, } I' = \frac{P_T}{E\cos\varphi'} = \frac{7{,}000 \text{ W}}{110 \text{ V} \cos 29.5°} \approx 73.1 \text{ A}$$

- Comparing with the original source current $I = 82.6$ A from step (d), after a capacitor is in parallel and the power factor is increased, the source current is $I' = 73.1$A.

- So, the source current can decrease 9.5 A ($I - I' = 82.6$ A – 73.1 A = 9.5 A). This can reduce the line power loss (I^2R) and utilize the capacity of the source more efficiently.

10.4 Methods of analyzing AC circuits

10.4.1 Mesh current analysis

Analysis methods

- All analysis methods that we have learned for analyzing DC circuits with one or two more sources can also be used for analyzing AC circuits, such as the branch current analysis, mesh analysis, node voltage analysis, superposition theorem, Thevenin's and Norton's theorems.
- The phasor form will be used to represent the circuit quantities in these analysis methods in AC circuits.
- Since these analysis methods have been discussed in detail for DC circuits (Chapter 4 and 5), some examples will be presented to use these methods in AC circuits or networks. (Reviewing Chapters 4 and 5 before reading the following contents is highly recommended.)

Mesh current analysis

The procedure for applying the mesh current analysis method in an AC circuit:

1. Identify each mesh and label the reference directions for each mesh current clockwise.
2. Apply KVL around each mesh of the circuit, and the numbers of KVL equations should be equal to the numbers of mesh. Sign each self-impedance voltage as positive and each mutual-impedance voltage as negative in KVL equations.
    - Self-impedance: An impedance that only has one mesh current flowing through it.
    - Mutual-impedance: An impedance that is located on the boundary of two meshes and has two mesh currents flowing through it.
3. Solve the simultaneous equations resulting from step 2, and determine each mesh current.

Note:

- Convert the current source to the voltage source first in the circuit, if there is any.
- If the circuit has a current source, the source current will be the same with the mesh current, so the number of KVL equations can be reduced.

10.4.2 Mesh current analysis example

Example 10.7: Use the mesh current analysis method to determine the mesh current I_1 in the circuit of Figure 10.26.

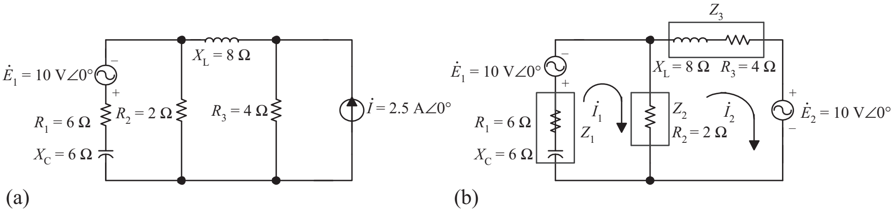

$\dot{E}_1 = 10\ \mathrm{V}\angle 0°$
$R_1 = 6\ \Omega$
$X_C = 6\ \Omega$
$R_2 = 2\ \Omega$
$X_L = 8\ \Omega$
$R_3 = 4\ \Omega$
$\dot{I} = 2.5\ \mathrm{A}\angle 0°$
(a)
$\dot{E}_1 = 10\ \mathrm{V}\angle 0°$
$R_1 = 6\ \Omega$
$X_C = 6\ \Omega$
Z_1
$\dot{I}_1$
Z_2
$R_2 = 2\ \Omega$
$\dot{I}_2$
Z_3
$X_L = 8\ \Omega$ $R_3 = 4\ \Omega$
$\dot{E}_2 = 10\ \mathrm{V}\angle 0°$
(b)

Figure 10.26 Figure for Example 10.7

Solution: Convert the current source to the voltage source (connect *R* and *X* to *Z*) as shown in Figure 10.26(b).

There, $\dot{E}_2 = \dot{I}R_3 = (2.5\mathrm{A}\angle 0°)(4\ \Omega) = 10\ \mathrm{V}\angle 0°$

1. Label all the reference directions for each mesh current $\dot{I}_1$ and $\dot{I}_2$ (clockwise), as shown in Figure10.26(b).
2. Write KVL around each mesh, and the number of KVL is equal to the number of meshes (there are two meshes in Figure 10.26(b)).

   Sign each self-impedance voltage as positive, and each mutual-impedance voltage as negative in KVL ($\Sigma\dot{V} = \Sigma\dot{E}$).

   Mesh 1: $(Z_1 + Z_2)\dot{I}_1 - Z_2\,\dot{I}_2 = -\dot{E}_1$

   Mesh 2: $-Z_2\,\dot{I}_1 + (Z_2 + Z_3)\dot{I}_2 = -\dot{E}_2$

   Substitute the following values of Z_1, Z_2, and Z_3 into the above equations,

   $$\begin{aligned} &Z_1 = (6 - j6)\ \Omega, Z_2 = 2\ \Omega, \text{ and } Z_3 = (4 + j8)\ \Omega \\ &\text{so,}\quad (8 - j6)\dot{I}_1 - 2\dot{I}_2 = -10\ \mathrm{V} \\ &\qquad\ -2\dot{I}_1 + (6 + j8)\dot{I}_2 = -10\ \mathrm{V} \end{aligned}$$

3. Solve the simultaneous equations resulting from step 2 using the determinant method, and determine the mesh current $\dot{I}_1$:

   $$\dot{I}_1 = \frac{\begin{vmatrix} -10\angle 0° & -2 \\ -10\angle 0° & 6 + j8 \end{vmatrix}}{\begin{vmatrix} 8 - j6 & -2 \\ -2 & 6 + j8 \end{vmatrix}} \approx 1.18\angle -151.9°\ \mathrm{A}$$

10.4.3 Node voltage analysis

The procedure for applying the node analysis method in an AC circuit

1. Label the circuit:
    - Label all the nodes and choose one of them to be the reference node.

      Usually ground or the node with the most branch connections should be chosen as the reference node (at which voltage is defined as zero).
    - Assign an arbitrary reference direction for each branch current (this step can be skipped if using the inspection method).
2. Apply KCL to all $n - 1$ nodes except for the reference node (n is the number of nodes).
    - Method 1: Write KCL equations and apply Ohm's law to the equations.
        - Assign a positive sign (+) to the self-impedance voltage and entering node current.
        - Assign a negative sign (−) for the mutual-impedance voltage and exiting node current.
    - Method 2: Convert voltage sources to current sources and write KCL equations using the inspection method.
3. Solve the simultaneous equations and determine each nodal voltage.

The procedure for applying the node voltage analysis method in an AC circuit is demonstrated in the following examples.

10.4.4 Node voltage analysis example

Example 10.8: Write node equations for the circuit in Figure 10.27(a).

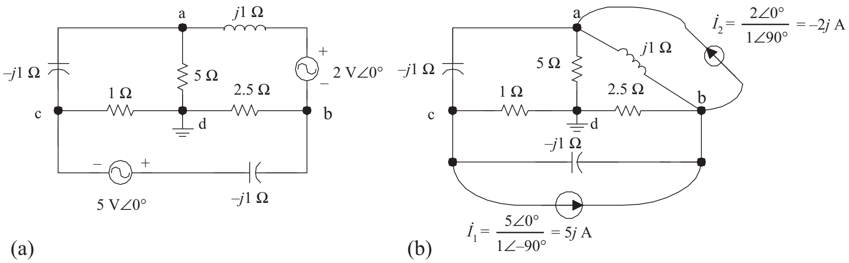

a
j1 Ω
+
2 V∠0°
−
−j1 Ω
5 Ω
1 Ω
2.5 Ω
c
d
b
−
+
5 V∠0°
−j1 Ω
(a)
a
$\dot{I}_2 = \frac{2\angle 0°}{1\angle 90°} = -2j$ A
j1 Ω
5 Ω
−j1 Ω
1 Ω
2.5 Ω
b
c
d
−j1 Ω
$\dot{I}_1 = \frac{5\angle 0°}{1\angle -90°} = 5j$ A
(b)

Figure 10.27 Figure for Example 10.8

1. Label nodes a, b, c, and d, and choose ground d to be the reference node as shown in Figure10.27(a).
2. Convert two voltage sources to current sources from Figure 10.27(a) to Figure 10.27(b), and write KCL equations to $n-1=4-1=3$ nodes by inspection (method 2):

$$\dot{V}_{\mathrm{a}} \qquad\qquad \dot{V}_{\mathrm{b}} \qquad \dot{V}_{\mathrm{c}} \qquad\qquad \dot{I}$$

- Node a $\left(\frac{1}{5}+\frac{1}{-j1}+\frac{1}{j1}\right)\dot{V}_{\mathrm{a}}-\frac{1}{j1}\dot{V}_{\mathrm{b}}-\frac{1}{-j1}\dot{V}_{\mathrm{c}}=-2j$
- Node b $\left(-\frac{1}{j1}\right)\dot{V}_{\mathrm{a}}+\left(\frac{1}{2.5}+\frac{1}{j1}+\frac{1}{-j1}\right)\dot{V}_{\mathrm{b}}-\frac{1}{-j1}\dot{V}_{\mathrm{c}}=5j-(-2j)$
- Node c $\left(-\frac{1}{-j1}\right)\dot{V}_{\mathrm{a}}-\frac{1}{-j1}\dot{V}_{\mathrm{b}}+\left(\frac{1}{1}+\frac{1}{-j1}+\frac{1}{-j1}\right)\dot{V}_{\mathrm{c}}=-5j$

After simplifying

$$\begin{array}{rcrcrcl}
0.2\dot{V}_{\mathrm{a}} & + & j\dot{V}_{\mathrm{b}} & - & j\dot{V}_{\mathrm{c}} & = & -j2 \\
j\dot{V}_{\mathrm{a}} & + & 0.4\dot{V}_{\mathrm{b}} & - & j\dot{V}_{\mathrm{c}} & = & j7 \\
-j\dot{V}_{\mathrm{a}} & - & j\dot{V}_{\mathrm{b}} & + & (1+2j)\dot{V}_{\mathrm{c}} & = & -j5
\end{array}$$

3. Three equations can solve three unknowns that are node voltages.

10.4.5 Superposition theorem

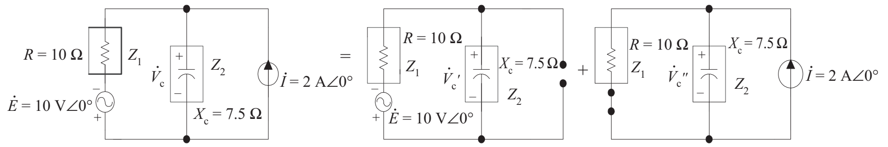
The procedure for applying the superposition theorem in an AC circuit

1. Turn off all power sources except one.
   - Replace the voltage source with the short circuit (placing a jump wire).
   - Replace the current source with an open circuit.
   - Redraw the original circuit with a single source.
2. Analyze and calculate this circuit by using the single source method.
3. Repeat steps 1 and 2 for the other power sources in the circuit.
4. Determine the total contribution by calculating the algebraic sum of all contributions due to single sources.

   Note: The result should be positive when the reference polarity of the unknown in the single source circuit is the same with the reference polarity of the unknown in the original circuit, otherwise it should be negative.

The procedure for applying the superposition theorem in an AC circuit is demonstrated in the following example.

Example 10.9: Determine $\dot{V}_C$ in circuit as shown in Figure 10.28(a) by using the superposition theorem.



(a) (b) (c)

Figure 10.28 Circuits for Example 10.9

Solution:

1. Choose $\dot{E}$ to apply to the circuit first, and use an open circuit to replace the current source $\dot{I}$ as shown in Figure 10.28(b).
2. Calculate $\dot{V}_C{}'$ in the circuit of Figure 10.28(b):

$$\dot{V}_C{}' = \dot{E}\frac{Z_2}{Z_1 + Z_2} = -10\angle 0^\circ\ \text{V}\frac{75\ \Omega\angle -90^\circ}{10\ \Omega - j7.5\ \Omega}$$

$$= \frac{-75\angle -90^\circ}{12.5\angle -36.87^\circ}\text{V} = -6\angle -53.13^\circ\ \text{V} \qquad \boldsymbol{A} = x + jy = \sqrt{x^2+y^2}\tan\frac{y}{x} = \boldsymbol{a}\angle\varphi$$

3. When the current source $\dot{I}$ is applied to the circuit only and the voltage source $\dot{E}$ is replaced by a short circuit, the circuit is as shown in Figure 10.28(c). Calculate $\dot{V}_C{}''$ in Figure 10.28(c):

$$\dot{V}_C{}'' = \dot{I}(Z_1//Z_2)$$

$$Z_1//Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{10(-j7.5)}{10 - j7.5}\ \Omega$$

$$\approx \frac{75\angle -90^\circ}{12.5\angle -36.87^\circ}\ \Omega = 6\angle -53.13^\circ\ \Omega$$

$$\dot{V}_C{}'' = \dot{I}(Z_1//Z_2) = (2\angle 0^\circ \text{A})(6\angle -53.13^\circ\ \Omega) = 12\angle -53.13^\circ\ \text{V}$$

4. Calculate the sum of voltages $\dot{V}_C{}'$ and $\dot{V}_C{}''$:

$$\dot{V}_C = \dot{V}_C{}' + \dot{V}_C{}'' = -6\angle -53.13^\circ\,\text{V} + 12\angle -53.13^\circ\,\text{V}$$

$$= [-6\cos(-53.13^\circ) - 6j\sin(-53.13^\circ) + 12\cos(-53.13^\circ) + 12j\sin(-53.13^\circ)]\,\text{V}$$

$$\approx [-3.6 + j4.8 + 7.2 - j9.6]\,\text{V} = (3.6 - j4.8)\,\text{V} = 6(-53.13^\circ)\,\text{V}$$

10.4.6 Thevenin's and Norton's theorems

The procedure for applying Thevenin's and Norton's theorems in an AC circuit

1. Open and remove the load branch (or any unknown current or voltage branch) in the network, and mark the letter a and b on the two terminals.
2. Determine the equivalent impedance Z_{TH} or Z_N: It should be equal to the equivalent impedance when you look at it from the a and b terminals when all sources are turned off or equal to zero in the network. i.e.

   $$\boxed{Z_{TH} = Z_N = Z_{ab}}$$

   - A voltage source should be replaced by a short circuit.
   - A current source should be replaced by an open circuit.
3.
   - Determine Thevenin's equivalent voltage V_{TH}: It equals the open circuit voltage from the original linear two-terminal network of a and b, i.e., $\boxed{V_{TH} = V_{ab}}$
   - Determine Norton's equivalent current I_N: It equals the short-circuit current for the original linear two-terminal network of a and b, i.e., $\boxed{I_N = I_{SC}}$ where "SC" means the short circuit.
4. Plot Thevenin's or Norton's equivalent circuit, and connect the load (or unknown current or voltage branch) to a and b terminals of the equivalent circuit. Then the load (or unknown) voltage or current can be determined.

The procedure for applying Thevenin's and Norton's theorems method in an AC circuit is demonstrated in the following example.

10.4.7 Thevenin's and Norton's theorems—an example

Example 10.10: Determine the current $\dot{I}_L$ in the load branch of Figure 10.29(a) by using Thevenin's theorem, and use Norton's theorem to check the answer.

Solution:

1. Open and remove the load branch Z_L, and mark a and b on the terminals of the load branch as shown in Figure 10.29(b).
2. Determine Thevenin's equivalent impedance Z_{TH} (the voltage source $\dot{E}$ is replaced by a short circuit) in Figure 10.29(b).

$$Z_{TH} = Z_{ab} = Z_3 + Z_4 + Z_1//Z_2$$

$$Z_{TH} = \left[1 - j1 + \frac{-j2.5(2.5 + j2.5)}{-j2.5 + (2.5 + j2.5)}\right]\Omega = \left[1 - j1 + \frac{6.25 - j6.25}{2.5}\right]\Omega$$

$$= (1 - j1 + 2.5 - j2.5)\,\Omega = (3.5 - j3.5)\,\Omega \approx 4.95\angle -45^\circ\,\Omega$$

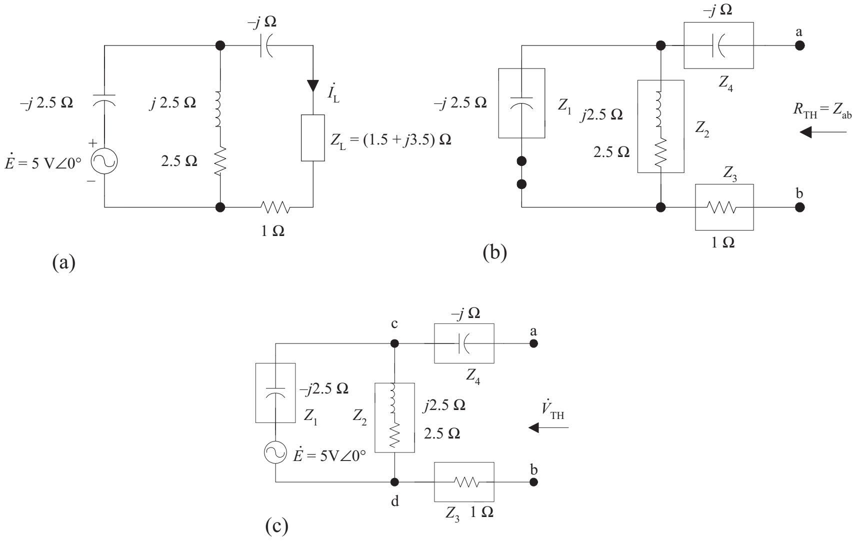

–j Ω
–j 2.5 Ω
j 2.5 Ω
$\dot{I}_L$
+
$\dot{E}$ = 5 V∠0°
–
2.5 Ω
Z_L = (1.5 + j3.5) Ω
1 Ω
(a)
–j Ω
a
Z_4
–j 2.5 Ω
Z_1
j2.5 Ω
Z_2
$R_{TH} = Z_{ab}$
2.5 Ω
Z_3
b
1 Ω
(b)
–j Ω
c
a
Z_4
–j2.5 Ω
Z_1
Z_2
j2.5 Ω
2.5 Ω
$\dot{V}_{TH}$
$\dot{E}$ = 5V∠0°
b
d
Z_3 1 Ω
(c)

Figure 10.29 Circuit for Example 10.10

3. Determine Thevenin's equivalent voltage $\dot{V}_{TH}$ by using Figure 10.29(c) to calculate the open-circuit voltage across the terminals a and b:

$$\dot{V}_{TH} = \dot{V}_{ab} = \dot{V}_{cd}$$

Since $\dot{I} = 0$ for Z_3 and Z_4 in Figure10.29(c), voltages across Z_3 and Z_4 are also zero,

$$\therefore \qquad \dot{V}_{TH} = \dot{V}_{cd} = \dot{E}\frac{Z_2}{Z_1 + Z_2} = 5\angle 0^\circ \text{ V}\frac{2.5 + j2.5}{-j2.5 + (2.5 + j2.5)}\ \Omega$$

$$\approx 5\angle 0^\circ \text{ V}(1.414\angle 45^\circ) \approx 7.07\angle 45^\circ \text{ V}$$

4. Plot Thevenin's equivalent circuit as shown in Figure 10.29(d). Connect the load Z_L to a and b terminals of the equivalent circuit and calculate the load current $\dot{I}_L$.

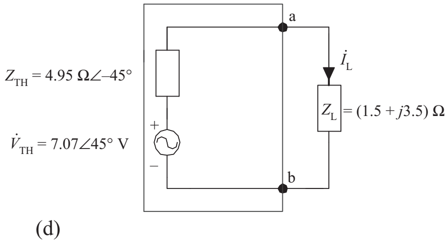

Z_{TH} = 4.95 Ω∠–45°
+
$\dot{V}_{TH}$ = 7.07∠45° V
–
a
b
$\dot{I}_L$
Z_L = (1.5 + j3.5) Ω
(d)

Figure 10.29(d) Thevenin's equivalent circuit for Example 10.10

$$\dot{I}_{\mathrm{L}} = \frac{\dot{V}_{\mathrm{TH}}}{Z_{\mathrm{TH}} + Z_{\mathrm{L}}} = \frac{7.07\angle 45\,^{\circ}\mathrm{V}}{(3.5 - j3.5)\,\Omega + (1.5 + j3.5)\,\Omega} = \frac{7.07\angle 45^{\circ}\ \mathrm{V}}{5\ \Omega} \approx \boxed{1.4\angle 45^{\circ}\mathrm{A}}$$

$Z_{\mathrm{TH}} = (3.5 - j3.5)\ \Omega$ (Step 2)

5. Determine Norton's equivalent circuit in Figure 10.29(a) as seen by Z_{L}.

- Norton's equivalent impedance Z_{N}:

$$Z_{\mathrm{N}} = Z_{\mathrm{TH}} = 3.5 - j3.5 = 4.95\angle - 45^{\circ}\ \Omega$$

$$\boldsymbol{A} = x + jy = \sqrt{x^2 + y^2}\tan\frac{y}{x} = \boldsymbol{a}\angle\varphi$$

- Norton's equivalent current I_{N}: It is equal to the short circuit current for the original two-terminal circuit of a and b (as shown in Figure 10.29(e) (I)).

$$\dot{I}_{\mathrm{N}} = \dot{I}_{\mathrm{SC}} = \dot{I}\,\frac{Z_2}{Z_2 + (Z_3 + Z_4)}$$

The current divider rule

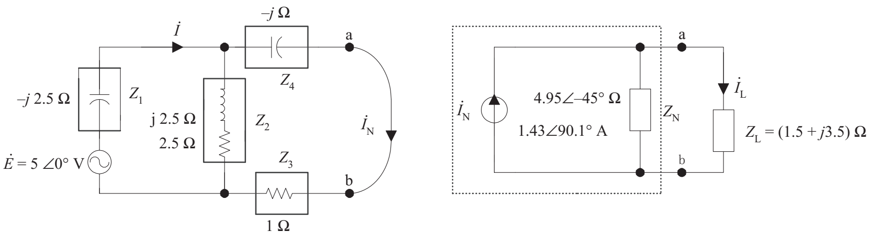
$$\text{There, } \dot{I} = \frac{\dot{E}}{Z_1 + Z_2//(Z_3 + Z_4)} = \frac{5\angle 0^{\circ}\ \mathrm{V}}{\left[-j2.5 + \frac{(2.5 + j2.5)(1 - j1)}{(2.5 + j2.5) + (1 - j1)}\right]\Omega}$$

$$= \frac{5\angle 0^{\circ}\ \mathrm{V}}{\left(-j2.5 + \frac{5}{3.5 + j1.5}\right)\Omega} \approx 1.54\angle 68.2^{\circ}\ \mathrm{A}$$

$$\boldsymbol{A} = a\angle\psi = x + jy = a(\cos\psi + j\,\sin\psi)$$

$$\text{Therefore,} \quad \dot{I}_{\mathrm{N}} = \dot{I}_{\mathrm{SC}} = \dot{I}\,\frac{Z_2}{Z_2 + (Z_3 + Z_4)}$$

$$= 1.54\angle 68.2^{\circ}\ \mathrm{A}\,\frac{(2.5 + j2.5)\ \Omega}{[2.5 + j2.5 + (1 - j1)]\ \Omega} \approx \boxed{1.43\angle 90^{\circ}\mathrm{A}}$$



$-j\ \Omega$
$\dot{I}$
a
Z_4
$-j\,2.5\ \Omega$
Z_1
$j\,2.5\ \Omega$
$2.5\ \Omega$
Z_2
$\dot{I}_{\mathrm{N}}$
$\dot{E} = 5\angle 0^{\circ}$ V
Z_3
b
$1\ \Omega$
a
$\dot{I}_{\mathrm{N}}$
$4.95\angle{-45^{\circ}}\ \Omega$
$1.43\angle 90.1^{\circ}$ A
Z_{N}
$\dot{I}_{\mathrm{L}}$
$Z_{\mathrm{L}} = (1.5 + j3.5)\ \Omega$
b

(e) (I) (II)

Figure 10.29(e) Norton's equivalent circuit for Example 10.10

6. Use Norton's theorem to check the load current $\dot{I}_L$: Determine the load current $\dot{I}_L$ on the terminals of a and b in Figure 10.29(e) (II) by using Norton's equivalent circuit.

$$\dot{I}_L = \dot{I}_N \frac{Z_N}{Z_N + Z_L} = 1.43\angle 90^\circ\ \text{A}\,\frac{4.95\angle -45^\circ\ \Omega}{[(3.5 - j3.5) + (1.5 + j3.5)]\ \Omega}$$

$$Z_N = Z_{TH} = 3.5 - j3.5$$

$$= 1.43\angle 90^\circ\ \text{A}\,\frac{4.95\angle -45^\circ\ \text{V}}{5} \approx \boxed{1.4\angle 45^\circ\text{A}}$$

Therefore, $\dot{I}_L$ is the same by Norton's theorem as the method by using Thevenin's theorem (checked).

Summary

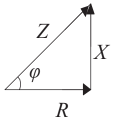
Impedance and admittance

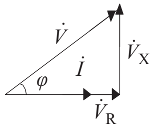
Component	**Impedance $\boldsymbol{Z} = \frac{\dot{V}}{\dot{I}}$**	**Admittance $Y = \frac{1}{\boldsymbol{Z}}$**	**Conductance (*G*) and susceptance (*B*)**
Resistor (R)	$Z_R = R$	$Y_R = G$	Conductance: $G = \frac{1}{R}$
Inductor (L)	$Z_L = jX_L$	$Y_L = -jB_L$	Inductive susceptance: $B_L = \frac{1}{X_L}$
Capacitor (C)	$Z_C = -jX_C$ $\quad j = \frac{1}{-j}$	$Y_C = jB_C$	Capacitive susceptance: $B_C = \frac{1}{X_C}$
Z, Y, X,* and *B	$\boldsymbol{Z} = z\angle\varphi = R + jX$ $z = \sqrt{R^2 + X^2}$ $\varphi = \tan^{-1}\frac{X}{R}$	$\boldsymbol{Y} = y\angle\varphi_y = G + jB$ $y = \sqrt{G^2 + B^2}$ $\varphi_y = \tan^{-1}\frac{B}{G}$	Reactance: $X = X_L - X_C$ Susceptance: $B = B_C - B_L$ $\left(X_L = \omega L, \quad X_C = \frac{1}{\omega L}\right)$

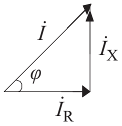
Quantity	**Quantity symbol**	**Unit**	**Unit symbol**
Impedance	Z	Ohm	Ω
Admittance	Y	Siemens (or mho)	S (or ℧)
Susceptance	B	Siemens (or mho)	S (or ℧)
Conductance	G	Siemens (or mho)	S (or ℧)

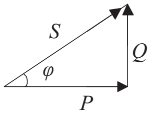
Impedance, voltage, current, and power triangles

- For a more inductive circuit:

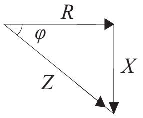


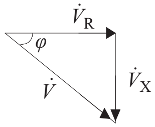

$\dot{V}$
$\dot{V}_X$
$\dot{I}$
φ
$\dot{V}_R$

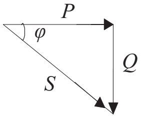

$\dot{I}$
$\dot{I}_X$
φ
$\dot{I}_R$

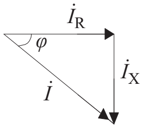


- For a more capacitive circuit:




$\dot{V}_R$
φ
$\dot{V}$
$\dot{V}_X$




$\dot{I}_R$
φ
$\dot{I}$
$\dot{I}_X$

- Impedance angle: $\varphi = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{\dot{V}_X}{\dot{V}_R} = \tan^{-1}\frac{\dot{I}_X}{\dot{I}_R} = \tan^{-1}\frac{Q}{P}$

Characteristics of impedance and admittance:

- The inductive load: $X > 0$ ($X_L > X_C$), $\varphi > 0$, $B < 0$ ($B_L > B_C$), $\varphi_y < 0$
- The capacitive load: $X < 0$ ($X_C > X_L$), $\varphi < 0$, $B > 0$ ($B_C > B_L$), $\varphi_y > 0$
- The resistive load: $X = 0$ ($X_C = X_L$), $\varphi = 0$, $B = 0$ ($B_L = B_C$), $\varphi_y = 0$

Impedances in series and parallel

- Impedances in series: $Z_{eq} = Z_1 + Z_2 + \ldots + Z_n$
- Impedances in parallel:

$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}} = Z_1//Z_2//\ldots//Z_n$$

$$Z_{eq} = \frac{1}{Y_{eq}} \qquad Y_{eq} = Y_1 + Y_2 + \ldots + Y_n$$

- Two impedances in parallel: $Z_{eq} = \frac{z_1 z_2}{z_1 + z_2} = Z_1//Z_2$

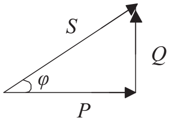
The phasor forms of VDR, CDL, KVL, and KCL

- VDR for impedance: $\dot{V}_1 = \frac{z_1}{z_1 + z_2}\dot{E}, \quad \dot{V}_2 = \frac{z_2}{z_1 + z_2}\dot{E}$
- CDR for impedance: $\dot{I}_1 = \frac{z_2}{z_1 + z_2}\dot{I}_T, \quad \dot{I}_2 = \frac{z_1}{z_1 + z_2}\dot{I}_T$
- The phasor form of KCL: $\Sigma\,\dot{I} = 0 \qquad \dot{I}_{in} = \dot{I}_{out}$
- The phasor form of KVL: $\Sigma\,\dot{V} = 0 \qquad \dot{V}_1 + \dot{V}_2 + \cdots \dot{V}_n = \dot{E}$

Power in AC circuits

Power	**General expression**	**R**	**L**	**C**
Instantaneous power	$p = VI\cos\varphi\,(1-\cos 2\omega t) + VI\sin\varphi\,\sin 2\omega t$	$p_R = VI(1-\cos 2\omega t)$	$p_L = VI\sin 2\omega t$	$p_C = -VI\sin 2\omega t$
Active power	$p = VI\cos\varphi$	$P_R = VI = \frac{1}{2}V_m I_m = I^2R = \frac{V^2}{R}$	$P_L = 0$	$P_C = 0$
Reactive power	$Q = VI\sin\varphi$	$Q_R = 0$	$Q_L = VI = I^2X_L = \frac{V^2}{X_L}$	$Q_C = -VI = -I^2X_C = -\frac{V^2}{X_C}$
Apparent power	$S = IV = I^2Z = \frac{V^2}{Z}$			

Quantity	**Quantity symbol**	**Unit**	**Unit symbol**
Instantaneous power	p	Watt	W
Active power	P	Watt	W
Reactive power	Q	Volt-amperes reactive	Var
Apparent power	S	Volt-amperes	VA

Power triangle	 $P = S\cos\varphi,\quad Q = S\sin\varphi,\quad S = \sqrt{P^2+Q^2}$ Impedance angle: $\varphi = \tan^{-1}\frac{Q}{P} = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{\dot{V}_X}{\dot{V}_R} = \tan^{-1}\frac{\dot{I}_X}{\dot{I}_R}$ Phasor power: $\dot{S} = \dot{V}\dot{I} = \dot{I}^2 Z = \frac{\dot{V}^2}{Z}, \dot{S} = P + jQ$

Power factor ($\cos\varphi$)	– Power factor PF: $\cos\varphi = \frac{P}{S}$ ($0 \le \cos\varphi \le 1$), $\cos\varphi$ without unit) – When $\cos\varphi = 1$: All energy supplied by the source is consumed by the load. – Power-factor correction: A capacitor in parallel with the inductive load can increase the power factor.

Total power	– Total active power: $P_T = P_1 + P_2 + \dots + P_n$ – Total reactive power: $Q_T = Q_{LT} - Q_{CT} = (Q_{L_1} + Q_{L_2} + \dots) - (Q_{C_1} + Q_{C_2} + \dots)$ Q_{LT}: the total reactive power for inductors. Q_{CT}: the total reactive power for capacitors. – Total apparent power: $S = \sqrt{P^2 + Q^2}$ – Total power factor: $PF_T = \cos\varphi_T = \frac{P_T}{S_T}$

Analysis methods for AC sinusoidal circuits

All analysis methods that are used to analyze DC circuits with one or two more sources can also be used to analyze AC circuits.

Practice problems

10.1

1. In an RLC series circuit, $R = 20\,\Omega$, $X_C = 10\,\Omega$, and $X_L = 15\,\Omega$. Calculate the input impedance of this circuit and plot its phasor diagram.
2. If the input impedance Z_{eq} of an RLC series circuit is $100\ \angle 30°\ \Omega$, $X_L = 36\,\Omega$, and $R = 47\,\Omega$. Calculate the capacitive impedance Z_C of this circuit.
3. In an RLC parallel circuit, $R = 5\ \text{k}\Omega$, $X_L = 2\ \text{k}\Omega$ and $X_C = 4\ \text{k}\Omega$. Calculate the input admittance and plot phasor diagram of this circuit.

10.2

4. Determine the input equivalent impedance Z_{eq} for the circuit in Figure 10.30 ($\omega = 10$ rad/s).

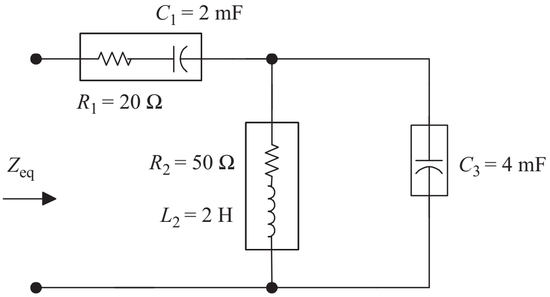


Figure 10.30

5. Determine the input equivalent impedance Z_{eq}, capacitor branch current $\dot{I}_C$, and inductor branch current $\dot{I}_L$ for the circuit in Figure 10.31.

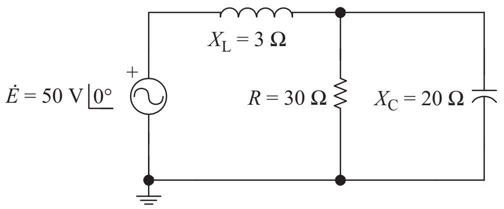

$X_L = 3\ \Omega$
+
$\dot{E} = 50\ \text{V}\angle 0°$
$R = 30\ \Omega$
$X_C = 20\ \Omega$

Figure 10.31

10.3

6. Calculate the total powers P_T, Q_T, S_T, the power factor cos φ, and plot the power triangle for the circuit of Figure 10.32.

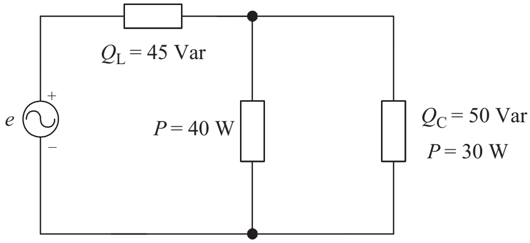

$Q_L = 45$ Var
+
e
−
$P = 40$ W
$Q_C = 50$ Var
$P = 30$ W

Figure 10.32

7. Calculate the active powers, reactive powers, total powers P_T, Q_T, S_T, power factor cosφ, and total current $\dot{I}_T$ for each component in the circuit of Figure 10.33, and also plot the power triangle for this circuit.

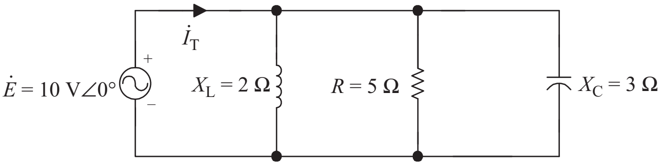

$\dot{I}_T$
+
−
$\dot{E} = 10\ \text{V}\angle 0°$
$X_L = 2\ \Omega$
$R = 5\ \Omega$
$X_C = 3\ \Omega$

Figure 10.33

10.4

8. Write the mesh equations for the circuit shown in Figure 10.34.

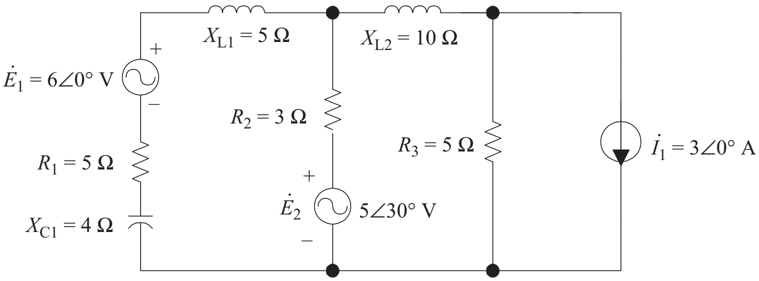

$X_{L1} = 5\ \Omega$
$X_{L2} = 10\ \Omega$
+
$\dot{E}_1 = 6\angle 0°$ V
−
$R_2 = 3\ \Omega$
$R_3 = 5\ \Omega$
$\dot{I}_1 = 3\angle 0°$ A
$R_1 = 5\ \Omega$
+
$\dot{E}_2$
$5\angle 30°$ V
−
$X_{C1} = 4\ \Omega$

Figure 10.34

9. Calculate the mesh current ***I*** for the circuit shown in Figure 10.35 using the superposition theorem.

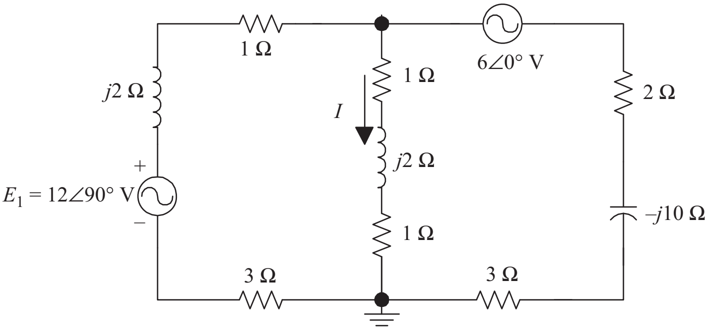

1 Ω
1 Ω
$6\angle 0°$ V
$j2\ \Omega$
2 Ω
I
$j2\ \Omega$
+
$E_1 = 12\angle 90°$ V
−
$-j10\ \Omega$
1 Ω
3 Ω
3 Ω

Figure 10.35

10. Determine the node voltages for the circuit shown in Figure 10.36.

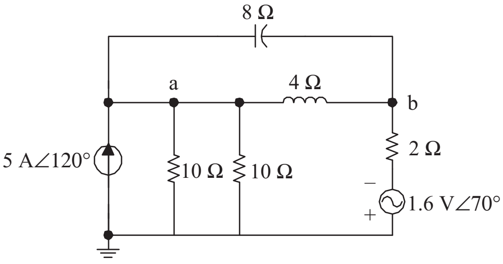

8 Ω
a
4 Ω
b
2 Ω
5 A∠120°
10 Ω
10 Ω
−
1.6 V∠70°
+

Figure 10.36

11. Determine the Thevenin equivalent circuit as viewed from terminals a and b for the circuit shown in Figure 10.37.

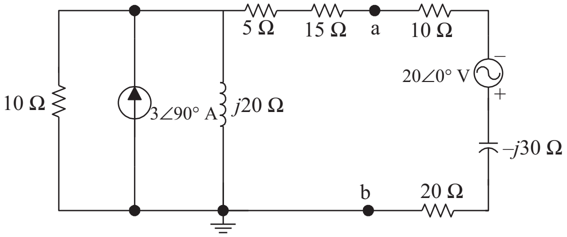

5 Ω
15 Ω
a
10 Ω
20∠0° V
−
+
10 Ω
3∠90° A
*j*20 Ω
−*j*30 Ω
b
20 Ω

Figure 10.37

Chapter 11

RLC circuits and resonance

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11.1 Series resonance

11.1.1 Introduction to series resonance

Introduction to resonance

The resonance phenomena have a wide range of applications in electrical and electronic circuits, particularly in communication systems and signal processing.

- Resonant circuits are simple combinations of inductors (L), capacitors (C), resistors (R), and a power source $(\dot{E})$.
- The capacitor or inductor voltage/current in a resonant circuit could be much higher than the source voltage or current, a small input signal can produce a large output signal when resonance appears in a circuit. This is why the resonant circuit has many important applications in communications systems.
- Resonance may also damage the circuit elements if it is not used properly. So, it is very important to analyze and study resonance phenomena and to know its pros and cons.

Series resonance

- Resonance may occur in a series RLC circuit, as shown in Figure 11.1, when the inductor reactance X_L is equal to the capacitor reactance X_C.
- When the magnitudes of X_L and X_C are equal ($X_L = X_C$), or when reactance X is zero ($X = X_L - X_C = 0$), the equivalent or total circuit impedance Z is equal to the resistance R, i.e., $\dot{Z} = R + j(X_L - X_C) = R$.
- When $X_L = X_C$, and $Z = R$, resonance will occur in the RLC series circuit.
- When resonance occurs in a series RLC circuit, the energy of the reactive components in the circuit will compensate each other ($X_L = X_C$), and the equivalent impedance Z of the series RLC circuit will be the lowest ($Z = R$). This is the characteristic of the series resonant circuit.

Series resonance	$X_L = X_C, \quad X = 0, \quad Z = R$

11.1.2 Frequency and impedance of series resonance

Frequency of series resonance

- The angular frequency of the series resonant circuit can be obtained from

$$X_L = X_C \qquad \text{or} \qquad \omega L = \frac{1}{\omega C}$$

- Solving for ω gives $\omega_r = \dfrac{1}{\sqrt{LC}}$ (The subnotation “r” stands for resonance.)

  Since $\omega = 2\pi f$

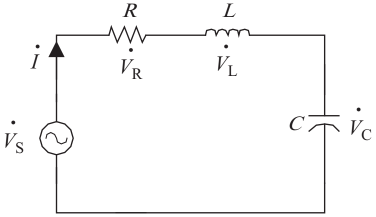


Figure 11.1 An RLC series circuit

Solving for f from the above equation gives the series resonant frequency as

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

- The resonant frequency f_r is dependent on the circuit elements L and C, meaning that it may produce or remove resonance by adjusting the inductance L or capacitance C in the RLC series circuit.

Frequency of series resonance	Resonant frequency: $f_r = \frac{1}{2\pi\sqrt{LC}}$ Resonant angular frequency: $\omega_r = \frac{1}{\sqrt{LC}}$

Impedance of series resonance

- When series resonance occurs, the circuit's equivalent impedance is at the minimum ($Z = R$). This is illustrated in Figure 11.2, which is the response curve of the impedance Z versus frequency f in the series resonant circuit.
- When $f = f_r$, the impedance Z is at the lowest point on the curve.

11.1.3 Current and phasor diagram of series resonance

Current of series resonance

- When resonance occurs in a series RLC circuit, the impedance of the circuit is equal to the resistance ($Z = R$), and the resonant current will be $\dot{I} = \frac{\dot{V}}{z} = \frac{\dot{V}}{R}$.
- When $f = f_r$, $X_L = X_C$, the only opposition to the flow of the current is resistance R, i.e., the impedance is minimum and current is maximum in a series resonant circuit.
- Figure 11.3 illustrates the response curve of current I versus frequency f in the series resonant circuit, and the current is at the highest point on the curve when $f = f_r$.

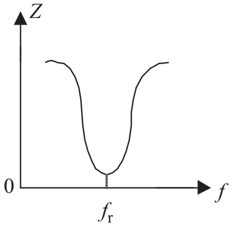


Figure 11.2 The response curve of Z vs. f for series resonance

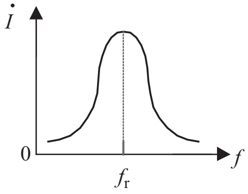


Figure 11.3 The response curve of I vs. f for series resonance

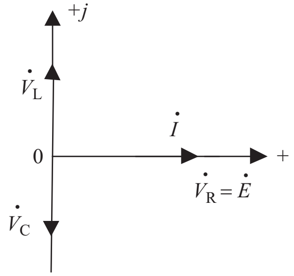


Figure 11.4 Phasor diagram of the series resonant circuit

I and ***Z*** of series resonance	– Impedance is minimum at series resonance: $\boldsymbol{Z = R}$ – Current is maximum at series resonance: $\dot{\boldsymbol{I}} = \frac{\dot{V}}{Z} = \frac{\dot{V}}{R}$

Phasor diagram of series resonance

- An RLC series resonant circuit is equivalent to a purely resistive circuit since $Z = R$.
- The capacitor and inductor voltages in the series resonant circuit are equal in magnitude but are opposite in phase, since $X_L = X_C$.

$\dot{v}_L = jX_L i_L$ and
$\dot{v}_C = jX_C i_C$

- The resistor voltage is equal to the source voltage $(\dot{V}_R = \dot{E})$ since $X = 0$ when series resonance occurs.
- The current $\dot{I}$ and source voltage $\dot{E}$ are also in phase (since $\dot{V}_R$ and $\dot{I}$ in phase), and the phase difference between $\dot{E}$ and $\dot{I}$ is zero ($\varphi = 0$).
- A phasor diagram of the series resonant circuit is illustrated in Figure 11.4.

Phasor relationship of series resonance	– $\dot{V}_L$ and $\dot{V}_C$ are equal in magnitude but opposite in phase. – $\dot{I}$ and $\dot{E}$ are in phase, and $\varphi = 0$.

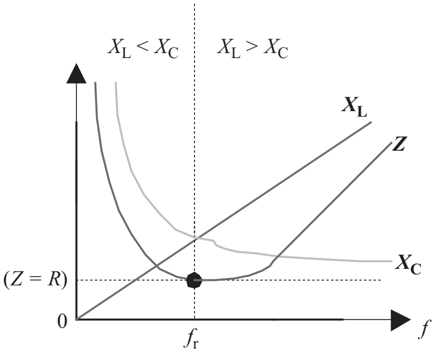


Figure 11.5 The response curves of X_L, X_C, and Z vs. f

11.1.4 Response curves of X_L, X_C, and Z versus f

Response curves

- The response curves of the inductive reactance X_L, capacitive reactance X_C, and impedance Z versus frequency f are illustrated in Figure 11.5.
- X_L and f are directly proportional ($X_L = 2\pi fL$), i.e., as frequency increases X_L increases.
- X_C and f are inversely proportional ($X_C = \frac{1}{2\pi fC}$), i.e., as frequency increases X_C decreases.
- When frequency f is zero in the circuit, $X_L = 0$, X_C and Z approach infinity,

$$\left(Z = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fc}\right)^2} = \sqrt{R^2 + (0 - \infty)^2} = \sqrt{R + \infty^2} \Rightarrow \infty\right)$$

Characteristics of series resonance

- The response curves of X_L, X_C, and Z versus f show that when the circuit frequency is below the resonant frequency f_r ($f < f_r$), the inductive reactance X_L is lower than the capacitive reactance X_C and the circuit appears capacitive.
- When the circuit frequency is above the resonant frequency f_r ($f > f_r$), the inductive reactance X_L is higher than the capacitive reactanceX_C, and the circuit appears more inductive.
- Only when the circuit frequency is equal to the resonant frequency f_r ($f = f_r$), the resonance occurs in the circuit. Impedance Z is equal to the circuit resistance R and has a minimum value, and the circuit appears purely resistive.

Characteristics of series resonance	– When $f < f_r$, $X_L < X_C$: the circuit is more capacitive. – When $f > f_r$, $X_L > X_C$: the circuit is more inductive. – When $f = f_r$, $X_L = X_C$, $I = I_{max}$, $Z = Z_{min} = R$: the circuit is purely resistive and resonance occurs.

11.1.5 Phase response of series resonance

Phase response of series resonance

- The phase response of the series resonant circuit can also be obtained from Figure 11.5.
- When the frequency of the circuit is above the resonant frequency f_r, the circuit is more inductive $X_L > X_C$, voltage leads current, and the phase difference is between zero and positive 90° ($0 \le \varphi \le +90°$).
- When the frequency of the circuit is below the resonant frequency f_r, the circuit is more capacitive $X_L < X_C$, the voltage lags current, and the phase difference is between zero and negative 90° ($-90° \le \varphi \le 0$).
- When the frequency of the circuit is equal to the resonant frequency f_r, $X_L = X_C$, $Z = R$, voltage and current are in phase, and the phase difference is zero ($\varphi = 0$).
- The phase response of the series resonant circuit can be illustrated in Figure 11.6.

Relation between frequency and phase angle

The following characteristics of the series resonant circuit can also be obtained from Figure 11.6.

- When the frequency increases from the resonant frequency f_r to infinity, the phase angle φ approaches positive 90°.
- When the frequency decreases from the resonant frequency f_r to zero, the phase angle φ approaches negative 90°.
- The equation of the phase angle φ is

$$\varphi = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{2\pi f L - \frac{1}{2\pi f C}}{R}$$

Phase response of series resonance	– When $f \to \infty$, $\varphi \to +90°$ – When $f \to 0$, $\varphi \to -90°$

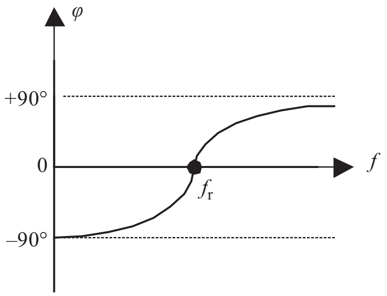


Figure 11.6 Phase response of the series resonant circuit

11.1.6 Quality factor

Quality factor Q

- There is an important parameter known as quality factor in the resonant circuit, which is denoted as Q.
- The quality factor is defined as the ratio of stored energy and consumed energy in physics and engineering, so it is the ratio of the reactive power stored by an inductor or a capacitor and average power consumed by a resistor in a resonant circuit,

  i.e., $$\text{Quality factor } Q = \text{Reactive power/average power} \qquad (11.1)$$

- The quality factor can be used to measure the energy that a circuit stores and consumes.

  The lower the energy consumption of a resistor (power loss) in a circuit, the higher the quality factor, and the better the quality of the resonant circuit.

- If substituting the equations of the reactive power and average power into the quality factor equation (11.1), the quality factor of the series resonance will be obtained as follows: $$Q = \frac{I^2X_L}{I^2R} = \frac{X_L}{R} = \frac{\omega L}{R}$$

  where R is the total or equivalent resistance in the series circuit.

- The quality factor Q can also be expressed by the capacitive reactance and the resistance as: $$\boldsymbol{Q} = \frac{X_C}{R} = \frac{1}{\omega CR}$$

Winding resistance R_w and Q

- The quality factor Q can be used to judge the quality of an inductor (or coil).
- A coil always contains a certain amount of winding resistance R_w, which is the resistance of the wire in the winding.
- The quality factor Q *for* a coil is defined as the ratio of the inductive reactance and the winding resistance, i.e., $\boldsymbol{Q} = \frac{X_L}{R_w}$
- The lower the winding resistance R_w of a coil, the higher the quality of the coil.

Quality factor Q	– Quality factor: the ratio of the reactive power and average power. – Quality factor of the series resonance: $Q = \frac{X_L}{R} = \frac{X_C}{R}$. – Quality factor of the coil: $Q = \frac{X_L}{R_w}$ (R_w—winding resistance). (The lower the R_w, the higher the quality of the coil.)

Note: Both the quality factor and reactive power are denoted by the letter Q, so be careful not to confuse them. The quality factor is a dimensionless parameter, and the unit of reactive power is Var, which can be used to distinguish between these two quantities.

11.1.7 Voltage of series resonance

Voltage resonance

- Multiplying current $\dot{I}$ for both the denominator and numerator of the quality factor equation $Q = \frac{X_L}{R}$, gives $Q = \frac{X_L}{R} = \frac{\dot{I}X_L}{\dot{I}R} = \frac{\dot{V}_L}{\dot{E}}$

  Similarly, for $Q = \frac{X_C}{R}$ $Q = \frac{X_C}{R} = \frac{\dot{I}X_C}{\dot{I}R} = \frac{\dot{V}_C}{\dot{E}}$
- When the resonance occurs in a RLC series circuit:

$$\dot{V}_L = \dot{V}_C = \dot{E}Q \tag{11.2}$$

- The quality factor Q is always greater than 1, so the inductor or capacitor voltage may greatly exceed the source voltage in a series resonant circuit, as can be seen from (11.2).
- This means that a lower input voltage may produce a higher output voltage; therefore, the series resonance is also known as the *voltage resonance*. That is one of the reasons that series resonant circuits have a wide range of applications.
- When choosing the storage elements L and C for a series resonant circuit, the affordability of their maximum voltage should be taken into account, or else the high resonant voltage may damage circuit components.

A small input force can produce a large output vibration

- The concept of circuit resonance is similar to resonance in physics, which is defined as a system oscillating at maximum amplitude at resonant frequency, so a small input force can produce a large output vibration.
- There are many examples of resonance in daily life,
  - such as pushing a child in a playground swing to the resonant frequency, which makes the swing go higher and higher to the maximum amplitude with very little effort.
  - Another example is bouncing a basketball. Once the ball is bounced to the resonant frequency, it will yield a smooth response, and the ball will reach maximum height since the small force produces a large vibration.
- Resonance may also cause damage.

  For example, a legend says that when a team of soldiers walking a uniform pace passed through a bridge, the bridge collapsed since the uniform pace reached resonant frequency resulted in a small force producing a large vibration.

Relationship of voltage and Q	– A lower input voltage may produce a higher output voltage. – Inductor or capacitor voltage may greatly exceed the supply voltage. $\dot{V}_L = \dot{V}_C = \dot{E}Q \quad (Q > 1)$

11.1.8 Series resonance example

Example 11.1: A series resonant circuit is shown in Figure 11.7. Determine the total equivalent impedance, quality factor, and inductor voltage of this circuit.

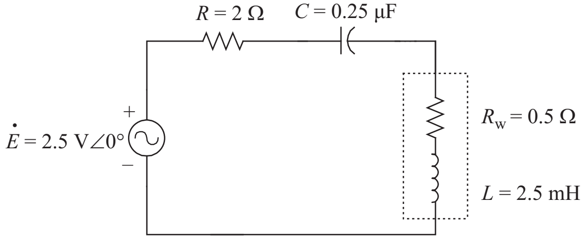


Figure 11.7 Circuit for Example 11.1

Solution:

- $$Z = R_T = R + R_W = (2 + 0.5)\,\Omega = 2.5\angle 0°\ \Omega$$
- $$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(2.5\ \text{mH})(0.25\mu\text{F})}} \approx 6366\ \text{Hz}$$
  $$X_L = 2\pi fL = 2\pi(6366)(2.5\ \text{mH}) \approx 100\ \Omega$$
  $$Q = \frac{X_L}{R_T} = \frac{100\ \Omega}{2.5\ \Omega} = 40$$
- $$\begin{aligned}\dot{V}_L &= jX_L \cdot \dot{I} = \frac{\dot{E}}{Z}\cdot jX_L \\ &= \frac{2.5\angle 0°\ \text{V}}{2.5\angle 0°\ \Omega} \times 100\angle 90°\ \Omega = 100\angle 90°\ \text{V}\end{aligned}$$
- This example shows that the inductor voltage of the series resonant circuit is indeed greater than the supply voltage.

  $$(\dot{V}_L = 100\angle 90°\ \text{V}) > (\dot{E} = 2.5\angle 0°\ \text{V})$$

11.2 Bandwidth and selectivity

11.2.1 The bandwidth of series resonance

Bandwidth (BW)

- When a RLC series circuit is in resonance, its impedance will reach the minimum value and the current will reach the maximum value. The curve of the current versus frequency of the series resonant circuit is illustrated in Figure 11.8.

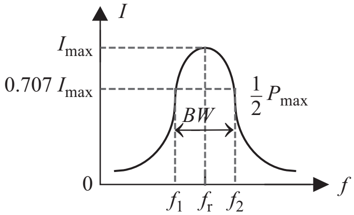


Figure 11.8 Bandwidth of a series resonant circuit

- The current reaches the maximum value $I_{\max}$ as the frequency closes in on the resonant frequency f_r, which is located at the center of the curve (as displayed in the diagram).
- The characteristic of the resonant circuit can be expressed in terms of its bandwidth (BW) or pass-band. The BW of the resonant circuit is the difference between two frequency points f_2 and f_1, i.e., $\text{BW} = f_2 - f_1$

Critical frequencies

- f_2 and f_1 are called critical, cutoff, or half-power frequencies.
- The BW of the resonant circuit is a frequency range between f_2 and f_1 when current I is equivalent to 0.707 of its maximum value $I_{\max}$, or 70.7% of the maximum value of the curve (as shown in Figure11.8).

Half-power frequency

- The power delivered by the source at the points f_1 and f_2 can be determined from the power formula $P = I^2R$

$$P_{f_1} = I_{f_1}^2 R = (0.707 I_{\max})^2 R \approx 0.5 I_{\max}^2 R = 0.5 P_{\max}$$

and

$$P_{f_2} = I_{f_2}^2 R = (0.707 I_{\max})^2 R = 0.5 I_{\max}^2 R = 0.5 P_{\max}$$

- At both points f_2 and f_1, the circuit power is only one-half of the maximum power that it is produced by the source at resonance frequency f_r, where f_2 is the upper critical frequency, and f_1 is the lower critical frequency.

Bandwidth (pass-band)	– Bandwidth ($\text{BW} = f_2 - f_1$) is the range of frequencies at $\dot{I} = 0.707\dot{I}_{\max}$. – f_2 and f_1 are critical or cutoff or half-power frequencies. $P_{f_{1,2}} = 0.5P_{\max}$.

11.2.2 The selectivity of series resonance

The selectivity curve of the series resonant circuit

- Figure 11.8 shows the frequency range between f_2 and f_1 at which the current is near its maximum value, and the series resonant circuits can select frequencies in this range.

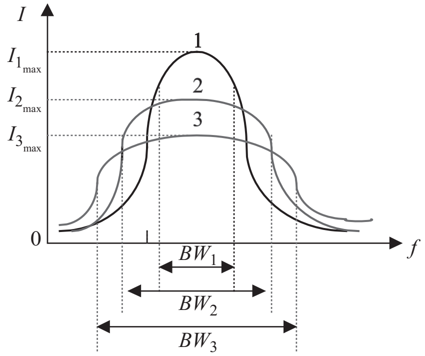


Figure 11.9 Selectivity of a series resonant circuit

- The curve in Figure 11.8 is called the selectivity curve of the series resonant circuit. The selectivity is the capability of a series resonant circuit to choose the maximum current that is closer to the resonant frequency f_r.
- The steeper the selectivity curve, the faster the signal attenuation (reducing), the higher the maximum current value, and the better the circuit selectivity.
- For example, in Figure 11.9, the selectivity curve 1 has a bandwidth of BW_1 and a maximum current $I_{1_{max}}$, which has a better current selectivity than selectivity curve 2 or 3.
- This means that the series resonant circuit of curve 1 has a higher quality and can be expressed as $Q = \frac{f_r}{BW}$. (Q is the quality factor of the series resonant circuit.)

Current selectivity

- The bandwidth BW is an important characteristic for the resonant circuit.
- A series resonant circuit with a *narrower* BW has a better current selectivity.
- A series resonant circuit with a *wider* BW is good for passing the signals.

Sometimes in order to take into account both aspects, the selectivity curve between narrow and wide curves may be chosen (such as the selectivity curve 2 (BW_2) in Figure 11.9.)

Therefore, the concepts of BW and selectivity may apply to different circuits with different design choices.

Selectivity of the series resonance	The capability of the circuit to choose the maximum current I_{max} closer to the resonant frequency f_r.

11.2.3 The quality factor and selectivity

Q and Selectivity

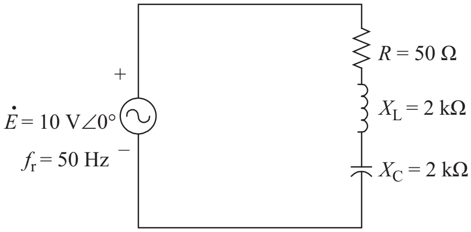
- The quality factor Q in the resonant circuit is a measure of the quality and selectivity of a resonant circuit.

- The higher the Q value, the narrower the BW ($\text{BW}\downarrow\ = \dfrac{f_\text{r}}{Q\uparrow}$), the higher the maximum current, and the better the current selectivity, which is desirable in many applications.

Q represents the quality of a resonant circuit

- The disadvantage of the narrower BW or higher Q is that the ability for passing signals in the circuit will be reduced.
- The lower the Q value, the wider the BW ($\text{BW}\uparrow = \dfrac{f_\text{r}}{Q\downarrow}$), and the better the ability to pass signals; however, it will have a poor current selectivity.
- Q is denoted as the "quality" factor since it represents the quality of a resonant circuit.

Example 11.2 Given a series resonant circuit shown in Figure 11.10(a), determine the BW and current $\dot{I}$ (phasor-domain) of this circuit with three resistors that are 50, 100, and 200 Ω and plot their selectivity curves.



+
$\dot{E} = 10\text{ V}\angle 0°$
$f_\text{r} = 50\text{ Hz}$
−
$R = 50\ \Omega$
$X_\text{L} = 2\text{ k}\Omega$
$X_\text{C} = 2\text{ k}\Omega$

Figure 11.10(a) The circuit for Example 11.2

Solution:

- When $R = 50\ \Omega$: $Q = \dfrac{X_\text{L}}{R} = \dfrac{2\text{ k}\Omega}{50\ \Omega} = 40, \qquad BW_1 = \dfrac{f_\text{r}}{Q} = \dfrac{50\text{ Hz}}{40} = 1.25\text{ HZ}$

$$\dot{I} = \frac{\dot{E}}{Z} = \frac{\dot{E}}{R} = \frac{10\angle 0°\text{ V}}{50\ \Omega} = 0.2\angle 0°\text{ A}$$

- When $R = 100\ \Omega$: $Q = \dfrac{X_\text{L}}{R} = \dfrac{2\text{ k}\Omega}{100\ \Omega} = 20, \qquad BW_2 = \dfrac{f_\text{r}}{Q} = \dfrac{50\text{ Hz}}{20} = 2.5\text{ Hz}$

$$\dot{I} = \frac{\dot{E}}{Z} = \frac{\dot{E}}{R} = \frac{10\angle 0°\text{ V}}{100\ \Omega} = 0.1\angle 0°\text{ A}$$

- When $R = 200\ \Omega$: $Q = \dfrac{X_\text{L}}{R} = \dfrac{2\text{ k}\Omega}{200\ \Omega} = 10, \qquad BW_3 = \dfrac{f_\text{r}}{Q} = \dfrac{50\text{ Hz}}{10} = 5\text{ Hz}$

$$\dot{I} = \frac{\dot{E}}{Z} = \frac{\dot{E}}{R} = \frac{10\angle 0°\text{ V}}{200\ \Omega} = 0.05\angle 0°\text{ A}$$

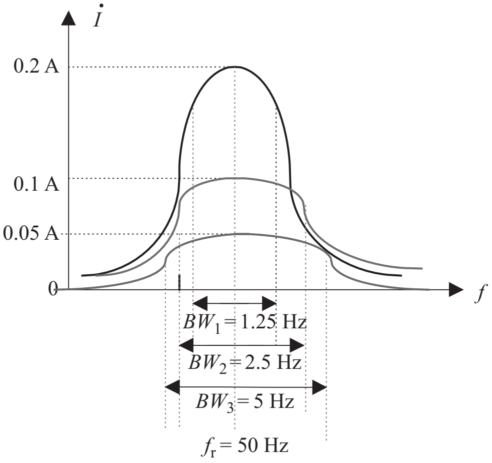


Figure 11.10(b) The selectivity curve for Example 11.2

- Example 11.2 shows that the selectivity curve of a resonant circuit depends greatly upon the amount of resistance in the circuit.
- When resistance R in a series resonant circuit has a smaller value, the selectivity curve of the circuit is steeper, the quality factor Q has a higher value, the current at the resonant frequency f_r has a higher value, and the selectivity is better.
- However, the pass-band (BW) of the circuit with a smaller R value is narrower, and the ability to pass signal will be poor.

Quality factor and selectivity	– Quality factor: a measure of the quality and selectivity of a resonant circuit, $Q = \frac{f_r}{\text{BW}}$. – $Q \uparrow = \frac{X_L}{R \downarrow} \Rightarrow \text{BW} \downarrow = \frac{f_r}{Q \uparrow}$: the steeper the selectivity curve, the better the current selectivity, but the worse the ability to pass signals. – $Q \downarrow = \frac{X_L}{R \uparrow} \Rightarrow \text{BW} \uparrow = \frac{f_r}{Q \downarrow}$: the flatter the selectivity curve, the worse the current selectivity, but the better the ability to pass signals.

The analysis method of the series resonant circuit can also be applied to the parallel resonant circuits.

Series resonance summary

Characteristics	Series resonance
Condition of resonance	$X_L = X_C, \quad X = 0, \quad Z = R$
Resonant frequency	$f_r = \dfrac{1}{2\pi\sqrt{LC}}$
Impedance	$Z = R$ minimum (admittance Y maximum)
Current	$\dot{I}_T = \dfrac{\dot{V}}{R}$ maximum
BW	$\mathrm{BW} = f_2 - f_1 = \dfrac{f_r}{Q}$
Quality factor	$Q = \dfrac{X_L}{R} = \dfrac{X_C}{R}$
Relationship of voltage and quality factor	$\dot{V}_L = \dot{V}_C = \dot{E}Q$

11.3 Parallel resonance

11.3.1 Introduction to parallel resonance

Parallel resonance

- Resonance may occur in a parallel resistor, inductor, and capacitor (RLC) circuit, as shown in Figure 11.11, when the circuit inductive susceptance B_L is equal to the capacitive susceptance B_C.
- The analysis method of the parallel resonance is similar to series resonance.

Admittance of parallel resonance

- When the magnitudes of the capacitive susceptance B_C and the inductive susceptance B_L are equal ($B_C = B_L$), or when the susceptance B is zero ($B = B_C - B_L = 0$), the circuit input equivalent (total) admittance Y is equal to the circuit conductance G, i.e.,

$$Y = G + jB = G$$

- Under the above condition, resonance will occur in the RLC parallel circuit.

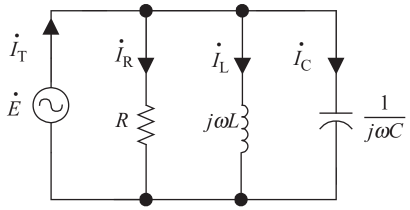


Figure 11.11 A parallel RLC circuit

The characteristic of the parallel resonant circuit

- When the resonance occurs in a RLC parallel circuit, the energy of the reactive components in the circuit will compensate each other ($B_C = B_L$).
- The equivalent admittance Y of the parallel RLC circuit is at the lowest ($Y = G$).

Parallel resonance	$B_C = B_L, \quad B = 0, \quad Y = G$

11.3.2 Frequency and admittance of parallel resonance

Frequency of parallel resonance

- The angular frequency of the parallel resonant circuit can be obtained from

$$Y = G + j(B_C - B_L) = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

- From $B_C = B_L$ or $\omega C = \dfrac{1}{\omega L}$

  solving for ω gives $\omega_r = \dfrac{1}{\sqrt{LC}}$

- Since $\omega = 2\pi f$

  the parallel resonant frequency is $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

- The parallel resonant angular frequency is ω_r and resonant frequency f_r are the same with those in the series resonant circuit.
- The resonant frequency f_r is dependent on the circuit elements L and C, meaning that by adjusting the inductance L or capacitance C in the RLC parallel circuit, resonance may be produced or removed.

Frequency of parallel resonance	– Resonant frequency: $f_r = \dfrac{1}{2\pi\sqrt{LC}}$ – Resonant angular frequency: $\omega_r = \dfrac{1}{\sqrt{LC}}$

Admittance

- When parallel resonance occurs, the equivalent admittance Y of the circuit is at the minimum ($Y = G$), $B = B_C - B_L = 0$, so the circuit equivalent impedance Z is at a maximum ($Z \uparrow = \dfrac{1}{Y\downarrow}$). (This is shown in Figure 11.12, which is the response curve of the impedance Z versus the frequency f in the parallel resonant circuit.)
- When $f = f_r$, the impedance Z is at the highest point on the curve and this is opposite to the series resonance.

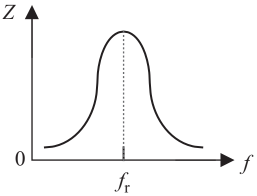


Figure 11.12 The response curve of Z vs. f for parallel resonance

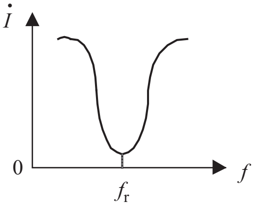


Figure 11.13 The response curve of I vs. f for parallel resonance

11.3.3 Current of parallel resonance

The total current in the parallel resonant circuit

- When resonance appears in a parallel RLC circuit, the impedance of the circuit is equal to the resistance ($Z = R$), and the total current in the circuit will be

$$\dot{I}_T = \frac{\dot{V}}{Z} = \frac{\dot{V}}{R}$$

- When $f = f_r$, in the parallel resonant circuit,
  - the admittance Y is at the minimum:

$$B_C = B_L, \qquad Y = G, \qquad\qquad Y = G + j(B_C - B_L)$$

  - the impedance Z is at the maximum:

$$Z\uparrow = \frac{1}{Y\downarrow}$$

  - the current is at the minimum:

$$\dot{I}_T\downarrow = \frac{\dot{V}}{\dot{Z}\uparrow} = \frac{\dot{V}}{R}$$

Response curve for parallel resonance

- Figure 11.13 illustrates the response curve of current I versus frequency f in the parallel resonant circuit.
- Current is at the lowest point on the curve when $f = f_r$. (This is opposite to series resonance.)

***I* and *Z* of parallel resonance**	– Impedance is maximum at parallel resonance: $Z = R \quad (B = 0, Y = G) \qquad Y = G + j(B_C - B_L)$ – Current is minimum at parallel resonance: $\dot{I}_T = \frac{\dot{V}}{\dot{Z}} = \frac{\dot{V}}{R}$

11.3.4 Phasor diagram of parallel resonance

The capacitor and inductor branch currents in the parallel resonant circuit

- An RLC parallel resonant circuit is equivalent to a purely resistive circuit since $Y = G$ and $Z = R$.
- The capacitor and inductor branch currents in the parallel resonant circuit are equal in magnitude but opposite in phase, since $B_L = B_C$ $\quad (B_L = \frac{1}{X_L}, B_C = \frac{1}{X_C})$

and
$$\dot{I}_L = \frac{V_L}{jX_L} = -j\frac{V_L}{X_L}, \qquad \dot{I}_C = \frac{V_C}{-jX_C} = j\frac{V_C}{X_C} \qquad +j = \frac{-1}{j}$$

i.e.,
$$\boxed{\dot{I}_L = -\dot{I}_C}$$

Phasor diagram of the parallel resonant circuit

- The resistor voltage is equal to the source voltage $\dot{V}_R = \dot{E}$ in the parallel resonant circuit of Figure 11.11.
- The total current $(\dot{I}_T)$ and the source voltage $(\dot{E})$ are in phase (since $\dot{V}_R$ and $\dot{E}$ are in phase).
- The phase difference between $\dot{E}$ and $\dot{I}_T$ is zero, i.e., the admittance angle $\varphi_y = 0$.
- A phasor diagram of the parallel resonant circuit is illustrated in Figure 11.14.

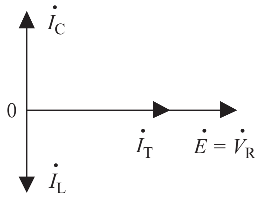


Figure 11.14 Phasor diagram of the parallel resonant circuit

Phasor relationship of parallel resonance	– $\dot{I}_L$ and $\dot{I}_C$ are equal in magnitude but opposite in phase. $\dot{I}_L = -\dot{I}_C$ – $\dot{I}_T$ and $\dot{E}$ are in phase, and $\varphi_y = 0$.

11.3.5 Quality factor of parallel resonance

Quality factor *Q*

- Recall: The quality factor is the ratio of the reactive power stored by an inductor or a capacitor and the average power dissipated by a resistor in a circuit, i.e.,

  Quality factor Q = Reactive power/average power

- The quality factor of a parallel resonance: If we substitute the equations of the reactive power ($P_L = \frac{\dot{V}_L^2}{X_L}$) and average power ($P_R = \frac{\dot{V}_R^2}{R}$) in Figure 11.11 into the quality factor equation, the quality factor of a parallel resonance will be obtained as follows:

$$Q = \frac{\dot{E}^2/X_L}{\dot{E}^2/R} = \frac{R}{X_L} \qquad (\because \quad \dot{V}_L = \dot{V}_R = \dot{E})$$

- The quality factor Q can be expressed by the capacitive reactance and the resistance as:

$$Q = \frac{R}{X_C}$$

- The quality factor of a parallel resonant circuit is inverted with the series resonant circuit.

(Recall the quality factor of a series resonant circuit: $Q = \frac{X_L}{R} = \frac{X_C}{R}$)

Quality factor *Q*	Quality factor of the parallel resonance: $Q = \frac{R}{X_L} = \frac{R}{X_C}$

11.3.6 Current of parallel resonance

The inductor and capacitor current

- Dividing the voltage $\dot{E}$ for both the denominator and numerator of the quality factor equation $Q = \frac{X_L}{R}$,

  gives $\quad Q = \frac{\dot{E}/X_L}{\dot{E}/R} = \frac{\dot{I}_L}{\dot{I}_T}$

- Similarly, for $Q = \dfrac{X_C}{R}$

$$Q = \frac{\dot{E}/X_C}{\dot{E}/R} = \frac{\dot{I}_C}{\dot{I}_T}$$

- The inductor or capacitor current: When resonance occurs in an RLC parallel circuit, the inductor or capacitor current

$$\dot{I}_L = \dot{I}_C = \dot{I}_T Q \tag{11.3}$$

Current resonance

- Usually the quality factor Q is always great than 1, the inductor or capacitor branch current may greatly exceed the total supply current in a parallel resonant circuit. (This can be seen from (11.3).)
- Current resonance: A lower input current may produce a higher output current in a parallel resonant circuit, and therefore the parallel resonance is also known as current resonance.
- The higher resonant current may damage circuit components: When choosing the storage elements L and C for a parallel resonant circuit, it should be taken into account the affordability of their maximum current, or else the higher resonant current may damage circuit components.

Relationship of current	A lower input current may produce a higher output current. The inductor or capacitor current may greatly exceed the supply current: $\dot{I}_L = \dot{I}_C = \dot{I}_T Q \qquad (Q > 1)$

11.3.7 The bandwidth of parallel resonance

BW of parallel resonance

- The characteristic of the parallel resonant circuit can be expressed in terms of its BW or pass-band. Recall

$$\mathrm{BW} = f_2 - f_1$$

or

$$\mathrm{BW} = \frac{f_r}{Q}$$

- The BW of the parallel resonant circuit is illustrated in Figure 11.15.
- When the RLC parallel circuit is in resonance, its current reaches the minimum value.
- The BW of the parallel resonant circuit is a frequency range between the critical or cutoff frequencies f_2 and f_1, when the current is equivalent to 0.707 of its maximum value I_{max}, or 70.7 % of the maximum value of the curve.

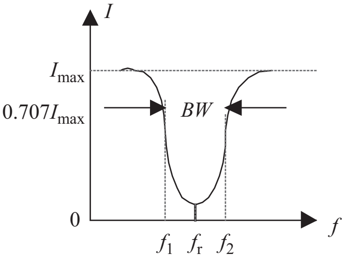


Figure 11.15 The BW of the parallel resonance

Parallel resonance summary

Characteristics	Parallel resonance
Conditions of resonance	$B_L = B_C,\ B = 0,\ Y = G$
Resonant frequency	$f_r = \dfrac{1}{2\pi\sqrt{LC}}$
Impedance	$Z = R$ maximum (admittance Y minimum)
Current	$\dot{I}_T = \dfrac{\dot{V}}{R}$ minimum
BW	$\mathrm{BW} = f_2 - f_1 = \dfrac{f_r}{Q}$
Quality factor	$Q = \dfrac{R}{X_L} = \dfrac{R}{X_C}$
Relationship of current and quality factor	$\dot{I}_L = \dot{I}_C = \dot{I}_T Q$

11.4 A practical parallel resonant circuit

11.4.1 Resonant admittance

A practical parallel circuit

- In practical electrical or electronic system applications, the parallel resonant circuit usually is formed by an inductor (coil) in parallel with a capacitor.
- A practical coil always has internal resistance (wounding resistance), an actual parallel resonant circuit will look like the one illustrated in Figure 11.16.

Resonant admittance

- The input equivalent admittance of the practical parallel circuit shown in Figure 11.16 is:

$$Y = \frac{1}{R + jX_L} + j\frac{1}{X_C} \qquad Y = \frac{1}{Z}$$

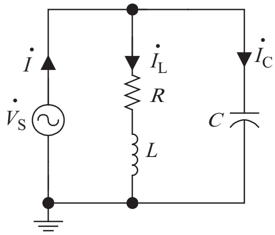


Figure 11.16 A practical parallel circuit

- Multiplying $(R - jX_L)$ to the numerator and denominator of the first term in the above equation gives

$$Y = \frac{R}{R^2 + {X_L}^2} - j\frac{X_L}{R^2 + {X_L}^2} + j\frac{1}{X_C}$$

or

$$Y = \frac{R}{R^2 + {X_L}^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2 + {X_L}^2}\right) \tag{11.4}$$

- Resonant admittance: The parallel resonance occurs when the circuit admittance Y is equal to the circuit conductance G $(Y = G)$, so when the resonance occurs for the practical parallel circuit in Figure 11.16, the resonant admittance should be:

$$\boxed{Y = G = \frac{R}{R^2 + {X_L}^2}} \qquad (\because Y = G + jB)$$

11.4.2 Resonant frequency

Resonant angular frequency and frequency

- According to the parallel resonant conditions, resonance occurs when the capacitive susceptance B_C is equal to the inductive susceptance B_L, i.e., $B_C = B_L$, thus (11.4) gives

$$\frac{X_L}{R^2 + {X_L}^2} = \frac{1}{X_C} \qquad X_L = \omega L, \quad X_C = \frac{1}{\omega C}$$

or

$$\frac{\omega L}{R^2 + (\omega L)^2} = \omega C \tag{11.5}$$

- The resonance frequency and angular frequency for the circuit in Figure 11.16 can be obtained from (11.5) as follows:
  - Resonance angular frequency: $\omega_r = \sqrt{\frac{L - CR^2}{L^2 C}} = \frac{1}{\sqrt{LC}}\sqrt{1 - \frac{CR^2}{L}}$
  - Resonance frequency: $f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{CR^2}{L}} \qquad \omega = 2\pi f$

Condition for resonance

- Resonance will occur in the circuit of Figure 11.16 only when

$$1-\frac{CR^2}{L}>0, \quad 1>\frac{CR^2}{L}, \quad R^2<\frac{L}{C}, \qquad \text{or} \quad R<\sqrt{\frac{L}{C}}$$

- If $1-\dfrac{CR^2}{L}<0$ or $R>\sqrt{\dfrac{L}{C}}$ resonance will not occur.

Practical parallel resonance	– Resonant admittance: $Y=\dfrac{R}{R^2+{X_L}^2}$ – Resonant angular frequency: $\omega_r=\dfrac{1}{\sqrt{LC}}\sqrt{1-\dfrac{CR^2}{L}}$ – Resonant frequency: $f_r=\dfrac{1}{2\pi\sqrt{LC}}\sqrt{1-\dfrac{CR^2}{L}}$ – When $1-\dfrac{CR^2}{L}>0$ or $R<\sqrt{\dfrac{L}{C}}$, resonance occurs.

11.4.3 Applications of the resonance

The purpose of resonant circuits

- Resonant circuits are used in a wide range of applications in communication systems, such as filters, and tuners.
- The purpose of resonant circuits are the same
  - to select a specific frequency (resonant frequency f_r) and reject all others,
  - or select signals over a specific frequency range that is between the cutoff frequencies f_1 and f_2.

A parallel tuning circuit

- The key circuit of a communication system is a tuned amplifier (tuning circuit).
- A simplified parallel radio tuning circuit: Figure 11.17 is a simplified parallel radio tuning circuit for a radio circuit. The combination of a practical parallel resonant circuit and an amplifier can select the appropriate signal to be amplified.

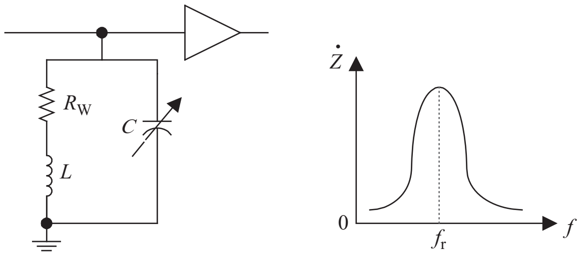


Figure 11.17 A simplified parallel radio tuner

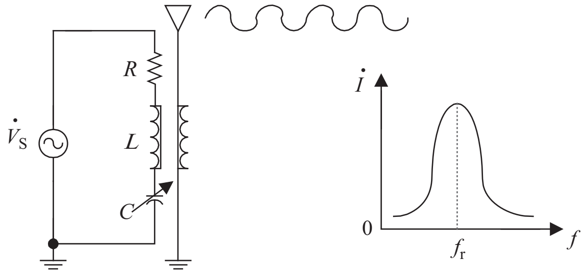


Figure 11.18 A simplified series radio tuner

- The input signals in the radio tuner circuit have a wide frequency range, because there are many different radio signals from different radio stations.
- When adjusting the capacitance of the variable capacitor in the practical parallel resonant circuit (i.e., adjusting the switch of the radio channel), the circuit resonant frequency f_r will consequently change.
- Once f_r matches the desired input signal frequency with the highest input impedance, the desired input signal will be passed, and this is the only signal that will be amplified.
- After it is amplified by the amplifier in the circuit, this signal of the corresponding station can be clearly heard.

A series tuning circuit

- Figure 11.18 is a simplified series radio tuning circuit. It is similar to the parallel tuning circuit.
- When adjusting the capacitance of the variable capacitor in the series resonant circuit, the circuit resonant frequency f_r will change.
- Once f_r matches the desired input signal frequency with the highest current, the desired input signal will be passed and amplified.

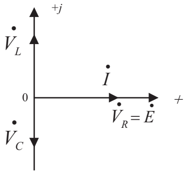
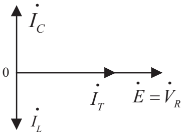
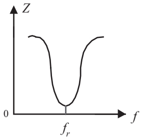
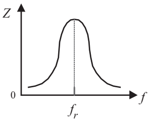
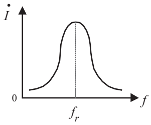
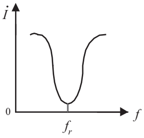
Summary

Series/parallel resonance

Characteristics	**Series resonance**	**Parallel resonance**
Conditions of resonance	$X_L = X_C, \quad X = 0, \quad Z = R$	$B_L = B_C, \quad B = 0, \quad Y = G$
Phasor relationship	– $\dot{V}_L$ and $\dot{V}_C$ are equal in magnitude but opposite in phase. – $\dot{I}$ and $\dot{E}$ in phase, and $\varphi = 0$.	– $\dot{I}_L$ and $\dot{I}_C$ are equal in magnitude but opposite in phase. – $\dot{I}_T$ and $\dot{E}$ in phase, and $\varphi_y = 0$.

(Continues)

(*Continued*)

Characteristics	**Series resonance**	**Parallel resonance**
Phasor diagram	 +j $\dot{V}_L$ $\dot{I}$ 0 + $\dot{V}_R = \dot{E}$ $\dot{V}_C$	 $\dot{I}_C$ 0 $\dot{I}_T$ $\dot{E} = \dot{V}_R$ $\dot{I}_L$
Resonant frequency	$f_r = \frac{1}{2\pi\sqrt{LC}}$	$f_r = \frac{1}{2\pi\sqrt{LC}}$
Impedance	$Z = R$ minimum (admittance Y maximum) 	$Z = R$ maximum (admittance Y minimum) 
Current	$\dot{I} = \frac{\dot{V}}{R}$ maximum 	$\dot{I}_T = \frac{\dot{V}}{R}$ minimum 
BW	$\mathrm{BW} = f_2 - f_1 = \frac{f_r}{Q}$	$\mathrm{BW} = f_2 - f_1 = \frac{f_r}{Q}$
Quality factor	$Q = \frac{X_L}{R} = \frac{X_C}{R}$ or $Q = \frac{f_r}{BW}$	$Q = \frac{R}{X_L} = \frac{R}{X_C}$ or $Q = \frac{f_r}{BW}$
Relationship of voltage/ current and *Q*	$\dot{V}_L = \dot{V}_C = \dot{E}Q$	$\dot{I}_L = \dot{I}_C = \dot{I}_T Q$

Practical parallel resonance	– Resonant admittance: $Y = \frac{R}{R^2 + {X_L}^2}$ – Resonant angular frequency: $\omega_r = \frac{1}{\sqrt{LC}}\sqrt{1 - \frac{CR^2}{L}}$ – Resonant frequency: $f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{CR^2}{L}}$ – Resonance occurs when $1 - \frac{CR^2}{L} > 0$, or $R < \sqrt{\frac{L}{C}}$

BW and selectivity

- BW (pass-band): the frequency range corresponding to $\dot{I} = 0.707\dot{I}_{\max}$.

  $$\mathrm{BW} = f_2 - f_1$$

- f_2 and f_1: critical or cutoff or half-power point frequencies.

  $$P_{f1,2} = 0.5P_{\max}$$

- Quality factor: a measure of the quality and selectivity of a resonant circuit.
- Selectivity: the capability of the circuit to choose the maximum current closer to the resonant frequency f_r.
- In a series resonant circuit, V_L or V_C may greatly exceed the supply voltage E, i.e., a lower input voltage may produce a higher output voltage.
- In a parallel resonant circuit, I_L or I_C may greatly exceed the total current I_T, i.e., a lower input current may produce a higher output current.

Practice problems

11.1

1. If the inductance in a series resonant RLC circuit is decreased, the resonant frequency (   ).
2. The total reactance of a series RLC circuit at resonance is (   ), and the impedance is (   ).
3. When a series RLC circuit is in resonance, its impedance will reach the (   ) value and the current will reach the (   ) value.
4. In a series RLC resonant circuit, the current is 20 mA, the inductor voltage is 60 V and the source voltage is 5 V. Determine the resonant impedance Z, inductive reactance X_L, and the capacitive reactance X_C for this circuit.
5. In a series RLC resonant circuit, $\dot{E} = 10\angle 0°$ V, $R = 9\ \Omega$, $L = 10$ mH, $R_W = 1\ \Omega$ and $C = 1\mu$F. Determine the resonant frequency, total resonant impedance, current, inductor voltage and quality factor Q for this circuit.

11.2

6. f_1 and f_2 are called (   ) frequencies or (   ) frequency points.
7. The BW is the frequency range when the current $\dot{I} = ($ $)\ \dot{I}_{\max}$.
8. The capability of the circuit to choose the maximum current closer to the resonant frequency f_r is called (   ).
9. The higher the Q value, the narrower the BW, the higher the maximum (   ), and the better the current selectivity.
10. In a RLC series resonant circuit, the resistance R is 10 Ω, the capacitive reactance X_C is 5 kΩ and the resonant frequency f_r is 60 Hz. Determine the BW of this circuit.

11.3

11. The RLC parallel resonant angular frequency ω_r and resonant frequency f_r are the (   ) with those in the RLC series resonant circuit.
12. When the RLC parallel resonance appears, a lower input current may produce a higher output current. The inductor or capacitor current may greatly exceed the (   ) current.

11.4

13. A practical parallel resonant circuit is formed by a capacitor in parallel with an (   ).
14. Resonance will occur in a practical parallel resonant circuit only when (   ) or (   ).
15. In a practical parallel resonant circuit, $V_S = 6.3$ V, $R = 20\ \Omega$, $L = 50$ mH, and $C = 47$ pF. Determine the resonant frequency f_r and the impedance Z for this circuit.

Chapter 12

Mutual inductance and transformers

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12.1 Mutual inductance

12.1.1 Mutual inductance and self-inductance

Induced voltage

- Recall: When a changing current flows through a coil (inductor), it will produce an electromagnetic field around the coil, and as a result an induced voltage v_L will across it.
- Self-inductance is the ability of a coil to produce an induced voltage due to the changing of the current in the coil itself.
- Mutual inductance is the ability of a coil to produce an induced voltage due to the changing of the current in another coil nearby.

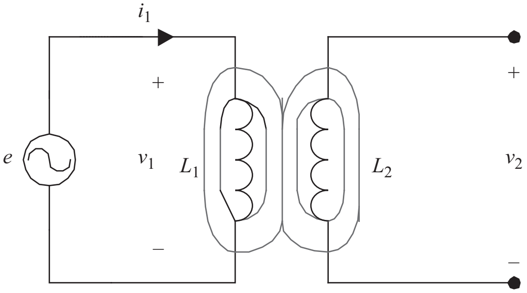


Figure 12.1 Magnetic coupling

The principle of mutual inductance

- Self-induced voltage v_1: In Figure 12.1, a coil L_1 is placed close to another coil L_2. When AC current i_1 flows through the first coil L_1, the changing of alternating current will produce a changing electromagnetic field and flux ϕ_1, resulting in a self-induced voltage v_1 across the first coil L_1.
- Induced voltage v_2: Since the two coils are very close, there is also a portion of magnetic flux, ϕ_{1-2}, that is produced by changing the electromagnetic field linked to the coil L_2, and consequently produces the induced voltage v_2 across the second coil L_2.
- Inductive coupling: The phenomenon of a portion of the flux of a coil linking to another coil is called inductive coupling, and this is the principle of mutual inductance.

12.1.2 Factors affecting mutual inductance

Factors affecting mutual inductance

- There are three factors that affect mutual inductance: inductances of the two coils L_1, L_2, and the coupling coefficient k.
- The coefficient of coupling k determines the degree of the coupling between the two coils, and it is the ratio of ϕ_{1-2} and ϕ_1:

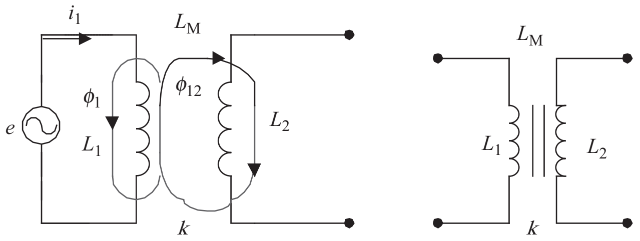
$$k = \frac{\phi_{1-2}}{\phi_1}$$

- ϕ_1 is the magnetic flux generated by the current i_1 in the first coil L_1.
- ϕ_{1-2} is the portion of the magnetic flux that is generated by the current i_1 in the first coil L_1 and linked to the second coil as shown in Figure 12.2(a). ϕ_{1-2} is called the cross-linking flux.

Mutual inductance

Mutual inductance is denoted by L_M and can be expressed mathematically using the following formula:

$$L_M = k\sqrt{L_1 L_2}$$



(a) (b)

Figure 12.2 Mutual inductance

Mutual inductance	An induced voltage in one coil due to a current change in a nearby coil. $L_M = k\sqrt{L_1 L_2}$

12.1.3 Coefficient of coupling

Induced voltages

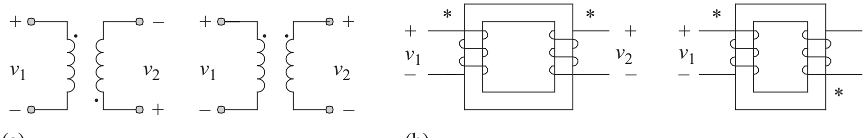
- The induced voltage in the coil L_1: The voltage generated by a changing current (AC) that flows through the coil L_1 is given by $v_1 - L\frac{di_1}{dt}$ (Chapter 6).
- The induced voltage in the coil L_2: When the AC current i_1 flows through the second coil L_2, the induced voltage in the coil L_2 is given by $v_2 = L_M \frac{di_1}{dt}$, or $\dot{V}_2 = jL_M\dot{I}_1$ in the phasor form.

Leakage flux

- Leakage flux: In practice, not all of the magnetic flux generated by current i_1 will pass through L_1 and L_2, and the portion of the magnetic flux that does not link with L_1 and L_2 is known as a leakage flux.
- Cross-linking flux $\text{Ø}_{1\text{–}2}$: The closer the two coils are placed (or if the two coils have a common core as shown in Figure 12.3(b)), the higher the cross-linking flux $\text{Ø}_{1\text{–}2}$ and the lower the leakage flux.

Coefficient of coupling

- The full-coupling occurs when $k = \frac{\phi_{1-2}}{\phi_1} = 1$, i.e., $\phi_{1-2} = \phi_1$
- When all of the flux link coils 1 and 2, there will be no leakage flux.
- If the gap between the two coils is large, it will cause
  - the cross-linking flux to decrease,
  - the leakage flux to increase,
  - the coupling coefficient k to decrease. k is in the range between 0 and 1 $(0 \leq k \leq 1)$.



(a) (b)

Figure 12.3 Dot convention

Coefficient of coupling	– The coefficient of the coupling: $k = \frac{\phi_{1-2}}{\phi_1}$ $(0 \leq k \leq 1)$ – ϕ_1: The flux generated by the current i_1 in the first coil L_1. – ϕ_{1-2}: The flux generated by the current i_1 in the coil L_1 cross-linking to coil L_2.

12.1.4 Dot convention

Dot convention method

- The dot convention method can be used to indicate whether the induced voltage in the second coil is in phase or out of phase with the voltage in the first coil.
- The dot convention method places two small phase dots (·) or asterisk (*), one on the coil L_1 and the other on the coil L_2, to indicate that polarities of the induced voltage v_1 in the coil L_1 and v_2 in the adjacent coil L_2 are the same at these points, as shown in Figure 12.3.

Corresponding terminals

- The polarity of the induced voltage across the mutually coupled coils can be determined by the dot convention method.
- Corresponding terminals: The dotted terminals of coils should have the same voltage polarity at all time, and dotted terminals are known as corresponding terminals.

Dot convention	Dotted terminals of coils have the same voltage polarity.

12.2 Basic transformer

12.2.1 Transformer

Introduction to transformers

- A transformer is an electrical device formed by two coils that are wound on a common core. You may have seen transformers on top of the utility poles.
- A transformer uses the principle of mutual inductance to convert AC electrical energy from input to output.

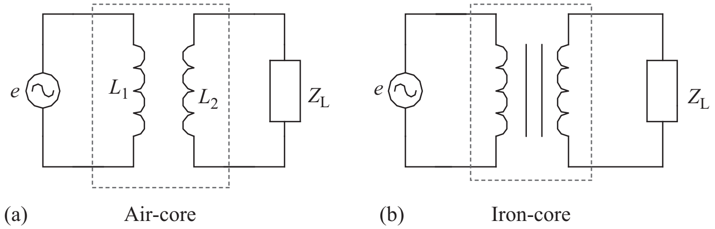


Figure 12.4 Simplified transformer circuits

- Simplified transformer circuits: Figure 12.4 shows two simplified transformer circuits. A changing current from the AC voltage source in the first coil produces a changing magnetic field, inducing a voltage in the second coil.
  - The first coil connected to a AC power source is called primary winding.
  - The second coil connected to the load Z_L is called secondary winding.
- Structurally, the transformers are categorized as two main types: the air-core and iron-core transformers. The symbols for them are shown in Figure 12.4(a) and (b), respectively.

The main applications of transformer

- Increase or decrease the voltage or current
- Transfer electric energy from one circuit to another
- Prevent DC from passing from one circuit to the other
- Isolate two circuits electrically
- Impedance matching

……

Transformer	A transformer uses the principle of mutual inductance to convert AC electrical energy from input to output.

12.2.2 Air-core transformer

Air-core transformer

- An air-core transformer is not necessary to have a physical core,
  - it can be obtained by placing the two coils L_1 and L_2 close to each other (Figure 12.5(a)),
  - or by winding both the coils L_1 (primary coil) and L_2 (secondary coil) on a hollow cylindrical-shaped core with isolating material (Figure 12.5(b)).
- The circuit of an air-core transformer is shown in Figure 12.5(a),
  - R_1 represents the primary winding resistance of the transformer.
  - R_2 represents the secondary winding resistance of the transformer.

Linear transformer

- The air-core transformer is also known as a linear transformer.
- When the core of the transformer is made by the insulating material with constant permeability such as air, plastic, wood, and cardboard, it is a linear transformer.

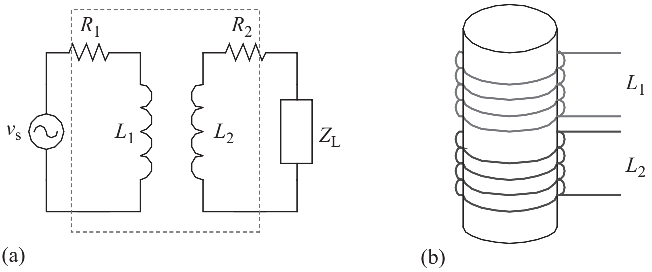


Figure 12.5 Air-core transformer

Air-core transformer	A linear transformer that can be obtained by placing the two coils close to each other or winding two (or more) coils around an insulating substance.

The air-core transformers are usually used in high-frequency circuits, such as in instrumentation, radio and TV circuits, and communication devices.

12.2.3 Iron-core transformer

Iron-core transformer

- The coils of the iron-core transformer are wound on the ferromagnetic material that are laminated sheets insulated to each other, as illustrated in Figure 12.6.
- When two coils are wound on a common core, it will have higher cross-linking flux and lower leakage flux.
- The ferromagnetic materials can provide an easy path for the magnetic flux.

The coupling coefficient $k \approx 1$

- If two coils are wound on a common core, the flux generated in the coil L_1 will almost all link with the coil L_2.
- This means that the coupling coefficient k is close to 1, and this is the reason that iron-core transformer is usually considered as the ideal transformer ($k = 1$).

Applications

- Iron-core transformers are widely used transformers in which the efficiency is high compared to the air-core-type transformer.
- Iron-core transformers are usually used in power systems, audio circuits, etc.

Iron-core transformer	– A transformer that has laminations of ferromagnetic material with coils wound on it. – The coupling coefficient k is close to 1 ($k \approx 1$).

12.2.4 Ideal transformer

Ideal transformer

- Ideal transformer: An ideal transformer has no losses (imaginary transformer). Efficiency of this transformer is considered as 100%.
- Full-coupling ($k = 1$): The coupling coefficient k of an ideal transformer is 1, i.e., ideal full-coupling, neglecting winding resistance and magnetic losses in the coils of the transformer.
- Figure 12.6(a) is a circuit of an ideal transformer with the voltage source, and the load. (The portion within the dashed line is the symbol of the ideal transformer.)
- An iron-core transformer is considered an ideal transformer because it uses ferromagnetic materials with high permeability as its core.
- Common core: The primary and secondary windings are wound on a common core, which have near zero leakage flux and can achieve a full-coupling ($k = 1$).

Transformer parameters

The parameters of an ideal transformer in Figure 12.6(a) are listed in Table 12.1.

The turns ratio of a transformer

- The turns ratio of a transformer is the ratio of the number of turns, i.e., the number of turns on the secondary coil N_S to the number of turns on the primary coil N_p.
- The turns ratio can be derived from the voltage ratio of the secondary and primary voltages.

Derive the turns ratio (*n*)

- From Faraday's law described in Chapter 6, $v_L = \mathrm{N}\dfrac{d\phi}{dt}$, we can get
  - The primary voltage $\quad v_p = N_p \dfrac{d\varphi}{dt}$
  - The secondary voltage $\quad v_S = N_S \dfrac{d\varphi}{dt}$
- Turns ratio n: Dividing v_S by v_P gives the transformer's turns ratio n:

$$\frac{v_S}{v_P} = \frac{N_S}{N_P} = n$$

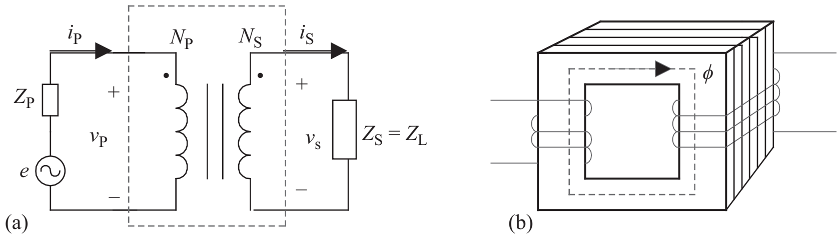


Figure 12.6 Iron-core transformer

Table 12.1 Parameters of an ideal transformer

Parameters	**Name**
v_p	Primary voltage
v_S	Secondary voltage
N_p	Number of turns on the primary coil
N_S	Number of turns on the secondary coil
i_p	Primary current
i_S	Secondary current
Z_p	Primary impedance
Z_S	Secondary impedance
Z_L	Load impedance ($Z_L = Z_S$)

Power

- If the transformer is an ideal transformer, i.e., that transformer has no power loss itself, the input power is equal to the output power,
- i.e., $P_P = P_S$ or $v_P i_p = v_S i_S$,

so $$\frac{i_P}{i_S} = \frac{v_S}{v_P} = n \tag{12.1}$$

Voltage and current

- The primary voltage $v_p = \dfrac{v_S}{n}$ can be obtained from (12.1). $n = \dfrac{v_S}{v_p}$
- The primary current $i_p = n\, i_S$ can also be obtained from (12.1). $\dfrac{i_p}{i_S} = n$

Impedance

- The primary impedance can be obtained by substituting v_p and i_p into Z_p as follows:

$$Z_P = \frac{v_P}{i_P} = \frac{v_S/n}{n\, i_S} = \frac{1}{n^2} Z_L \qquad Z = \frac{v}{i}$$

or $$n^2 = \frac{Z_L}{Z_P}, \quad n = \sqrt{\frac{Z_L}{Z_p}}$$

- The secondary impedance is the load impedance Z_L: $Z_L = \dfrac{v_S}{i_S}$

Turns ratio	– Instantaneous form:	$n = \frac{N_S}{N_P} = \frac{v_S}{v_P} = \frac{i_P}{i_S} = \sqrt{\frac{Z_L}{Z_P}}$, or $Z_L = n^2 Z_p$
	– Phasor form:	$n = \frac{N_S}{N_P} = \frac{\dot{V}_S}{\dot{V}_P} = \frac{\dot{I}_P}{\dot{I}_S} = \sqrt{\frac{Z_L}{Z_P}}$

Power	– Instantaneous form: $p_S = i_S\, v_S, \quad p_P = i_P\, v_P$ – Phasor form: $\dot{P}_S = \dot{I}_S\, \dot{V}_S, \quad \dot{P}_P = \dot{I}_P\, \dot{V}_P$

12.2.5 Transformer parameters conversion

- Conversion of the voltage, current, and impedance: The expressions of the transformer's turns ratio indicate that a transformer can be used to convert voltage, current, and impedance.
- Voltage conversion:
  - Convert from the primary to the secondary, multiplying by n:

$$v_S = n v_P \qquad\qquad n = \frac{v_S}{v_p}$$

  - Convert from the secondary to the primary, multiplying by $\frac{1}{n}$:

$$v_p = \frac{1}{n} v_S \qquad\qquad n = \frac{v_S}{v_p}$$

- Current conversion:
  - Convert from the primary to the secondary, multiplying by $\frac{1}{n}$:

$$i_S = \frac{1}{n} i_p \qquad\qquad n = \frac{i_p}{i_S}$$

  - Convert from the secondary to the primary, multiplying by n:

$$i_p = n\, i_S \qquad\qquad n = \frac{i_p}{i_S}$$

- Impedance conversion:
  - Convert from the primary to the secondary, multiplying by $\frac{1}{n^2}$:

$$Z_P = \frac{1}{n^2} Z_L \qquad\qquad Z_L = n^2 Z_P$$

  - Convert from the secondary to the primary, multiplying by n^2:

$$Z_L = n^2 Z_P$$

The converted impedance is also called the reflected impedance, meaning the reflection of the primary impedance results in the secondary impedance.

Transformer parameters conversion	– Voltage conversion: $v_S = nv_P, \quad v_p = \frac{1}{n}v_S$ – Current conversion: $i_S = \frac{1}{n}i_p, \quad i_p = ni_S$ – Impedance conversion: $Z_P = \frac{1}{n^2}Z_L, \quad Z_L = n^2 Z_P$

12.2.6 Transformer parameters conversion—an example

Example 12.1: The number of turns on the primary is 40 for an ideal transformer, and the number of turns on the secondary is 100. $\dot{V}_p = 50$ V, $\dot{I}_{\mathbf{p}} = 5$ A, and $Z_L = 2\ \Omega$. Determine the transformer's turns ratio, secondary voltage, secondary current, primary impedance (reflected from the secondary), and the primary power (the amplitude only).

Solution:

$N_p = 40, \quad N_S = 100, \quad \dot{V}_P = 50\text{ V}, \quad \dot{I}_{\mathbf{P}} = 5\text{ A}, \quad$ and $\quad Z_L = 2\ \Omega$

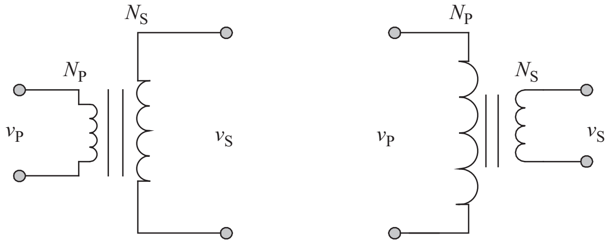
- Turns ratio: $n = \dfrac{N_S}{N_P} = \dfrac{100}{40} = 2.5$
- Secondary voltage: $\dot{V}_S = n\dot{V}_P = (2.5)(50\text{ V}) = 125\text{ V}$
- Secondary current: $\dot{I}_S = \dfrac{\dot{I}_P}{n} = \dfrac{5\text{ A}}{2.5} = 2\text{ A}$
- Primary impedance: $Z_P = \dfrac{Z_L}{n^2} = \dfrac{2\ \Omega}{2.5^2} = 0.32\ \Omega$
- Primary power: $\dot{P}_S = \dot{I}_S\dot{V}_S = (2\text{ A})(125\text{ V}) = 250\text{ W} = 0.25\text{ kW}$

12.3 Step-up and step-down transformers

12.3.1 Step-up transformer

Characteristics of a step-up transformer

- A step-up transformer is a transformer that can increase its secondary voltage, since a step-up transformer always has more secondary winding turns than the primary.
- $v_S > v_p$: The secondary voltage of a step-up transformer (v_S) is always higher than the primary voltage (v_p), i.e., $v_S > v_p$.



(a) Step-up (b) Step-down

Figure 12.7 Step-up and step-down transformers

- The value of the secondary voltage depends on the turns ratio (n).
- $N_S > N_P$: The equation of $n = \frac{N_S}{N_P} = \frac{v_S}{v_P}$ indicates that to have a higher secondary voltage, the number of turns on the secondary winding must be greater than that of the primary's, i.e., $N_S > N_P$ as illustrated in Figure 12.7(a).
- $n > 1$: $N_S > N_P$ means the turn ratio $n = \frac{N_S}{N_P} > 1$. This is an important characteristic of a step-up transformer.

Step-up transformer	– $v_S > v_p$ – $N_S > N_P$ – $n > 1$

12.3.2 Step-down transformer

Characteristics of a step-down transformer

- A step-down transformer is a transformer that can decrease its secondary voltage.
- $v_S < v_p$: Since a step-down transformer always has less turns on the secondary winding than the primary, the secondary voltage of a step-down transformer (v_S) is always lower than the primary voltage (v_p), i.e., $v_S < v_p$.
- The value of the secondary voltage depends on the turns ratio (n).
- $N_S < N_P$: The equation of $n = \frac{N_S}{N_P} = \frac{v_S}{v_P}$ indicates that to have a voltage that is lower in secondary than primary, the number of turns on the secondary coil must be less than primary's, i.e., $N_S < N_P$ as illustrated in Figure 12.7(b).
- $n < 1$: $N_S < N_P$ means the turns ratio $n = \frac{N_S}{N_P} < 1$. This is an important characteristic of a step-down transformer, which is opposite of a step-up transformer.

Step-down transformer	– $v_S < v_P$ – $N_S < N_P$ – $n < 1$

Example 12.2: If a transformer has 125 turns of secondary windings and 250 turns of primary windings, calculate its turn ratio and determine if it is a step-up or a step-down transformer.

Solution:

$N_S = 125, \quad N_P = 250, \quad N_S < N_P$

- Turns ratio: $n = \dfrac{N_S}{N_P} = \dfrac{125}{250} = \dfrac{1}{2} = 0.5 < 1$
- $n = 0.5 < 1$

  It is a step-down transformer.

12.3.3 *Applications of step-up and step-down transformers*

The functions of step-up and step-down transformers

- Transformers can be used to convert voltage, current, and impedance.
- In the power system, the basic usage of transformers is stepping up or stepping down the voltage or current, which will require converting voltage or current from primary to secondary winding.
- The functions of step-up and step-down transformers are to increase or decrease the voltage of their secondary windings, and have important applications in the power transmission system.

Power transmission system

- A simplified power transmission system is illustrated in Figure 12.8.
- Step-up voltage: The voltage generated from the generator of a power plant needs to rise to a very high value through the step-up transformer so that it can be delivered through long distance transmission lines.
- Improve efficiency: Step-up voltage can reduce the loss of energy or power created due to the winding resistance in the line ($I^2 R_w = P_{Loss}$) for a long distance line transmission, and improve the efficiency of the electricity transmission.

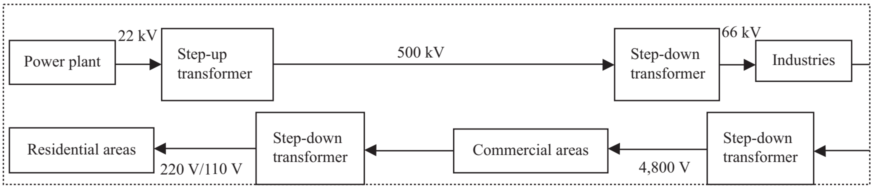

Power plant
22 kV
Step-up transformer
500 kV
Step-down transformer
66 kV
Industries
Residential areas
220 V/110 V
Step-down transformer
Commercial areas
4,800 V
Step-down transformer

Figure 12.8 Power transmission system

- Step-down current: Decreasing the current to reduce the power loss on the transmission line may reduce the output power ($p = iv$) of the transmission system.
- Reduce the power loss: If the voltage is increased through the step-up transformer before the transmission, it can maintain the same output power, but reduce the power loss on the line (decrease the current), i.e.,

$$\vec{P}^{\rightarrow} = (I\downarrow)(V\uparrow) \quad \Rightarrow \quad (I^2\downarrow)(R) = P_{\text{Loss}}\downarrow$$

- Example: If a step-up transformer is used to increase the voltage by 100 V, then the current will reduce by 100 A [$v_S\uparrow\ = (n\uparrow)(v_P),\ i_S\downarrow = \left(\dfrac{1}{n\uparrow}\right)(i_P)$], and the loss of the power due to the winding resistance in the line will reduce to 10,000 W, since

$$I^2R_w = P_{\text{Loss}}, \quad \text{and} \quad I^2\downarrow R_w \Rightarrow P_{\text{Loss}}\downarrow.$$

- Step-down voltage: The local distribution stations require step-down transformers to reduce the very high voltage by the long-distance transmission and can send it to commercial or residential areas.

12.3.4 Other types of transformers

There are other types of commonly used transformers listed as follows:

- Center-tapped transformer: It has a tap (connecting point) in the middle of the secondary winding, and it can provide two balanced output voltages with the same value, as shown in Figure 12.9(a).
- Multiple-tapped transformer: It has multiple taps in the secondary winding, and it can provide several output voltages with different values, as shown in Figure 12.9(b).
- Adjustable (or variable) transformer: The output voltage of adjustable transformer across the secondary winding is adjustable. The secondary winding of the adjustable transformer can provide an output voltage that may be variable in a range of zero to the maximum values. An adjustable transformer is shown in Figure 12.9(c).
- Autotransformer: It is a transformer with only a single winding, which is a common coil for both the primary and the secondary coils, and a portion of the common coil acts as part of both the primary and secondary coils, as shown in Figure 12.9(d). An autotransformer can be made smaller and lighter.

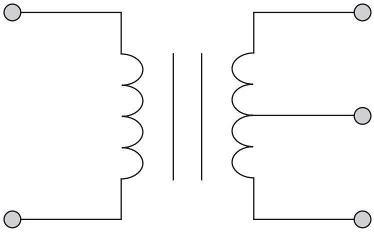


Figure 12.9(a) Center-tapped transformer

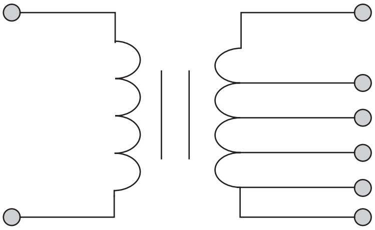


Figure 12.9(b) Multiple-tapped transformer

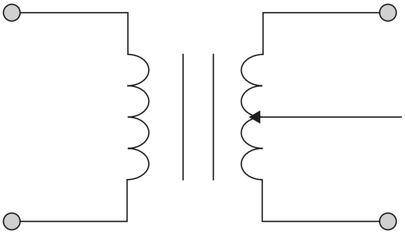


Figure 12.9(c) Adjustable transformer

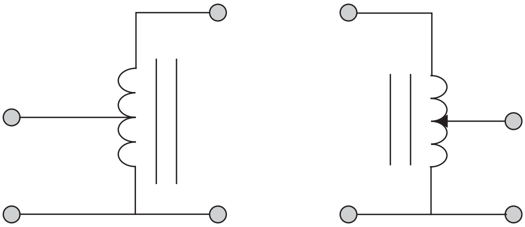


Figure 12.9(d) Autotransformer

12.4 Impedance matching

12.4.1 Maximum power transfer

Impedance matching

In addition to stepping up and stepping down voltages, a transformer has another important application, matching the load and source impedance in a circuit to achieve the maximum power transfer from the source to the load. It is known as impedance matching.

Maximum power transfer in DC circuits

- $R_L = R_S$: The maximum power delivered from a source to a load in a DC circuit can be achieved when the load resistance is equal to the internal resistance of the source ($R_L = R_S$).

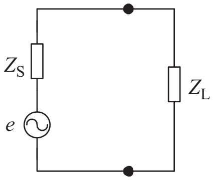


Figure 12.10 Impedance matching

- $R_L = R_{TH} = R_N$: The maximum power delivered from a source to a load in a DC circuit can also be achieved when the load resistance is equal to the Thevenin/Norton's equivalent resistance of the network ($R_L = R_{TH} = R_N$). The theory of maximum power transfer in DC circuits was introduced in Chapter 5.

Maximum power transfer in AC circuits

- $R \to Z$: Maximum power transfer theory can also be applied to an AC circuit by replacing the resistance R with impedance Z.
- $Z_L = Z_S$: When the load impedance Z_L is equal to the source internal impedance Z_S, the power received by the load from the source reaches the maximum, this is shown in Figure 12.10.

Maximum power transfer	When $Z_L = Z_S$, the power delivered from the source to the load reaches the maximum.

12.4.2 Impedance matching

To achieve the maximum power transfer

- Internal resistance is fixed: In the practical circuits (or Thevenin's equivalent circuits), the internal resistance of the source is fixed, usually is not matching with the load impedance, and also not adjustable.
- Impedance matching: A transformer with an appropriate turns ratio n can be placed between the load and source to make the load impedance and the source internal resistance equal, and to achieve the maximum power transfer, i.e., $n = \sqrt{\frac{Z_L}{Z_p}}$.

Impedance matching	Place a transformer with $n = \sqrt{\frac{Z_L}{Z_p}}$ between the source and the load to achieve maximum power transfer.

Example 12.3: A simplified amplifier circuit is illustrated in Figure 12.11(a). The circuit within dashed lines is Thevenin's equivalent circuit for the amplifier circuit, and its internal resistance is 100 Ω. How do we deliver the maximum power to the speaker if the resistance of the speaker is 4 Ω (so that the speaker can have the maximum volume)?

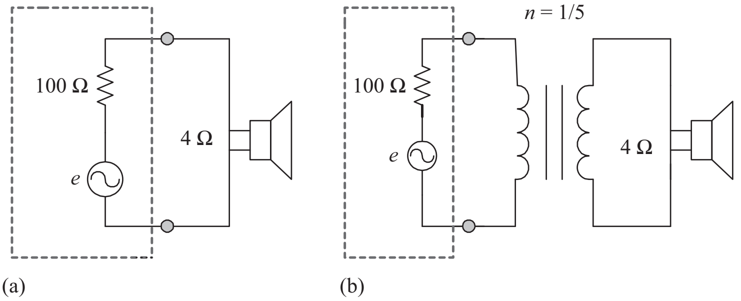


Figure 12.11 Circuit for Example 12.2

Solution:

- Since the load impedance ($Z_L = R_L = 4\ \Omega$) does not match with the source internal impedance ($Z_S = R_S = 100\ \Omega$) currently, the maximum power cannot be delivered to the speaker if the source and load are connected directly.
- Choose an audio transformer with the appropriate turns ratio n, i.e.,

$$n = \sqrt{\frac{Z_L}{Z_p}} = \sqrt{\frac{4}{100}} = 0.2 = \frac{1}{5}$$

- Therefore, if placing an impedance matching transformer with the turns ratio of 1/5 between the amplifier and speaker as illustrated in Figure 12.11(b), the speaker will have the maximum volume.

Summary

Mutual inductance

- Self-inductance is the ability of a coil to produce an induced voltage due to the changing of the current in the coil itself.
- Mutual inductance is the ability of a coil to produce an induced voltage due to the changing of the current in another coil nearby.

$$L_M = k\sqrt{I_1 I_2}$$

- Coefficient of coupling: $k = \dfrac{\phi_{1-2}}{\phi_1} \qquad (0 \leq k \leq 1)$
  - ϕ_1: The flux generated by the current i_1 in the first coil L_1.
  - ϕ_{1-2}: The flux generated by the current i_1 in the coil L_1 cross-linking to coil L_2.
- Dot conversion: dotted terminals of coils have the same voltage polarity.

Transformers

- Transformer: It uses the principle of mutual inductance to convert AC electrical energy from input to output.
- Air-core transformer: A linear transformer that can be obtained by placing the two coils close to each other or winding two (or more) coils around an insulating substance.
- Iron-core transformer: A transformer that has laminations of ferromagnetic material with coils wound on it. The coupling coefficient k is close to 1.
- Ideal transformer: An ideal transformer has no losses (imaginary transformer). Efficiency of this transformer is considered as 100%.
- An iron-core transformer is considered the ideal transformer because it uses ferromagnetic materials with high permeability as its core.

The parameters of an ideal transformer ($k = 1$):

Parameters	Name
v_p	Primary voltage
v_S	Secondary voltage
N_p	Number of turns on the primary coil
N_S	Number of turns on the secondary coil
i_p	Primary current
i_S	Secondary current
Z_p	Primary impedance
Z_S	Secondary impedance
Z_L	Load impedance $\quad Z_L = Z_S$

- Turns ratio: $$n = \frac{N_S}{N_P} = \frac{v_S}{v_P} = \frac{i_P}{i_S} = \sqrt{\frac{Z_L}{Z_P}}, \quad \text{or} \quad Z_L = n^2 Z_p$$

  In phasor form: $$n = \frac{N_S}{N_P} = \frac{\dot{V}_S}{\dot{V}_P} = \frac{\dot{I}_P}{\dot{I}_S} = \sqrt{\frac{Z_L}{Z_P}}$$

- Power: $$p_S = i_S\, v_S, \quad p_P = i_P\, v_P$$ $$\dot{P}_S = \dot{I}_S \dot{V}_S, \quad \dot{P}_P = \dot{I}_P \dot{V}_P$$

- Transformer parameters conversion (V, I, and Z):
  - Voltage conversion: $v_S = nv_p, \quad v_P = \frac{1}{n} v_S$
  - Current conversion: $i_S = \frac{1}{n} i_P, \quad i_P = n\, i_S$
  - Impedance conversion: $Z_L = n^2 Z_P, \quad Z_P = \frac{1}{n^2} Z_L$
- Impedance matching: Place a transformer with the turns ratio $n = \sqrt{\frac{Z_L}{Z_p}}$ between the source and the load to achieve maximum power transfer from the source to the load.

Step-up and step-down transformers

- Step-up transformer:

$$v_S > v_p$$
$$N_S > N_P$$
$$n > 1$$

- Step-down transformer:

$$v_S < v_p$$
$$N_S < N_P$$
$$n < 1$$

Practice problems

12.1

1. (   ) inductance is the ability of a coil to produce an induced voltage due to the changing of the current in another coil nearby.
2. The closer the two coils are, the greater the flux linkage, and the (   ) the value of the coupling coefficient k.
3. The polarity of the induced voltage across the mutually coupled coils can be determined by the (   ) convention method.
4. $\phi_{1\text{–}2}$ represents the portion of the magnetic flux that is generated by the current i_1 in the first coil L_1 and linked to the (   ) coil.
5. $L_M = 1.5$ μH, $L_1 = 1$ μH, and $L_2 = 4$ μH for a mutual inductor. Determine the coupling coefficient of this mutual inductor.

12.2

6. Structurally, the transformers are categorized as two main types: the (   )-core and (   )- core transformers.
7. A transformer uses the principle of mutual inductance to convert (   ) electrical energy from input to output.

8. If the turns ratio of a transformer is 5:1 and the secondary voltage is 20 V, the primary voltage is (   ) V.
9. If the turns ratio of a transformer is 0.3 and the number of turns in the primary coil is 200, the number of turns in the secondary coil is (   ).

12.3

10. If the primary voltage of a transformer is 110 V and the turns ratio is 4, what is the secondary voltage for this transformer? Is it a step-up or step-down transformer?
11. What is the turns ratio of a step-down transformer if a 220 V voltage is reduced to 110 V?

12.4

12. In what condition can an AC source transfer maximum power to its load?
13. Determine the turns ratio for the transformer in the circuit shown in Figure 12.12 that can transfer maximum power from the AC source to the speaker.

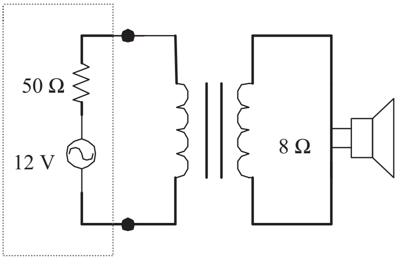


Figure 12.12

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Chapter 13

Circuits with dependent sources

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13.1 Dependent sources

13.1.1 Introduction to dependent sources

Independent sources

- Independent voltage/current source: A voltage source or a current source that acts independently and provides fixed voltage/current in a branch. It will not be affected by other voltages and currents in the circuit.
- Symbols of independent sources: The DC and AC circuits we have discussed in the previous chapters have independent voltage/current sources (Figure 13.1).

Dependent (or controlled) sources

- Dependent (or controlled) source: A voltage source or a current source whose value is controlled by or dependents on other voltage or current somewhere else in the circuit (or network).
- Applications: Dependent sources are a useful concept in modeling and analyzing electronic components such as transistors, amplifiers, and filters.

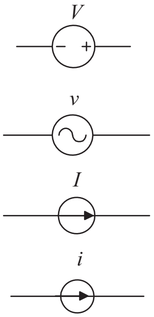


Figure 13.1 Independent sources

- The circuits we will analyze in this chapter have dependent (or controlled) sources, in which the source voltage or current is a function of other voltage or current in the circuit.

Dependent (controlled) sources	A voltage or a current source whose value is controlled by or dependents on other voltage or current somewhere else in the circuit.

13.1.2 Types of dependent sources

Four types of dependent sources

The dependent sources can be categorized into the following four types according to whether it is controlled by a circuit voltage or current, as well as whether the dependent source itself is a voltage source or current source:

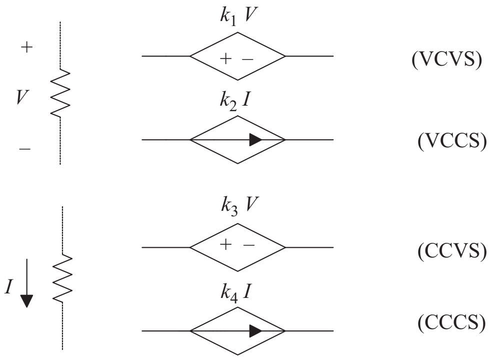
- Voltage-controlled voltage source (VCVS)
- Voltage-controlled current source (VCCS)
- Current-controlled voltage source (CCVS)
- Current-controlled current source (CCCS)

Symbols of independent sources

The above dependent sources can be represented by the symbols in Figure 13.2.

Control coefficients

- k_1, k_2, k_3, and k_4: In Figure 13.2, k_1, k_2, k_3, and k_4 are called control coefficients or gain parameters.
- A voltage-controlled source has a *voltage* across its two terminals that equals to a control coefficient k multiplied by a controlling voltage or current elsewhere in the same circuit.
- A current-controlled source has a *current* in its branch that equals to a control coefficient k multiplied by a controlling voltage or current elsewhere in the same circuit.



(a) Controlling sources (b) Controlled sources

Figure 13.2 Dependent sources

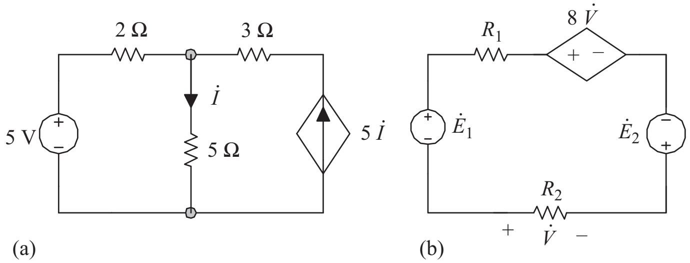


Figure 13.3 Circuits with dependent sources

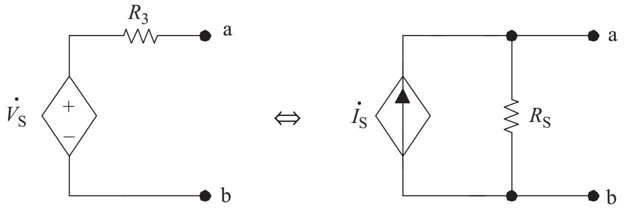
13.1.3 Circuits with dependent sources

Circuits with dependent sources

- CCCS: $5\,\dot{I}$ in the circuit of Figure 13.3(a) represents a CCCS.
  - Its control coefficient k is 5.
  - The current $\dot{I}$ is a controlling current through the 5 Ω resistor branch in the same circuit.
- VCVS: $8\,\dot{V}$ in the circuit of Figure 13.3(b) is a VCVS.
  - Its control coefficient is 8.
  - The voltage $\dot{V}$ is a controlling voltage across the resistor R_2 in the same circuit.

Applications

- CCCS: After you take an analog electronics course, you will understand that a good example for modeling a CCCS is a transistor circuit.
- Transistor: Based on the property of a bipolar transistor, a current amplifier, its large collect current i_c is proportional to the small base current i_b according to the relationship $i_c = \beta i_b$. In this equation, the current gain β is the same as the control coefficient k in the dependent source.



(a) $R_S = R_S,\ \dot{V}_S = \dot{I}_S R_S$

(b) $R_S = R_S,\ \dot{I}_S = \dfrac{\dot{V}_S}{R_S}$

Figure 13.4 Equivalent conversion

13.1.4 Equivalent conversion of dependent sources

Equivalent conversion

- Equivalent conversion of dependent sources is the same as the equivalent conversion of independent sources.
  – Internal resistance R_S of the source does not change before and after the conversion.
  – Apply Ohm's law to convert the source.
- Controlled current source → controlled voltage source:

$$R_S = R_S, \quad \dot{V}_S = \dot{I}_S R_S$$

- Controlled voltage source → controlled current source:

$$R_S = R_S, \quad \dot{I}_S = \frac{\dot{V}_S}{R_S}$$

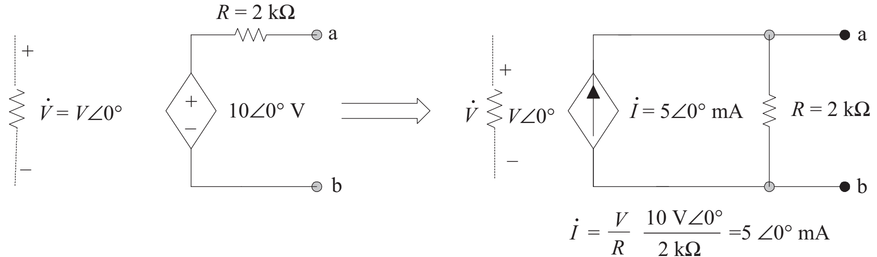
Voltage-controlled source ↔ current-controlled source

For instance, the voltage-controlled source in Figure 13.4(a) can be converted equivalently to a current-controlled source as shown in Figure 13.4(b), and vice versa.

Equivalent conversion of dependent sources	The same as the equivalent conversion of independent sources: – Internal resistance R_S of the source does not change. – Apply Ohm's law to convert the source.

13.1.5 Examples of equivalent conversion

Example 13.1: The voltage-controlled voltage source (VCVS) in Figure 13.5(a) can be converted equivalently to a voltage-controlled current source (VCCS) as shown in Figure 13.5(b).

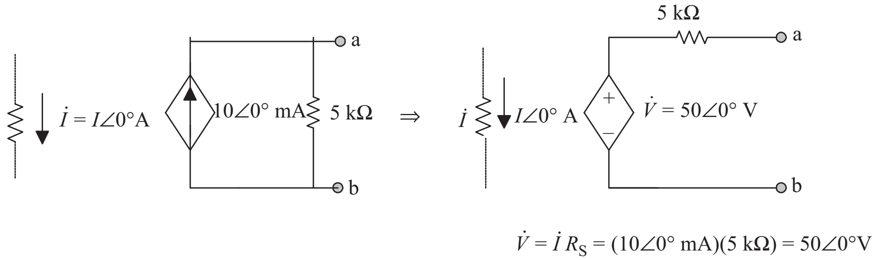

R = 2 kΩ
a
+
$\dot{V} = V\angle 0°$
−
+
−
10∠0° V
b
a
+
$\dot{V}$
$V\angle 0°$
−
$\dot{I} = 5\angle 0°$ mA
R = 2 kΩ
b
$\dot{I} = \frac{V}{R}\ \frac{10\ \text{V}\angle 0°}{2\ \text{k}\Omega} = 5\ \angle 0°$ mA

(a) VCVS

(b) VCCS

Figure 13.5 Circuits for Example 13.1

Example 13.2: The current-controlled current source (CCCS) in Figure 13.6(a) can be converted equivalently to a current-controlled voltage source (CCVS) as shown in Figure 13.6(b).


$\dot{I} = I\angle 0°$A
10∠0° mA
5 kΩ
a
b
⇒
5 kΩ
a
$\dot{I}$
$I\angle 0°$ A
+
−
$\dot{V} = 50\angle 0°$ V
b
$\dot{V} = \dot{I}\,R_S = (10\angle 0°\ \text{mA})(5\ \text{k}\Omega) = 50\angle 0°\text{V}$

(a) CCCS

(b) CCVS

Figure 13.6 Circuit for Example 13.2

13.2 Analyzing circuits with dependent sources

13.2.1 KVL and KCL

- The analyzing methods for circuits with dependent sources are similar to that of circuits with independent sources.
- The following examples will describe these methods.

Example 13.3: Determine the current *I* in the circuit of Figure 13.7(a).

Solution:

- Simplify and convert the circuit of Figure 13.7(a) to that in Figure 13.7(b).

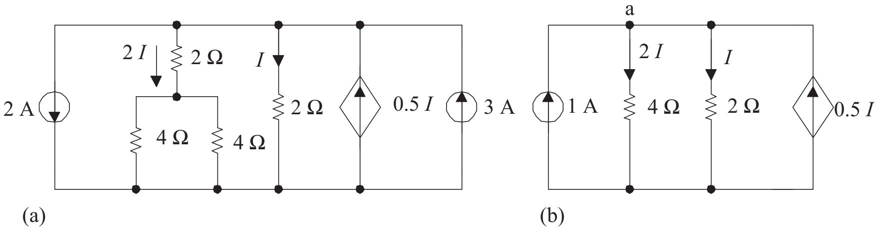


Figure 13.7 Circuit for Example 13.3

There, $3\text{ A} - 2\text{ A} = 1\text{ A}$

$2\,\Omega + 4\,\Omega // 4\,\Omega = 4\,\Omega$

Note: This circuit has a CCCS, simplify the circuit without changing the CCCS (both controlling branches and controlled source).

- Write the KCL equation for the node a in Figure 13.7(b):

$$\Sigma I = 0: \qquad 1\text{ A} - 2I - I + 0.5I = 0$$

- Current I can be solved from the above equation:

$$-2.5\,I = -1\text{ A}$$

$$\therefore \boxed{I = 0.4\text{ A}}$$

Example 13.4: Determine the voltage V in the circuit of Figure 13.8.

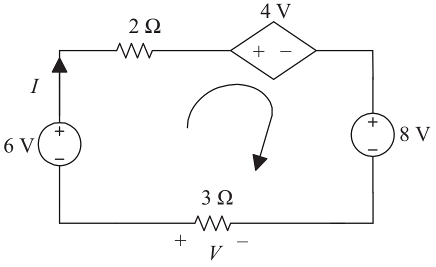


Figure 13.8 Circuit for Example 13.4

Solution:

- Applying KVL, $\Sigma V = 0$: $\quad -6 + 2I + 4V + 8 + 3I = 0$

  That is, $\quad 2 + 5I + 4V = 0$

- Substituting $V = -3I$ into the above equation gives:

$$2 + 5I + 4(-3I) = 0$$

- Solving for I:

$$2 + 5I - 12I = 0$$
$$2 - 7I = 0$$
$$I \approx 0.29 \text{ A}$$

- Solving for V:

$$\begin{aligned} V &= -3I \\ &= (-3\ \Omega)(0.29 \text{ A}) \\ &= \boxed{-0.87 \text{ V}} \end{aligned}$$

Analyzing circuits with dependent sources	Analyzing circuits with dependent sources is similar to the methods of analysis for circuits with independent sources.

13.2.2 Node voltage analysis

Example 13.5: Write node voltage equations for the circuit in Figure 13.9 using the node voltage analysis method.
Tip: Write KVL by treating the dependent source as an independent source first, and then represent the control quantity as node voltages.

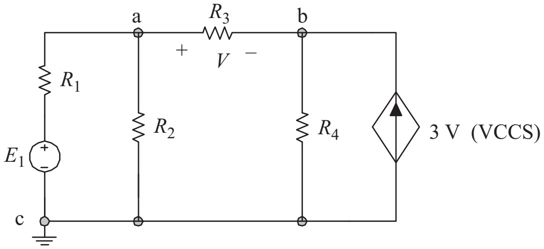


Figure 13.9 Circuit for Example 13.5

Solution: The procedure for applying the node voltage analysis method (Chapter 4, Section 4.4) to the above circuit is as follows:

1. Label nodes a, b, and c, and choose ground c as the reference node as shown in Figure 13.9.

2. Write KCL equations to $n - 1 = 3 - 1 = 2$ nodes (nodes a and b) by inspection.

Node a:
$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_a - \frac{1}{R_3}V_b = \frac{1}{R_1}E_1 \tag{13.1}$$

Node b:
$$-\frac{1}{R_3}V_a + \left(\frac{1}{R_3} + \frac{1}{R_4}\right)V_b = 3\ \text{V} \tag{13.2}$$

Substituting the control voltage $V = V_a - V_b$ to (13.2) gives,

$$-\frac{1}{R_3}V_a + \left(\frac{1}{R_3} + \frac{1}{R_4}\right)V_b = 3(V_a - V_b) \tag{13.3}$$

3. Solving (13.1) and (13.3) can determine the node voltage V_a and V_b (if R_1, R_2, R_3, and E_1 are given).

13.2.3 Mesh current analysis

Example 13.6: Use the mesh current analysis method to write mesh equations for the circuit in Figure 13.10.
Tip: Write KVL by treating the dependent source as an independent source first, and then represent the controlling quantity as mesh current.

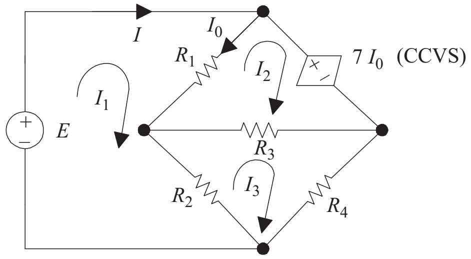


Figure 13.10 Circuit for Example 13.6

Solution: The procedure for applying the mesh current analysis method (Chapter 4, Section 4.3) to the above circuit is as follows:

1. Label all the reference directions for each mesh current I_1, I_2, and I_3 (clockwise) as shown in Figure 13.10.
2. Apply KVL around each mesh, and ensure the number of KVL equations is equal to the number of meshes (there are three meshes in Figure 13.10).

Mesh 1:
$$(R_1 + R_2)I_1 - R_1I_2 - R_2I_3 = E \tag{13.4}$$

Mesh 2: $$-R_1I_1 + (R_1 + R_3)I_2 - R_3I_3 = -7I_0 \tag{13.5}$$

Mesh 3: $$-R_2I_1 - R_3I_2 + (R_2 + R_3 + R_4)I_3 = 0$$

Substituting the controlling current $I_0 = I_1 - I_2$ to (13.5) yields,

$$-R_2I_1 + (R_1 + R_3)I_2 - R_3I_3 = -7(I_1 - I_2) \tag{13.6}$$

3. Solve three simultaneous equations (13.4), (13.5), and (13.6) resulting from step 2 can determine three mesh currents I_1, I_2, and I_3.

13.2.4 Superposition theorem

Example 13.7: Determine the branch current I for the circuit in Figure 13.11 by using the superposition theorem.
Tip: The dependent source will not act separately in the superposition theorem. Do not change the dependent source in the circuit when another independent source is acting in the circuit.

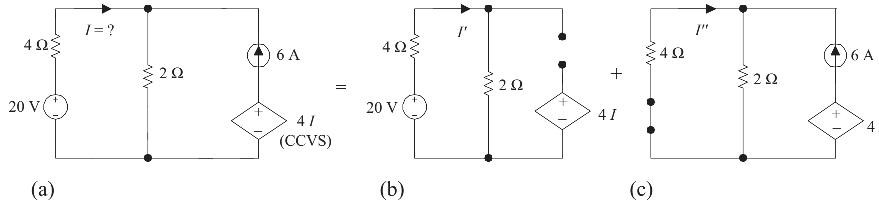


Figure 13.11 Circuit for Example 13.7

Solution: The procedure for using the superposition theorem (Chapter 5, Section 5.1) to the above circuit is as follows:

1. Choose 20 V voltage source applied to the circuit first, replace the 6 A current source with an open circuit as shown in Figure 13.11(b), and calculate I':

$$I' = \frac{20\text{V}}{4\,\Omega + 2\,\Omega} \approx 3.33\text{ A} \qquad I = \frac{V}{R}$$

2. When a 6 A current source is applied to the circuit, replace the 20 V voltage source with a short circuit as shown in Figure 13.11(c), and calculate I'':

$$I'' = -6\text{A}\frac{2\,\Omega}{2\,\Omega + 4\,\Omega} = -2\text{ A} \qquad I_1 = I_T\frac{R_2}{R_1 + R_2}$$

(The 6 A current is negative due to its assumed direction to be opposite to I''.)

3. Calculate the sum of currents I' and I'':

$$I = I' + I'' = 3.33\text{ A} + (-2\text{ A}) = \boxed{1.33\text{ A}}$$

13.2.5 Thevenin's theorem

Example 13.8: Determine the voltage across the two terminals a and b in Figure 13.12(a) by using Thevenin's theorem (plot Thevenin's equivalent circuit).

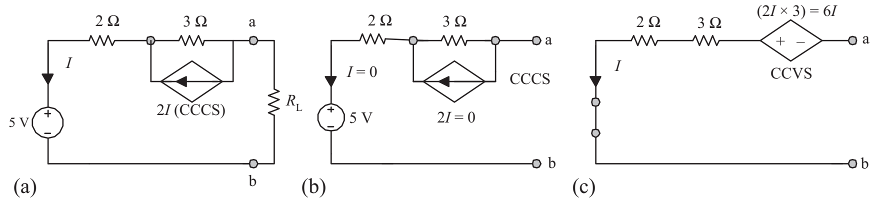


Figure 13.12 Circuits for Example 13.8

Solution: The procedure for using Thevenin's theorem (Chapter 5, Section 5.2) to the above circuit is as follows:

1. Open and remove the load branch resistor R_L, and mark a and b on the terminals of the load branch as shown in Figure 13.12(b).
2. Determine Thevenin's equivalent voltage V_{TH}: Since the branch a and b is open, $I = 0$, and the CCCS is also 0 ($2I = 0$) in the circuit of Figure 13.12(b),

$$\text{so:} \quad V_{TH} = V_{ab} = \boxed{5\text{ V}}$$

3. Determine Thevenin's equivalent resistance R_{TH}: Replace the 5 V voltage source with a short circuit and convert CCCS to CCVS as shown in Figure 13.12(c).

$$R_{TH} = R_{ab} = \frac{V_{ab}}{I} = \frac{6I + (2\,\Omega + 3\,\Omega)I}{I} = 11\,\Omega$$

4. Plot Thevenin's equivalent circuit as shown in Figure 13.12(d).

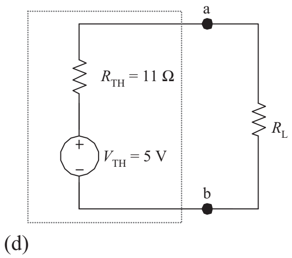
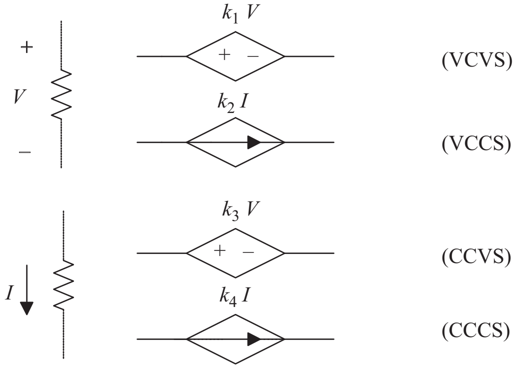


Figure 13.12(d) Thevenin's equivalent circuit

Summary

Dependent (controlled) source: A voltage or a current source whose value is controlled by or dependents on other voltage or current somewhere else in the circuit.

- Voltage-controlled voltage source (VCVS)
- Voltage-controlled current source (VCCS)
- Current-controlled voltage source (CCVS)
- Current-controlled current source (CCCS)



Equivalent conversion of dependent sources is the same as the equivalent conversion of independent sources:

- Controlled current source → controlled voltage source:

$$R_S = R_S, \qquad \dot{V}_S = \dot{I}_S R_S$$

- Controlled voltage source → controlled current source:

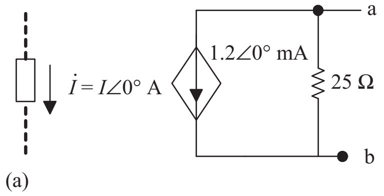
$$R_S = R_S, \qquad \dot{I}_S = \frac{\dot{V}_S}{R_S}$$

The method of analysis for circuits with dependent sources is similar to the methods of analysis for circuits with independent sources.

Practice problems

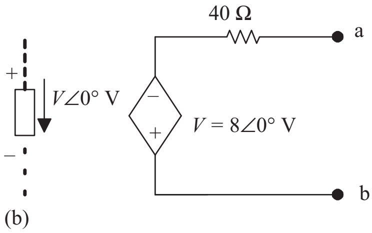
13.1

1. Convert the dependent source circuits in Figure 13.13(a) and (b).



$\dot{I} = I\angle 0°$ A
1.2∠0° mA
25 Ω
a
b
(a)

Figure 13.13(a)

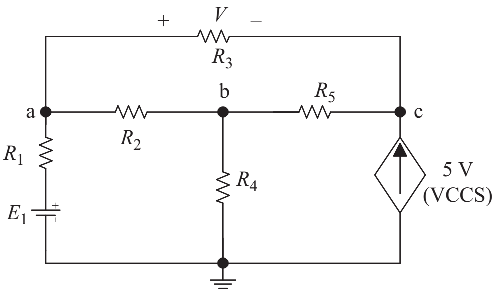


40 Ω
a
b
+
−
V∠0° V
V = 8∠0° V
(b)

Figure 13.13(b)

13.2

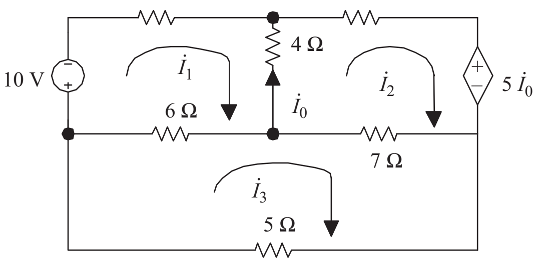
2. Write the node equations for the circuit shown in Figure 13.14.



+ V −
R_3
b
R_5
a
c
R_2
R_1
R_4
5 V
(VCCS)
E_1

Figure 13.14

3. Write the mesh equations for the circuit shown in Figure 13.15.



2 Ω
3 Ω
4 Ω
10 V
$\dot{I}_1$
$\dot{I}_2$
$5\dot{I}_0$
6 Ω
$\dot{I}_0$
7 Ω
$\dot{I}_3$
5 Ω

Figure 13.15

4. Determine the Norton's equivalent circuit in the circuit of Figure 13.16.

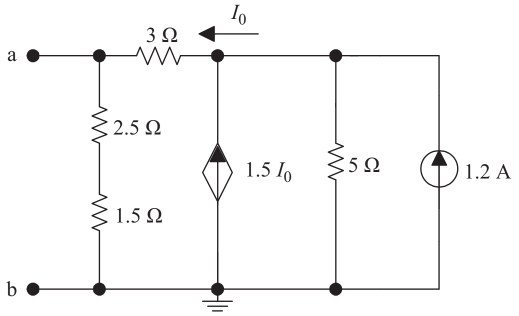

I_0
3 Ω
a
b
2.5 Ω
1.5 Ω
1.5 I_0
5 Ω
1.2 A

Figure 13.16

5. Determine the voltage V_0 in the circuit of Figure 13.17 using the superposition theorem.

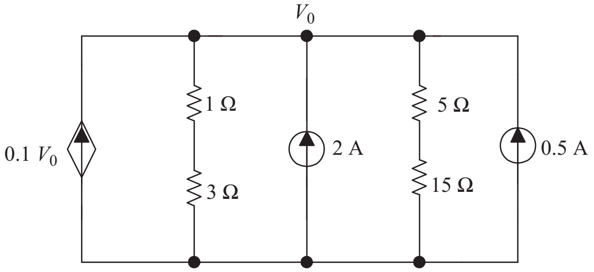

V_0
0.1 V_0
1 Ω
3 Ω
2 A
5 Ω
15 Ω
0.5 A

Figure 13.17

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Chapter 14

Three-phase systems

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14.1 Three-phase circuits

14.1.1 Introduction to three-phase systems

Two-phase AC power systems

There are two types of systems in an electric circuit, single-phase and three-phase AC power systems.

- In a single-phase AC circuit, there is only one phase (one coil in a magnetic field).
- In a three-phase AC circuit, there will be three phases carrying AC voltages that are offset by 120° (three coils in a magnetic field).

Three-phase power system

It is a common method of AC power generation, transmission, and distribution.

- A three-phase power system gives the three-phase voltage of equal magnitude and frequency.
- The three windings (coils) in a magnetic field are placed at 120° apart. The individual voltage (and current) will be 120° apart.
- Most industrial and commercial electrical power systems use a three-phase configuration.

The reason for using three-phase power system

- A three-phase circuit is more economical than three single-phase circuits because it uses less conductor material to transmit and distribute the same amount of power.
- A three-phase power system is more efficient and reliable to produce, transmit, and consume electricity.
- The power produced by a three-phase AC voltage source is less pulsating (smooth) than a single-phase AC power.

Three-phase power system	– A common method of AC power generation, transmission and distribution. – It gives the three-phase voltage of equal magnitude and frequency. – Three-phase voltage (and current) offset by 120°.

14.1.2 Two connection methods

Two connection methods

Both the three-phase source and the three-phase load can be connected either wye (Y) or delta (Δ) in a three-phase AC system (Figure 14.1).

The wye (Y) or star configuration

The starting (the generator side) or finishing (the load side) points of three phases are connected together at a single neutral point (it can be earthed) (Figure 14.2).

- There is a neutral point in the wye-connected system.
- Wye connection is used where it requires neutral terminal to obtain phase voltage.
- The wye-connected system is general and typical used in power transmission system.

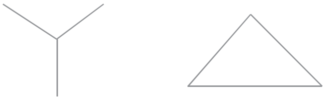
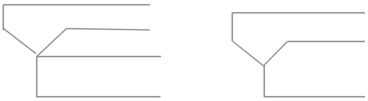


Figure 14.1 Y and Δ connections



(a) 3-phase 4-line (b) 3-phase 3-line

Figure 14.2 Y configuration

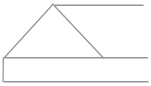


Figure 14.3 Δ configuration

- Three-phase four-wire or three-wire systems can be derived from the wye connection.
- In wye configuration, the phase voltage is low as $1/\sqrt{3}$ of the line voltage, so, it needs less number of turns, hence, saving in conductor material.

The delta (Δ) configuration is when three phases in an AC power system are connected like a triangle (Figure 14.3). (Three wires are taken out from the coil joints.)

- There is no neutral point in the delta-connected system.
- Delta connection is general and typically used in the distribution system.
- Three-phase three-wire system is derived from the delta connection.
- In delta configuration, the phase voltage is equal to the line voltage, hence, it needs more number of turns (compare with the wye connection).

Balanced three-phase circuit

- All three sources are balanced (three voltages of the same amplitude, frequency but apart by 120°).
- Source and load impedances are equal in all three phases. $Z = Z_A = Z_B = Z_C$

14.2 Analysis of the three-phase sources

14.2.1 *Wye-connected voltage sources*

Phase voltages and currents

- Phase voltage ($\boldsymbol{V}_p$) is the voltage measured between any phase and neutral (across a single component) in a three-phase circuit. Phase voltage: line-to-neutral
- Phase current ($\boldsymbol{I}_p$) is the current through any one component in a three-phase circuit.

Line voltages and currents

- Line voltage ($\boldsymbol{V}_L$) is the voltage measured between any two lines (line-to-line voltage) in a three-phase circuit (Figure 14.4). Line voltage: line-to-line
- Line current (I_L) is the current through any one line in a three-phase circuit.

Relationship between phase current and line current

- Line current ($\boldsymbol{I}_L$) is equal to phase current ($\boldsymbol{I}_p$) in a balanced wye circuit.
- Line current = phase current $\boxed{I_L = I_p}$

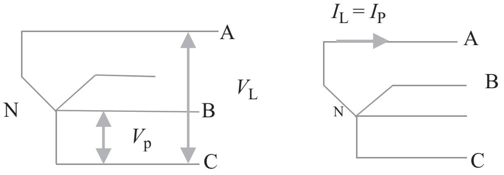


Figure 14.4 Voltages and currents in Y configuration

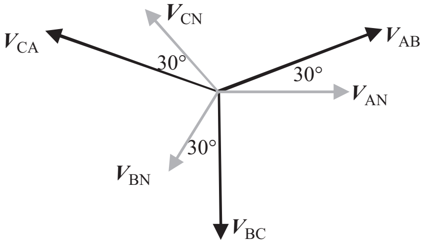


Figure 14.5 Phasor diagram of voltages in Y configuration

Relationship between phase voltage and line voltage

- Line voltage (V_L) is equal to phase voltage (V_p) times the square root of 3 in a balanced wye circuit.

  Line voltage $= \sqrt{3}$ phase voltage $\boxed{V_L = \sqrt{3}V_p}$
- Line voltage leads phase voltage by 30° (Figure 14.5). $\boxed{V_L = \sqrt{3}V_p\angle 30°}$

The polar equations for the phase voltages

$V_{AN} = V_{AN}\angle 0°$
$V_{CN} = V_{CN}\angle 120°$
$V_{BN} = V_{BN}\angle -120°$

Lines A, B, C, and neutral N.

Phasor diagram of voltages (Figure 14.5)

Example 14.1: Given the phase voltage $V_{AN} = 220\angle 0°$ V in a balanced three-phase wye source. Determine each phase and line voltage.

Solution:

- Phase voltages: $V_{AN} = \boxed{220\angle 0°\ \text{V}}$

$V_{CN} = \boxed{220\angle 120°\ \text{V}}$

$V_{BN} = \boxed{220\angle -120°\ \text{V}}$

- Line voltages: $$\begin{aligned} \boldsymbol{V}_{\mathrm{AB}} &= \sqrt{3}\boldsymbol{V}_{\mathrm{p}} = \sqrt{3}\boldsymbol{V}_{\mathrm{AN}} \\ &= \sqrt{3}(220\angle 30^\circ\ \mathrm{V}) \approx \boxed{381\angle 30^\circ\ \mathrm{V}} \qquad V_{\mathrm{L}} = \sqrt{3}V_{\mathrm{p}}\angle 30^\circ \\ \boldsymbol{V}_{\mathrm{CA}} &= \sqrt{3}\boldsymbol{V}_{\mathrm{p}} = \sqrt{3}\boldsymbol{V}_{\mathrm{CN}} \\ &= \sqrt{3}(220\angle(30^\circ + 120^\circ))\mathrm{V} \\ &\approx \boxed{381\angle 150^\circ\ \mathrm{V}} \\ \boldsymbol{V}_{\mathrm{BC}} &= \sqrt{3}\boldsymbol{V}_{\mathrm{p}} = \sqrt{3}\ V_{\mathrm{BN}} \\ &= \sqrt{3}(220\angle(30^\circ - 120^\circ))\mathrm{V} \\ &\approx \boxed{381\angle -90^\circ\ \mathrm{V}} \end{aligned}$$

Y-connected source

Quantity	Formula
Voltages	$V_{\mathrm{L}} = \sqrt{3}V_{\mathrm{p}}$, $\boldsymbol{V}_{\mathrm{L}} = \sqrt{3}V_{\mathrm{p}}\angle 30^\circ$
Currents	$\boldsymbol{I}_{\mathrm{L}} = \boldsymbol{I}_{\mathrm{P}}$

14.2.2 Delta-connected sources

Relationship between phase voltage and line voltage

- Line voltage ($\boldsymbol{V}_{\mathrm{L}}$) is equal to phase voltage ($\boldsymbol{V}_{\mathrm{p}}$) in a balanced delta circuit (Figure 14.6).
- Line voltage = phase voltage $\boxed{\boldsymbol{V}_{\mathrm{L}} = \boldsymbol{V}_{\mathrm{p}}}$

Relationship between phase current and line current

- Line current ($\boldsymbol{I}_{\mathrm{L}}$) is equal to phase current ($\boldsymbol{I}_{\mathrm{p}}$) times the square root of 3 in a balanced delta circuit. Line current = $\sqrt{3}$ phase current $\boxed{I_{\mathrm{L}} = \sqrt{3}I_{\mathrm{p}}}$
- Line current leads phase current by 30°. $\boxed{\boldsymbol{I}_{\mathrm{L}} = \sqrt{3}\boldsymbol{I}_{\mathrm{p}}\angle 30^\circ}$

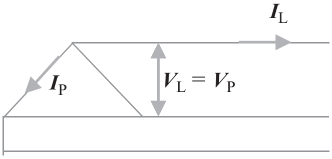

$\boldsymbol{I}_{\mathrm{L}}$
$\boldsymbol{I}_{\mathrm{P}}$
$V_{\mathrm{L}} = V_{\mathrm{P}}$

Figure 14.6 Voltages and currents in Δ configuration

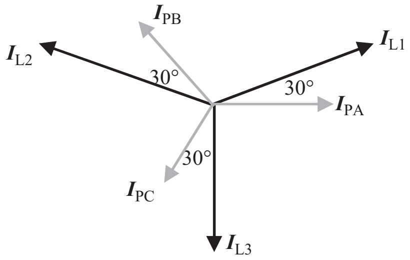


Figure 14.7 Phasor diagram of currents in Δ configuration

The polar equations for the line voltages

- $V_{AB} = V_P\angle 0°$
- $V_{CA} = V_P\angle 120°$
- $V_{BC} = V_P\angle -120°$

The polar equations for the phase currents

- $I_{PA} = I_P\angle 0°$
- $I_{PC} = I_P\angle 120°$
- $I_{PB} = I_P\angle -120°$

Phasor diagram of currents (Figure 14.7)

Example 14.2: Given the phase current $I_{PA} = 13\angle 0°$ A in a balanced delta circuit. Determine each phase and line current.

Solution:

- Phase currents: $I_{PA} = \boxed{13\angle 0°\ \text{A}}$

$$I_{PC} = \boxed{13\angle 120°\ \text{A}}$$

$$I_{PB} = \boxed{13\angle -120°\ \text{A}}$$

- Line currents: $I_{L1} = \sqrt{3}I_p = \sqrt{3}I_{PA}$

$$= \sqrt{3}(13\angle 30°\ \text{A}) \approx \boxed{22.5\angle 30°\ \text{A}} \qquad 0° + 30°$$

$$I_{L2} = \sqrt{3}I_p = \sqrt{3}I_{PC}$$

$$= \sqrt{3}(13\angle 150°\ \text{A}) \approx \boxed{22.5\angle 150°\ \text{A}} \qquad 30° + 120°$$

$$I_{L3} = \sqrt{3}I_p = \sqrt{3}I_{PB}$$

$$= \sqrt{3}(13\angle -90°\text{A}) = \boxed{22.5\angle -90°\ \text{A}} \qquad 30° - 120°$$

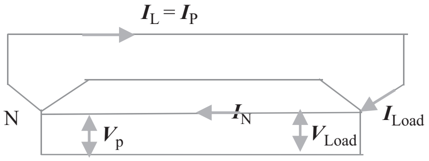


Figure 14.8 Y–Y connection

Δ-connected source

Quantity	Formula
Voltages	$V_L = V_P$
Currents	$I_L = \sqrt{3} I_P$, $\quad I_L = \sqrt{3} I_P \angle 30°$

14.3 Analysis of the Y–Y and Y–Δ systems

14.3.1 Y–Y system

Four configurations

Source-load can be connected in four possible configurations.

- Y–Y, Y–Δ, Δ–Y, Δ–Δ.
- The source and load can be either Y- or Δ-connected.

General Y–Y connection (Figure 14.8)

Voltages and currents

(They are true for both a balanced and an unbalanced load.)

- Voltages: Load voltage = phase voltage $\boxed{V_{Load} = V_P}$ $\quad V_L = \sqrt{3} V_P$
- Currents: Load Current = phase current = line current
  $\boxed{I_{Load} = I_P = I_L}$ $\quad I_L = I_P$
- Neutral current: $\boxed{I_N = I_{Load\ A} + I_{Load\ B} + I_{Load\ C}}$ $\quad I_N = 0$ in a balanced system.

Example 14.3: Given $V_{AN} = 120\angle 0°$ V, $R_A = R_B = R_C = 30\ \Omega$, and $X_A = X_B = X_C = 40\ \Omega$ in a Y–Y, three-phase four-wire system. Determine each load and phase voltage, and the line and load currents.

Solution:

- Load and phase voltages:

$$\boldsymbol{V}_{\mathrm{AN}} = \boldsymbol{V}_{\mathrm{Load\ A}} = \boxed{120\angle 0^\circ\ \mathrm{V}} \qquad \boldsymbol{V}_{\mathrm{Load}} = \boldsymbol{V}_{\mathrm{P}}$$

$$\boldsymbol{V}_{\mathrm{BN}} = \boldsymbol{V}_{\mathrm{Load\ B}} = \boxed{120\angle 120^\circ\ \mathrm{V}}$$

$$\boldsymbol{V}_{\mathrm{CN}} = \boldsymbol{V}_{\mathrm{Load\ C}} = \boxed{120\angle -120^\circ\ \mathrm{V}}$$

- Impedance:

$$Z_{\mathrm{A}} = Z_{\mathrm{B}} = Z_{\mathrm{C}} = R + jX = 30\ \Omega + j\,40\ \Omega = \boxed{50\angle 53.13^\circ\ \Omega}$$

- Line and load currents:

$$\boldsymbol{I}_{\mathrm{AB}} = \boldsymbol{I}_{\mathrm{Load\ A}} = \frac{\boldsymbol{V}_{\mathrm{Load\ A}}}{Z_{\mathrm{A}}} = \frac{120\angle 0^\circ}{50\angle 53.13^\circ\ \Omega} = \boxed{2.4\angle -53.13^\circ\ \mathrm{A}} \qquad \boldsymbol{I}_{\mathrm{L}} = \boldsymbol{I}_{\mathrm{p}}$$

$$\boldsymbol{I}_{\mathrm{CA}} = \boldsymbol{I}_{\mathrm{Load\ C}} = \frac{\boldsymbol{V}_{\mathrm{Load\ C}}}{Z_{\mathrm{C}}} = \frac{120\angle 120^\circ\ \mathrm{V}}{50\angle 53.13^\circ\ \Omega} = \boxed{2.4\angle 66.87^\circ\ \mathrm{A}}$$

$$\boldsymbol{I}_{\mathrm{BC}} = \boldsymbol{I}_{\mathrm{Load\ B}} = \frac{\boldsymbol{V}_{\mathrm{Load\ B}}}{Z_{\mathrm{B}}} = \frac{120\angle -120^\circ\ \mathrm{V}}{50\angle 53.13^\circ\ \Omega} = \boxed{2.4\angle -173.13^\circ\ \mathrm{A}}$$

Y–Y system

Quantity	**Formula**
Voltages	$\boldsymbol{V}_{\mathrm{Load}} = \boldsymbol{V}_{\mathrm{P}}$
Currents	$\boldsymbol{I}_{\mathrm{L}} = \boldsymbol{I}_{\mathrm{P}} = \boldsymbol{I}_{\mathrm{Load}}$, $\boldsymbol{I}_{\mathrm{N}} = \boldsymbol{I}_{\mathrm{Load\ A}} + \boldsymbol{I}_{\mathrm{Load\ B}} + \boldsymbol{I}_{\mathrm{Load\ C}}$

14.3.2 Y–Δ system

General Y–Δ connection (Figure 14.9)

Voltages and currents

- Voltages: Load voltage = line voltage

$$\boxed{\boldsymbol{V}_{\mathrm{Load}} = \boldsymbol{V}_{\mathrm{L}}}$$

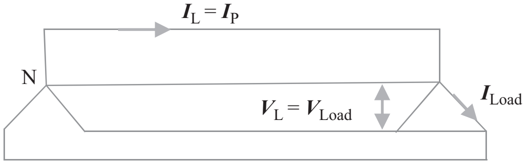


Figure 14.9 Y–Δ connection

- Currents: Line current = $\sqrt{3}$ times phase current = $\sqrt{3}$ times load current

  $\boxed{\boldsymbol{I}_{\mathrm{L}} = \sqrt{3}\boldsymbol{I}_{\mathrm{P}} = \sqrt{3}\boldsymbol{I}_{\mathrm{Load}}}$

  Load current = Load voltage divided by load impedance

  $\boxed{\boldsymbol{I}_{\mathrm{Load}} = \frac{V_{\mathrm{Load}}}{Z}}$

Example 14.4: Given $\boldsymbol{V}_{\mathrm{AB}} = 110\angle 0°$ V (line voltage), $Z = 55\angle 45°\ \Omega$ in a balanced Y–Δ system. Determine each load voltage and current.

Solution:

1. Load voltages:

$$\boldsymbol{V}_{\mathrm{Load\ A}} = \boldsymbol{V}_{\mathrm{AB}} = \boxed{110\angle 0°\ \mathrm{V}} \qquad \boldsymbol{V}_{\mathrm{Load}} = \boldsymbol{V}_{\mathrm{L}}$$

$$\boldsymbol{V}_{\mathrm{Load\ C}} = \boldsymbol{V}_{\mathrm{CA}} = \boxed{110\ \mathrm{V}\ 120°\ \mathrm{V}}$$

$$\boldsymbol{V}_{\mathrm{Load\ B}} = \boldsymbol{V}_{\mathrm{BC}} = \boxed{110\ \mathrm{V}\ \angle -120°\ \mathrm{V}}$$

2. Load currents:

$$\boldsymbol{I}_{\mathrm{Load\ A}} = \frac{\boldsymbol{V}_{\mathrm{Load\ A}}}{Z} = \frac{110\angle 0°\ \mathrm{V}}{55\angle 45°\ \Omega} = \boxed{2\angle -45°\ \mathrm{A}} \qquad \boldsymbol{I}_{\mathrm{Load}} = \frac{\boldsymbol{V}_{\mathrm{Load}}}{Z}$$

$$\boldsymbol{I}_{\mathrm{Load\ C}} = \frac{\boldsymbol{V}_{\mathrm{Load\ C}}}{Z} = \frac{110\angle 120°\ \mathrm{V}}{55\angle 45°\ \Omega} = \boxed{2\angle 75°\ \mathrm{A}}$$

$$\boldsymbol{I}_{\mathrm{Load\ B}} = \frac{\boldsymbol{V}_{\mathrm{Load\ B}}}{Z} = \frac{120\angle -120°\ \mathrm{V}}{55\angle 45°\ \Omega} = \boxed{2\angle -165°\ \mathrm{A}}$$

Y–Δ system:

Quantity	**Formula**
Voltages	$\boldsymbol{V}_{\mathrm{Load}} = \boldsymbol{V}_{\mathrm{L}}$
Currents	$\boldsymbol{I}_{\mathrm{L}} = \sqrt{3}\boldsymbol{I}_{\mathrm{P}} = \sqrt{3}\boldsymbol{I}_{\mathrm{Load}}, \qquad \boldsymbol{I}_{\mathrm{Load}} = \frac{V_{\mathrm{Load}}}{Z}$

14.4 Power in balanced three-phase systems

14.4.1 Power in balanced Y- or Δ-connected systems

Recall: Power in single-phase AC circuits (Figure 14.10)

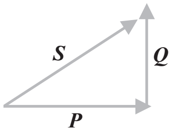


Figure 14.10 Power in AC circuits

Symbol	**Quantity**	**Formula**	**Unit**
P	Real power	$P = VI\cos\theta = \sqrt{S^2 - Q^2}$	Watt (W)
Q	Reactive power	$Q = VI\sin\theta = \sqrt{S^2 - P^2}$	Volt-ampere-reactive (VAR)
S	Apparent power	$S = VI = \sqrt{P^2 + Q^2}$	Volt-ampere (VA)

Power in balanced wye and delta circuits

Real power

- The real power per phase: $P_P = V_P I_P \cos\theta$
- The total three-phase real power: $P_T = 3P_P = 3V_P I_P \cos\theta$

  or $P_T = \sqrt{3}V_L I_L \cos\theta$

  - In a balanced Y-connected load: $V_L = \sqrt{3}V_P,\ I_L = I_P,$

    $$P_T = 3V_P I_P \cos\theta = 3\frac{v_L}{\sqrt{3}}I_L \cos\theta = \sqrt{3}V_L I_L \cos\theta$$

  - In a balanced Δ-connected load: $V_L = V_P,\ I_L = \sqrt{3}I_P,$

    $$P_T = 3V_P I_P \cos\theta = 3V_L\frac{I_L}{\sqrt{3}}\cos\theta = \sqrt{3}V_L I_L \cos\theta$$

Apparent power

- The apparent power per phase: $S_P = V_P I_P$
- The total three-phase apparent power: $S_T = 3V_P I_P = \sqrt{3}V_L I_L$

Reactive power

- The reactive power per phase: $Q_P = V_P I_P \sin\theta = \sqrt{S_p^2 - P_p^2}$
- The total three-phase reactive power: $Q_T = 3Q_P = \sqrt{3}V_L I_L \sin\theta = \sqrt{S_T^2 - P_T^2}$

Power factor: $\text{PF} = \cos\theta = \dfrac{P}{S}$

Power in balanced wye and delta circuits

Symbol	**Formula**	**Unit**
P_P	$P_P = V_P I_P \cos\theta$	Watt (W)
P_T	$P_T = 3P_P = 3V_P I_P \cos\theta = \sqrt{3} V_L I_L \cos\theta$	Watt (W)
Q_P	$Q_P = V_P I_P \sin\theta = \sqrt{{S_P}^2 - {P_T}^2}$	Volt-ampere-reactive (VAR)
Q_T	$Q_T = 3Q_P = \sqrt{3}\, V_L I_L \sin\theta = \sqrt{{S_T}^2 - {P_T}^2}$	Volt-ampere-reactive (VAR)
S_P	$S_P = V_P I_P$	Volt-ampere (VA)
S_T	$S_T = 3V_P I_P = \sqrt{3} V_L I_L$	Volt-ampere (VA)
PF	$\text{PF} = \cos\theta = \frac{P}{S}$	

14.4.2 Three-phase power examples

Example 14.5: The phase voltage is 110 V, the phase current is 15 A, and the total real power is 4.5 kW in a balanced wye-connected load circuit. Determine the line voltage and current, apparent powers, power factor, real power per phase, and reactive powers.

Solution: $\boldsymbol{V}_P = 110\text{ V},\ \boldsymbol{I}_P = 15\text{ A},\ P_T = 4.5\text{ kW}$

- Line voltage: $\boldsymbol{V}_L = \sqrt{3}\boldsymbol{V}_P = \sqrt{3}\,110\text{ V} \approx \boxed{190.5\text{ V}}$
- Line current: $\boldsymbol{I}_L = \boldsymbol{I}_P = \boxed{15\text{ A}}$
- Apparent power per phase: $S_P = V_P I_P = (110\text{ V})(15\text{ A}) = \boxed{1{,}650\text{ VA}}$
- Total apparent power: $S_T = 3S_P = 3(1{,}650\text{ VA}) = \boxed{4{,}950\text{ VA}}$
- Power factor: $\text{PF} = \cos\theta = \frac{P}{S} = \frac{4{,}500\text{ W}}{4{,}950\text{ VA}} \approx 0.91 = \boxed{91\%}$
- Real power per phase: $P_P = V_P I_P \cos\theta = (110\text{ V})(15\text{ A})(0.91) = \boxed{1{,}501.5\text{ W}}$
- Reactive power per phase: $Q_P = \sqrt{{S_P}^2 - {P_P}^2} = \sqrt{1{,}650^2 - 1{,}501.5^2} = \boxed{684.1\text{ VAR}}$
- Total three-phase reactive power: $Q_T = 3Q_P = 3(684.1\text{ VAR}) = \boxed{2{,}052.3\text{ VAR}}$

Example 14.6: The phase voltage is 110 V, the phase current is 15 A, and the total real power is 4.5 kW in a balanced delta-connected load circuit. Determine the line voltage and current, apparent powers, power factor, real power per phase, and reactive powers.

Solution: $V_P = 110\ \text{V},\ I_P = 15\ \text{A},\ P_T = 4.5\ \text{kW}$

- Line voltage: $V_L = V_P = \boxed{110\ \text{V}}$
- Line current: $I_L = \sqrt{3}I_P = \sqrt{3}(15\ \text{A}) \approx \boxed{25.98\ \text{A}}$
- Apparent power per phase: $S_P = V_P I_P = (110\ \text{V})(15\ \text{A}) = \boxed{1{,}650\ \text{VA}}$
- Total apparent power: $S_T = 3S_P = 3(1{,}650\ \text{VA}) = \boxed{4{,}950\ \text{VA}}$
- Power factor: $\text{PF} = \cos\theta = \frac{P}{S} = \dfrac{4{,}500\ \text{W}}{4{,}950\ \text{VA}} \approx 0.91 = \boxed{91\%}$
- Real power per phase: $P_P = V_P I_P \cos\theta = (110\ \text{V})(15\ \text{A})(0.91) = \boxed{1{,}501.5\ \text{W}}$
- Reactive power per phase: $Q_P = \sqrt{S_P{}^2 - P_P{}^2} = \sqrt{1{,}650^2 - 1{,}501.5^2} = \boxed{684.1\ \text{VAR}}$
- Total three-phase reactive power: $Q_T = 3Q_P = 3(684.1\ \text{VAR}) = \boxed{2{,}052.3\ \text{VAR}}$

Note that the results here are the same as for Example 14.5.

Summary

Three-phase power systems

- A common method of AC power generation, transmission, and distribution.
- It gives the three-phase voltage of equal magnitude and frequency.
- Three-phase voltage (and current) offset by 120°.

Phase voltages and currents

- Phase voltage (V_p) is the voltage measured between phase and neutral in a three-phase circuit. Phase voltage: line-to-neutral
- Phase current (I_p) is the current through any one component in a three-phase circuit.

Balanced three-phase circuit

- All three sources are balanced (three voltages of the same amplitude, frequency but apart by 120°).
- Source and load impedances are equal in all three phases. $Z = Z_A = Z_B = Z_C$

Line voltages and currents

- Line voltage ($\boldsymbol{V}_{\mathrm{L}}$) is the voltage measured between any two lines (line-to-line voltage) in a three-phase circuit.

Line voltage: line-to-line

- Line current (I_{L}) is the current through any single line in a three-phase circuit.

The wye (Y) or star configuration

- The starting or finishing points of three phases are connected together at a single neutral point.
- Y-connected source:

Quantity	**Formula**
Voltages	$V_{\mathrm{L}} = \sqrt{3}\boldsymbol{V}_{\mathrm{P}}, \quad \boldsymbol{V}_{\mathrm{L}} = \sqrt{3}V_{\mathrm{p}}\angle 30^\circ$
Currents	$\boldsymbol{I}_{\mathrm{L}} = \boldsymbol{I}_{\mathbf{P}}$

The delta (Δ) configuration

- It is when three phases in an AC power system are connected like a triangle. (Three wires are taken out from the coil joints.)
- **Δ-connected source**

Quantity	**Formula**
Voltages	$\boldsymbol{V}_{\mathrm{L}} = \boldsymbol{V}_{\mathrm{P}}$
Currents	$I_{\mathrm{L}} = \sqrt{3}\boldsymbol{I}_{\mathrm{P}}, \quad \boldsymbol{I}_{\mathrm{L}} = \sqrt{3}\boldsymbol{I}_{\mathrm{P}}\angle 30^\circ$

Y–Y system

Quantity	**Formula**
Voltages	$\boldsymbol{V}_{\mathrm{Load}} = \boldsymbol{V}_{\mathrm{P}}$
Currents	$\boldsymbol{I}_{\mathrm{L}} = \boldsymbol{I}_{\mathrm{P}} = \boldsymbol{I}_{\mathrm{Load}}, \boldsymbol{I}_{\mathrm{N}} = \boldsymbol{I}_{\mathrm{Load\ A}} + \boldsymbol{I}_{\mathrm{Load\ B}} + \boldsymbol{I}_{\mathrm{Load\ C}}$

Y–Δ system

Quantity	**Formula**
Voltages	$\boldsymbol{V}_{\mathrm{Load}} = \boldsymbol{V}_{\mathrm{L}}$
Currents	$\boldsymbol{I}_{\mathrm{L}} = \sqrt{3}\boldsymbol{I}_{\mathrm{P}} = \sqrt{3}\boldsymbol{I}_{\mathrm{Load}}, \boldsymbol{I}_{\mathrm{Load}} = \dfrac{\boldsymbol{V}_{\mathrm{Load}}}{Z}$

Power in balanced wye and delta circuits

Symbol	Quantity	Formula	Unit
P_P	Real power per phase	$P_P = V_P I_P \cos\theta$	Watt (W)
P_T	Total three-phase real power	$P_T = 3P_P = 3V_P I_P \cos\theta$	Watt (W)
Q_P	Reactive power per phase	$Q_P = V_P I_P \sin\theta = \sqrt{S_p^2 - P_p^2}$	Volt-ampere-reactive (VAR)
Q_T	Total three-phase reactive power	$Q_T = 3Q_p = \sqrt{3} V_L I_L \sin\theta = \sqrt{S_T^2 - P_T^2}$	Volt-ampere-reactive (VAR)
S_P	Apparent power per phase	$S_P = V_P I_P$	Volt-ampere (VA)
S_T	Total three-phase apparent power	$S_T = 3S_p = 3V_P I_P = \sqrt{3} V_L I_L$	Volt-ampere (VA)
PF	Power factor	$\text{PF} = \cos\theta = \frac{p}{s}$	

Practice problems

14.1

1. The three windings in a magnetic field are placed at (   ) apart in a three-phase power system.
2. A three-phase circuit is more economical than three single-phase circuit because it uses less conductor material to transmit and (   ) the same amount of power.
3. Three-phase four-wire or three-wire systems can be derived from the (   ) connection.
4. The delta configuration is when three phases in an AC power system are connected like a (   ).

14.2

5. Phase voltage is the voltage measured between phase and (   ) in a three-phase circuit.
6. Phase current is the current through any one (   ) in a three-phase circuit.
7. Line voltage is the voltage measured between any two (   ) in a three-phase circuit.
8. Line current is equal to phase current in a balanced (   ) circuit.
9. Line current is equal to phase current times the square root of 3 in a balanced (   ) circuit.
10. Line voltage is equal to phase voltage in a balanced (   ) circuit.
11. Given $V_{AN} = 110\angle 0^\circ$ V in a balanced three-phase wye source. Determine each phase and line voltage.

12. Given $\boldsymbol{I}_{\mathrm{AN}} = 6\angle 0^\circ$ A in a balanced delta circuit. Determine each phase and line voltage.

14.3

13. Given $\boldsymbol{V}_{\mathrm{AN}} = 110\angle 0^\circ$ V, $R_{\mathrm{A}} = R_{\mathrm{B}} = R_{\mathrm{C}} = 25\ \Omega$ and $X_{\mathrm{A}} = X_{\mathrm{B}} = X_{\mathrm{C}} = 30\ \Omega$ in a Y–Y, three-phase four-wire system. Determine each load and phase voltage, and line and load current.
14. Given $\boldsymbol{V}_{\mathrm{AB}} = 120\angle 0^\circ$ V, $Z = 54\angle 35^\circ\ \Omega$ in a balanced Y–Δ system. Determine each load voltage and current.
15. Given the phase voltage is 130 V, the phase current is 12 A, the total real power $P_{\mathrm{T}} = 4$ kW in a balanced wye-connected load circuit. Determine the line voltage and current, apparent powers, power factor, real power per phase, and reactive powers.
16. Given the phase voltage is 130 V, the phase current is 12 A, the total real power $P_{\mathrm{T}} = 4$ kW in a balanced delta-connected load circuit. Determine the line voltage and current, apparent powers, power factor, real power per phase, and reactive powers.

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Appendix A

Greek alphabets

Uppercase/lowercase	**Letter**	**Uppercase/lowercase**	**Letter**
Α α	Alpha	Ν ν	Nu
Β β	Beta	Ξ ξ	Xi
Γ γ	Gamma	Ο o	Omicron
Δ δ	Delta	Π π	Pi
Ε ε	Epsilon	Ρ ρ	Rho
Ζ ζ	Zeta	Σ σ or ς	Sigma
Η η	Eta	Τ τ	Tau
Θ θ	Theta	Υ υ	Upsilon
Ι ι	Iota	Φ ϕ	Phi
Κ κ	Kappa	Χ χ	Chi
Λ λ	Lambda	Ψ ψ	Psi
Μ μ	Mu	Ω ω	Omega

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Appendix B

Differentiation of the phasor

- For a sinusoidal function $f(t) = F_{\mathrm{m}} \sin(\omega t + \psi)$, taking the derivative of the $f(t)$ with respect to t gives:

  $$\frac{\mathrm{d}f(t)}{\mathrm{d}t} = F_{\mathrm{m}}\, \omega \cos(\omega t + \psi)$$

- and

  $$\begin{aligned}\frac{\mathrm{d}f(t)}{\mathrm{d}t} &= F_{\mathrm{m}}\, \omega \sin(\omega t + \psi + 90^\circ) \\ &= J_{\mathrm{m}}\left[\omega F_{\mathrm{m}} \mathrm{e}^{\mathrm{j}(\omega t + \psi + 90^\circ)}\right] = J_{\mathrm{m}}\left(\omega F_{\mathrm{m}} \mathrm{e}^{\mathrm{j}\omega t} \mathrm{e}^{\mathrm{j}\psi} \mathrm{e}^{\mathrm{j}90^\circ}\right) = J_{\mathrm{m}}\left(j\omega \boldsymbol{F} \mathrm{e}^{\mathrm{j}\omega t}\right)\end{aligned}$$

- where $\boldsymbol{F} = F_{\mathrm{m}} \mathrm{e}^{\mathrm{j}\psi}$,
- and $\mathrm{e}^{\mathrm{j}90^\circ} = j$ (from Euler's formula, $\mathrm{e}^{j90^\circ} = \cos 90^\circ + j \sin 90^\circ = j$).
- Therefore, the phasor of $\frac{\mathrm{d}f(t)}{\mathrm{d}t}$ is $j\omega \boldsymbol{F}$ (where $\mathrm{e}^{j\omega t}$ is the rotating factor), i.e. $\frac{\mathrm{d}f(t)}{\mathrm{d}t} \Leftrightarrow j\omega \boldsymbol{F}$
- Therefore, the derivative of the sinusoidal function with respect to time can be obtained by its phasor $\boldsymbol{F}$ multiplying with $j\omega$, this is equivalent to a phasor that rotates counterclockwise by 90° on the complex plane (since $+j = +90^\circ$).

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Answers

Answers: Practice problems

Chapter outline

Answers to selected odd-numbered problems 419

Answers to selected odd-numbered problems

Chapter R

1.
   - (a) 0.439 m
   - (b) 223.6 A
   - (c) 0.0000483 kV
   - (d) 25 hW
   - (e) 890 μV
   - (f) 0.167 kW
   - (g) 30 μA

3.
   - (a) 4,000 mA
   - (b) 63,006 V
   - (c) 5,290 mA
   - (d) 28.87 kΩ

5.
   - (a) 3,578
   - (b) 0.000043

7.
   - (a) 36.7×10^6
   - (b) 4.56×10^{-6}

9.
   - (a) 17.447×10^9
   - (b) 2.2857×10^{-6}

Chapter 1

5. Coulomb (C)
7. Joule (J)
9. Flux; 1 Weber = 10^8 Maxwell
11. Source voltage, wire, and load
13. See Table 1.1
15. Ammeter, Ⓐ
17. 2 amperes (A)
19. positive, negative
21. potential difference, V
23. load
25. ohmmeter $\textcircled{\Omega}$
27. *T*
29. $I = 0.2$ A
33. the same

Chapter 2

1. Work
3. Power
5. $P = 9$ W
7. $I \approx 0.0316$ A
9.
    (a) $P = -\ 0.5$ W (Releasing energy)
    (b) $P = 2.4$ W (Absorption energy)
11. $V_{\mathrm{ab}} = 4\,\mathrm{V}$
13. $P_2 = 0.16\,\mathrm{W}$
15. $I_1 = 5$ A, $I_2 = 2$ A, $I_3 = 1$ A
17. $I_1 = 4$ A, $I_2 = 3$ A, $I_3 = 6$ A, $I_4 = 7$ A
19.
    (a) $V_{\mathrm{AB}} = -RI - E$; (b) $V_{\mathrm{AB}} = RI - E$
21.
    (a) $R_{\mathrm{S}} = 108.8\,\Omega$; (b) $V_{\mathrm{ab}} = 9.2\,\mathrm{V}$

Chapter 3

3.
    (a) $I \approx 0.59\,\mathrm{A}$; (b) $V_{R_2} = 5.9\,\mathrm{V}$; (c) $P = 5.9\,\mathrm{W}$
5. $V_{R_1} \approx 3.81\,\mathrm{V}$, $V_{R_2} \approx 1.9\mathrm{V}$, $V_{R_3} \approx 2.86\,\mathrm{V}$
    $V_{R_4} \approx 4.76\,\mathrm{V}$, $V_{R_5} \approx 6.67\,\mathrm{V}$
7. $R_1 = 3\,\mathrm{k}\Omega$, $R_2 = 9\,\mathrm{k}\Omega$

$$(R_1 = \frac{V_{R_1}?}{E} R_T?, \qquad R_T = \frac{E}{I}, \qquad V_{R_1} = E - V_{R_2}, \qquad V_{R_1} = \frac{1}{3} V_{R_2})$$

11. $I_T \approx 17\,\text{mA}$, $I_1 = 2\,\text{mA}$, $I_3 = 5\,\text{mA}$, $P_T = 0.17\,\text{W}$
13.
    (a) $R_2 = 5\,\Omega$; (b) $R_2 = 150\,\text{k}\,\Omega$

17.
    (a) $R_{\text{eq}} = R_1 + (R_3 // R_2) + (R_4 // R_5) + R_6$
    (b) $R_{\text{eq}} = R_1 + R_2 // R_3 // (R_5 + R_6 + R_7) + R_4$

19. $V_{R_4} \approx 2.66\,\text{V}$; $V_{R_5} \approx 5.145\,\text{V}$
21.
    (a) $R_a = 34\ \Omega$, $R_b = 20.4\ \Omega$, $R_c \approx 22.7\ \Omega$
    (b) $R_a \approx 6.33\ \text{k}\Omega$, $R_b = 19\ \text{k}\Omega$, $R_c = 4.75\ \text{k}\Omega$

23. $R_X \approx 26.7\,\text{k}\Omega$

Chapter 4

1. $I_S = 3\ \text{A}$, $R_S = 6\ \Omega$
3. $I_L \approx 0.455\ \text{mA}$, $I_L \approx 0.455\ \text{mA}$
5. $n = 4$, $b = 6$, Mesh $= 3$
7. $I_2 \approx 3.26\ \text{A}$
9. $I_1 \approx -5.11\ \text{mA}$, $I_2 \approx -3.52\ \text{mA}$, $V_{R_2} = -0.89\,\text{V}$
11. $I_a \approx 23.3\ \text{mA}$, $I_b \approx -170\ \text{mA}$
13. $V_a = -14.86\,\text{V}$

Chapter 5

1. $I_{R_1} \approx 1.715\ \text{mA}$, $I_{R_3} \approx 1.278\ \text{mA}$
3. $I \approx 3.06\ \text{A}$
5. $I \approx -0.447\ \text{mA}$
7.
    (a) $R_L = R_S = 47\ \Omega$ (b) $R_L = R_S = 8.2\ \Omega$

9. $I \approx 719\ \mu\text{A}$

Chapter 6

1. (b)
3. $Q = 150\ \mu\text{C}$
5. $C_{\text{eq}} \approx 5.88\ \mu\text{F}$, $Q = 147\ \mu\text{C}$
7. $C_2 \approx 1.29\,\mu\text{F}$
9. $C_{\text{eq}} = 3\ \mu\text{F}$
11. conductor; electromagnetic field
13. N, A, μ; l
15. $w_L = 1\ \text{J}$
17. $L_{\text{eq}} = 30.075\,\text{mH}$
19. $L_{\text{eq}} \approx 3.71\,\text{H}$

Chapter 7

1. RL; RC; differential
3.
   (a) after (b) before
5.
   (a) 0^-; inductor (b) 0^+; capacitor
7. $v_C = 50\left(1 - e^{\frac{-t}{10.5\times10^{-3}}}\right)$ V; $i_C \approx 14.29e^{\frac{-t}{10.5\times10^{-3}}}$ mA
9. $t = 1\tau = 15$ ms
11.
   (a) $\tau = 0.05$ s
   (b) $u_C = 10\,e^{-20t}$ V, $i = 2\,e^{-20t}$ mA
   (c) 3.68 V, 1.35 V, 0.5 V, 0.18 V, 0.067 V
13. $\tau = 1.33$ µs, $v_L = 20\,e^{\frac{-t}{1.33\ \mu s}}$ V
   $v_R = 20\left(1 - e^{\frac{-t}{1.33\ \mu s}}\right)$ V, $i_L \approx 4.44\,e^{\frac{-t}{1.33\ \mu s}}$ mA
15. $v_L = -10e^{\frac{-t}{0.32\ ms}}$ V, $i_L = 4\,e^{\frac{-t}{0.32\ ms}}$ mA

Chapter 8

1. Electromagnetic
3. inside
5. lines
7. Tesla (T)
9. Domain
11. 0.51 T
13. without
15. field
17. Electromagnetism
19. electric
21. reluctance
23. 0.255×10^{-2}
25. 137.1×10^{-7} T
27. intensity
29. curve

Chapter 9

1. direction
3. $I_m = 20$ A, $\psi = 45°$, $\omega = 30$ rad/s, $T \approx 0.21$ s , $f \approx 4.76$ Hz
5. out of phase, orthogonal
7. $V_{Avg} = 9.555$ V,
9. $V_{pk} = 20$ V, $V_{p\text{-}p} = 40$ V, $V_{Avg} = 12.74$ V, $V \approx 14.14$ V

11. $\dot{V} = 30\angle{-45^\circ}$ V, $\dot{I} = 15\angle 35^\circ$ A
13.
    (a) $v = 10\sin(\omega t - 45^\circ)$ V
    (b) $i = 10.08\sin(\omega t + 14.95^\circ)$ A
15. $17.6\angle 156.6^\circ$ A
17. $v_L = 2.4\sin(60t + 120^\circ)$ V

Chapter 10

1. $Z \approx 20.62\angle 14.04^\circ\ \Omega$
3. $Y \approx 0.32\angle 1 - 51.34^\circ$ mS
5. $Z_{eq} \approx 14.2\angle{-49.6}\ \Omega$, $\dot{I}_L = 3.52\angle 49.6^\circ$ A, $\dot{I}_C = 2.92\angle 83.3^\circ$ A
7. $P_T = 20$ W
   $Q_T = 16.7$ Var
   $S_T \approx 26.1$ VA
   $\cos\varphi \approx 0.77$
   $\dot{I}_T = 2.61\angle 39.7^\circ$ A
9. $\dot{I} \approx 1.61\angle 35.7^\circ$ A
11. $Z_{TH} \approx 18.9\angle{-12.8^\circ}\ \Omega$, $\dot{V}_{TH} \approx 16.1\angle 125.5^\circ$ V

Chapter 11

1. increases
3. minimum, maximum
5. $f_r \approx 1{,}592$ Hz, $Z_T = 10\ \Omega$, $\dot{I} = 1\angle 0^\circ$A
   $\dot{V}_L = 100\angle 90^\circ$ V, $Q = 10$
7. 0.707
9. current
11. same
13. inductor (coil)
15. $f_r \approx 104$ kHz, $Z \approx 53.4\ \mathrm{M}\Omega$

Chapter 12

1. Mutual
3. dot
5. $k = 0.75$
7. AC
9. $N_S = 60$ turns
11. $n = 0.5$
13. $n = 0.4$

Chapter 13

1\.

(a) $30\angle 0^\circ$ mV, $25\ \Omega$
(b) $0.2\angle 0^\circ$ A, $40\ \Omega$

3\.

$$\begin{aligned}
&\text{Mesh 1}: && (2+4+6)\dot{I}_1 - 4\dot{I}_2 - 6\dot{I}_3 = -10\,\text{V}\\
&\text{Mesh 2}: && -4\dot{I}_1 + (3+4+7)\dot{I}_2 - 7\dot{I}_3 = -5\dot{I}_0\\
&\text{Mesh 3}: && -6\dot{I}_1 - 7\dot{I}_2 + (6+7+5)\dot{I}_3 = 0\\
& && \qquad\qquad\qquad\qquad (\dot{I}_0 = \dot{I}_2 - \dot{I}_1)
\end{aligned}$$

5. 12.5 V

Chapter 14

1. 120°
3. wye
5. neutral
7. lines
9. delta
11. – Phase voltages: $\boldsymbol{V}_{\text{AN}} = 110\angle 0^\circ$ V
$\boldsymbol{V}_{\text{BN}} = 110\angle 120^\circ$ V
$\boldsymbol{V}_{\text{CN}} = 110\angle{-120^\circ}$ V
– Line voltages: $\boldsymbol{V}_{\text{AB}} = 110\sqrt{3}\mathbf{V}_{\text{P}} = 190.5\angle 30^\circ$ V
$\boldsymbol{V}_{\text{CA}} = 190.5\angle 150^\circ$ V
$\boldsymbol{V}_{\text{BC}} = 190.5\angle{-90^\circ}$ V

13\.

– Load and phase voltages: $\boldsymbol{V}_{\text{AN}} = \boldsymbol{V}_{\text{LoadA}} = 110\angle 0^\circ$
$\boldsymbol{V}_{\text{BN}} = \boldsymbol{V}_{\text{LoadB}} = 110\angle 120^\circ$ V
$\boldsymbol{V}_{\text{CN}} = \boldsymbol{V}_{\text{LoadC}} = 110\angle{-120^\circ}$ V
– Line and load currents: $\boldsymbol{I}_{\text{AB}} = \boldsymbol{I}_{\text{LoadA}} = 2.82\angle{-50.19^\circ}$ A
$\boldsymbol{I}_{\text{CA}} = \boldsymbol{I}_{\text{LoadB}} = 2.82\angle 69.81^\circ$ A
$\boldsymbol{I}_{\text{BC}} = \boldsymbol{I}_{\text{LoadC}} = 2.82\angle{-170.19^\circ}$ A

15\.

– Line voltage: $V_{\text{L}} \approx 225.2$ V
– Line current: $\boldsymbol{I}_{\text{L}} = \boldsymbol{I}_{\text{P}} = 12$ A
– Apparent power per phase: $S_{\text{P}} = 1{,}560$ VA
– Total apparent power: $S_{\text{T}} = 4{,}680$ VA
– Power factor: $\text{PF} = \cos\theta \approx 85.47\%$
– Real power per phase: $P_{\text{P}} \approx 1{,}333.3$ W
– Reactive power per phase: $Q_{\text{P}} \approx 809.88$ VAR
– Total three-phase reactive power: $Q_{\text{T}} = 3Q_{\text{P}} = 2{,}429.65$ VAR

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2nd Edition

In this digital age, as the role of electronic circuits becomes ever broader and more complex, a thorough understanding of the key concepts of circuits is a great advantage. This book offers a thorough reference guide to the theory, elements and design of basic electric circuits, providing a solid foundation for those who plan to move into the field of electronics engineering, and essential information for anyone who uses electric circuitry in their profession or research. The book is designed to be accessible to newcomers to the field while also providing a useful review for more advanced readers. It has been extensively revised and expanded for this new edition to provide a clear source of information on this complex topic. Materials are presented visually with less text and more outlines so that readers can quickly get to the heart of each topic, making studying and reviewing more effective.

About the Author

Meizhong Wang has been an instructor at the College of New Caledonia (CNC) for 29 years. She currently teaches computing and mathematical courses and has lectured in electric circuits, electronics, physics, etc. at CNC and other college and university in Canada and China. She is also the author of several books, including *Algebra I & II Key Concepts, Practice, and Quizzes*, *Math Made Easy*, *Key Concepts of Intermediate Level Math*, *Legends of Four Chinese Sages* and *Understandable Electric Circuits*.

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